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1) You are looking for the number that satisfies BOTH conditions. It has to be even and a multiple of 3. Choice 2, or the number 12, is both even and a multiple of 3. **ANSWER: (2)**

2) Angles 6 and 8 are congruent as they form a pair of alternate interior angles. **ANSWER: (1)**

3) This is simply an example requiring you to use the counting principle. If one thing can be done m number of ways, and the other can be done in n number of ways, then both can be done in mn number of ways. For this problem multiply: 5X2X3 or **30**. **ANSWER: (3)**

4) Add the frequencies for each interval to compute the total number of students in the class:
2 + 4 + 5 + 4 + 1 = **16** **ANSWER: (3)**

5) Of the given transformations, a **dilation** is the only one that does not preserve size. Using the other transformations would always result in two triangles that are the exact image of each other with equal perimeters. **ANSWER: (1)**

6) Given an odd integer, you would add 2 to arrive at the next larger odd integer. Therefore, given n + 4, the next odd integer would be **n + 6**. **ANSWER: (4)**

7) The problem asks for the solution set of the following equation:

$\frac{x}{5} + \frac{x}{2} = 14$ Make life easier by multiplying all terms by the common denominator of 10.

$(10)\frac{x}{5} + (10)\frac{x}{2} = (10)(14)$ Simplify

$2x + 5x = 140$ Combine like terms

$7x = 140$ Divide both sides by 7.

x = 20 **ANSWER: (3)**

8) The diagram shown in the problem is in essence a right triangle. You are given two of its legs and asked to find its hypotenuse. Use the Pythagorean theorem

$c^2 = a^2 + b^2$ The legs are given as being 30ft. and 75ft. Let the hypotenuse = x

$x^2 = (30)^2 + (75)^2$ Simplify

$x^2 = 900 + 5625$ Combine

$x^2 = 6525$ Find the square root of 6525

$x = 80.774$

81ft. to the nearest foot. **ANSWER: (2)**

9) A translation of (x-1,y-3) means that 1 is subtracted from the x coordinate, and 3 is subtracted from the y coordinate. You are given the point P whose coordinates are (3,-5). The x coordinate is now 3 and will become 3 – 1 or 2. The y coordinate is now -5 and will become -5 – 3 or -8. The image will therefore be **(2, -8).** **ANSWER: (4)**

10) The letters H, S, and X all have point symmetry -- they will look the same when turned upside down. However, the H and X also have line symmetry. The S has no line symmetry and therefore satisfies the given conditions. **ANSWER: (2)**

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11) Any expression raised to a negative exponent is equal to 1 divided by the same expression raised to that positive exponent. In other words, $x^{-a} = \frac{1}{x^a}$. In our case, therefore, $x^{-4} = \frac{1}{x^4}$

ANSWER: (1)

12) Given $x^3 < x < \frac{1}{x}$, only $\frac{1}{5}$ will work as a solution set for x. $\frac{1}{5}$ cubed is $\frac{1}{125}$. Therefore you would have $\frac{1}{125} < \frac{1}{5} < 5$ (1 divided by 1/5 will equal 5) **ANSWER: (4)**

13) The contrapositive of a statement will be logically equivalent to the statement. To arrive at the contrapositive of a statement you would find the inverse and converse (or converse and inverse) of the original statement. In our case, the statement is "If you are an elephant, then you do not forget." The inverse of that statement would be "If you are not an elephant then you do forget." The converse of this would be "If you forget, then you are not an elephant." **ANSWER: (4)**

14) The sum of the measures of the interior angles of a polygon will be equal to 180 times the number of sides minus 2. A pentagon has 5 sides. 5 – 2 = 3. 180 time 3 = 540. The sum, in degrees of the measures of the interior angles of a pentagon will equal 540. **ANSWER: (3)**

15) This problem requires you to figure out the value of ${}_7C_3$.
${}_7C_3 = {}_7P_3$ divided by 3!
You may use your calculator or simply compute (7)(6)(5) divided by (3)(2)(1)

ANSWER: (2)

16) The product of a number and its multiplicative inverse equals 1.
$\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses as their product is 1. **ANSWER: (2)**

17) The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b and h are the base and height.
This problem asks you to solve for h.

$A = \frac{1}{2}bh$ Multiply both sides by 2.

$2A = bh$ Divide both sides by b.

$\frac{2A}{b} = h$ **ANSWER: (4)**

18) $|-7|$ is asking for the absolute value of -7. The absolute value of -7 is 7.
$-|-7|$ the negative of the absolute value of –7 is therefore –7. **ANSWER: (4)**

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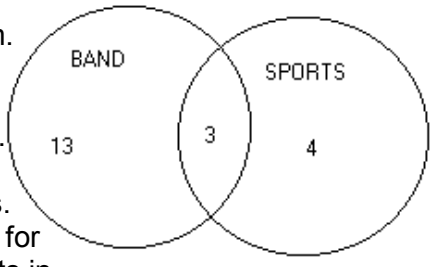
19) This problem is easily pictured and done using a Venn diagram. The diagram at the right pictures the given information. You can easily see that 3 students participated in both activities. 16 participated in band (13+3) and 7 (3+4) participated in sports. This accounts for a total of 20 students (13+3+4). You are also given that 9 students are not in band and also do not play sports. This means there are another 9 students that are not accounted for in the Venn diagram. 20 + 9 = 29. There are a total of 29 students in Mr. Wright's English class.



ANSWER: (3)

20) You are given the equation:

$x^2 - 5x + 6 = 0$ Factor

$(x - 3)(x - 2) = 0$ Set both factors equal to 0.

$x - 3 = 0$ $x - 2 = 0$ Solve these equations.

$x = 3$ **$x = 2$**

ANSWER: (4)

21) Let us assume that the base and height of the original triangle are represented by b and h. The area of this triangle would be $\frac{1}{2}$ **bh**. If we connect the midpoints of the sides of this triangle, then the new base would be $\frac{1}{2}$ b, and the new height would be $\frac{1}{2}$ h. This would result in a new area of $\frac{1}{2}\left(\frac{1}{2}b\right)\left(\frac{1}{2}h\right)$ or $\frac{1}{8}$ **bh**. The answer is $\frac{1}{4}$, because $\frac{1}{8}$ is $\frac{1}{4}$ of $\frac{1}{2}$.

Remember in general that the ratio of perimeters of similar triangles will be equal to the ratio of their sides, while the ratio of the areas of similar triangles will be equal to the square of the ratio of their sides.

ANSWER: (1)

22) You are presented with the expression with the following equation:

$2x + 3y = 12$ Transform into y=mx+b form, where m represents the slope. Subtract 2x from both sides.

$3y = -2x + 12$ Divide both sides by 3.

$y = -\frac{2}{3}x + 4$ You therefore know that the slope of this line is $-\frac{2}{3}$.

You also know that parallel lines have equal slopes, so you now have to figure out which of the given choices has a slope equal to $-\frac{2}{3}$.

(1) $6y - 4x = 2$ $6y = 4x + 2$ $y = \frac{4}{6}x + \frac{2}{6}$

(2) $6y + 4x = 2$ $6y = -4x + 2$ $y = -\frac{4}{6}x + \frac{2}{6}$

(3) $4x - 6y = 2$ $-6y = -4x + 2$ $y = \frac{-4}{-6}x - \frac{2}{6}$

(4) $6x + 4y = -2$ $4y = -6x - 2$ $y = -\frac{6}{4}x - \frac{2}{4}$

The slopes are:

(1) $\frac{4}{6}$ or $\frac{2}{3}$ (2) $-\frac{4}{6}$ or $-\frac{2}{3}$ (3) $\frac{-4}{-6}$ or $\frac{2}{3}$ (4) $= -\frac{6}{4}$ or $-\frac{3}{2}$

ANSWER: (2)

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23) $(2x^2+3x)-(3x^2-8x)=?$ Change the sign of all the terms in the second parenthesis and combine: $(2x^2+3x)+(-3x^2+8x)= 2x^2+3x -3x^2+8x= - x^2 + 11x$ **ANSWER: (1)**

24) You are given the coordinates for points R and T and asked for the length of line segment RT. R(-3,2) T(4,1) Here is the distance formula:

$d= \sqrt{(x1-x2)^2 + (y1-y2)^2}$ Let x1= -3 x2= 4 y1= 2 y2= 1

$d= \sqrt{(-3-4)^2 + (2-1)^2}$

$d = \sqrt{(-7)^2 + (1)^2}$

$d = \sqrt{49 + 1}$

$d = \sqrt{50}$ Simplify

$d = \sqrt{25} + \sqrt{2} = 5\sqrt{2}$ **ANSWER: (2)**

25) There are 5 girls and 2 boys. The probability of first choosing a girl is $\frac{5}{7}$. The probability for now choosing a boy would be $\frac{2}{6}$. The probability of first choosing a girl and then a boy would be $\frac{5}{7} \cdot \frac{2}{6}$ or $\mathbf{\frac{10}{42}}$ **ANSWER: (1)**

26) $-\pi$ is approximately –3.14

$-\sqrt{10}$ is approximately –3.16

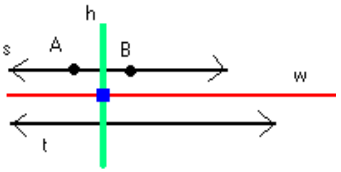
-16/5 is -3.2

the last choice is –3.02

-3.2 which can be thought of as –3.20 would be farthest left on the number line and is therefore smallest value of the given values. Therefore **–16/5** is the answer.

ANSWER: (3)

27) This is a locus problem. It should be worked out in two steps. First figure out the locus of points equidistant from two parallel lines. Let us assume that the two black horizontal lines, s and t at the right, are parallel. Then the red horizontal line w would be the points equidistant from these two lines. (We are assuming that I am a good artist, and that the red line is actually midway between the two "alleged" black parallel lines and parallel to them). Next, I picked two points on line s and named them A and B. The green vertical line h would be the locus of points equidistant from those two points as long as it is the perpendicular bisector of line segment AB. The intersection of the red horizontal line and the green vertical line is that one point highlighted in blue. This is the point that is both equidistant from the two parallel lines and equidistant from the two points. In other words, there is only one point that satisfies both conditions. **ANSWER: (1)**



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28) You are shown the graph of two inequalities and asked to choose a point which is in the solution set. In general, the solution set involving a system of inequalities will be the area contained in the shading of both inequalities. The tricky choice that can throw you off in this particular case is the point (0,4). This point does satisfy one inequality -- the one depicted by the solid line, but not the other – the one depicted by the dashed line. The correct choice happens to be (-4,1) as it lies in the area shaded by both inequalities.

ANSWER: (3)

29) You are asked to express the following in simplest form:

$(3x^3)(2y)^2(4x^4)$ First simplify $(2y)^2$ as being $4y^2$

$(3x^3)(4y^2)(4x^4)$ Multiply

$48x^7y^2$

ANSWER: (4)

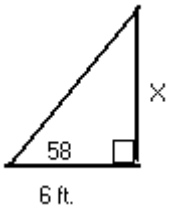
30) You are asked to express $\sqrt{72}$ in simplest $a\sqrt{b}$ form.

$\sqrt{72}$ Factor using a perfect square.

$\sqrt{36} \cdot \sqrt{2}$ Simplify $\sqrt{36}$

$6\sqrt{2}$ In $a\sqrt{b}$ form, 6 corresponds to the a. **ANSWER: (1)**

31) This is a simple trig problem. Look at the diagram at the right. Let the hypotenuse be the ladder leaning against a building, while x indicates the side of the building the ladder is leaning against. We want to know how far up the building the ladder reaches. We are given that the ladder makes and angle of 58 with the level ground, and that the foot of the ladder is 6 feet from the building. Relative to the angle of 58º, the side indicated with the x is considered opposite, while the side of 6 ft. is considered adjacent. The trigonometric relationship that includes both the opposite and adjacent is "tangent."



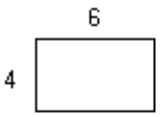
$\tan 58^\circ = \frac{\text{opposite}}{\text{adjacent}}$ $\tan 58^\circ = \frac{X}{6}$ Multiply both sides by 6

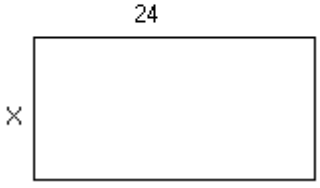
$6 \tan 58 = X$ Use your calculator to multiply tan 58 by 6.

$X = 9.602007$ Round to the nearest foot

ANSWER: 10 feet

32) Pictured at the right, and not necessarily drawn to scale, is Fran's favorite photograph which has a length of 6 inches and a width of 4 inches. She wants it made into a poster with a length of 24 as shown also to the right. What will be the width X of the poster? You are told that dimensions of the poster will be similar to those of the photograph. This means that the sides will be in proportion:





$$\frac{\text{Photograph dimension}}{\text{Poster dimension}} = \frac{6}{24} = \frac{4}{X}$$

$6X = (24)(4)$

$6X = 96$

$X = 16$ **ANSWER: 16 inches**

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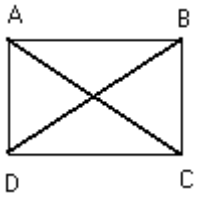
33) You are given that in rectangle ABCD, AC = 3x + 15 and BD = 4x – 5. You are asked to find the length of AC. As pictured at the right, you see AC and BD are diagonals. You should know that the diagonals of a rectangle are congruent. So set up your equation:



3x + 15 = 4x – 5 Subtract 3x from both sides.
15 = x – 5 Add 5 to both sides.
20 = x Once you know that x = 20, substitute for AC.

AC = 3x + 15 = 3 (20) + 15 = 60 + 15 = 75.
(You should also check to see that the other diagonal is also equal to 75.
BD = 4x – 5 = 4(20) – 5 = 80 – 5 = 75).

ANSWER: AC has a length of 75

34) José cannot build a triangular pen using boards with lengths of 7 feet, 8 feet, and 16 feet, because **in a triangle, the sum of the lengths of any two sides has to be greater than the length of the third side. In the case given here, 7 + 8 is 15, which would be less then the third side of 16**.

35) In order to construct a stem-and-leaf plot you first enter the data as you see it in the list and then order the data from lowest to highest. Put a key at the bottom of the plot.

1	5 9
2	5 8 6
3	2 9
4	0 3
5	0 5 0
6	5
7	5 2

1	5 9
2	5 6 8
3	2 9
4	0 3
5	0 0 5
6	5
7	2 5

Key: 1| 5 = 15

36) You are asked to find all negative odd integers that satisfy the following inequality:

$-3x + 1 \leq 17$ Subtract 1 from both sides.

$-3x \leq 16$ Divide both sides by –3 (inequality will change).

$x \geq \frac{16}{-3}$

$x \geq -5.3333\ldots$

We are looking for negative **odd** integers that satisfy the above x.

They are **–5, -3, and –1**.

37) You are given the dimensions of a fish tank as being 24 inches, 16 inches, and 18 inches. You are also told that you have a hose that can fill the tank at the rate of 500 cubic inches per minute. The question is how long will it take to fill the tank to a depth of 15 inches. The answer simply involves your figuring out what the volume of this tank is and then dividing this volume by 500. But recall that the question is not asking how long it will take to fill the tank completely. So in reality you will use not the given height of 18 inches, but the actual height you wish to fill the tank to. This is given as 15 inches. So in essence, the tank has a length of 24 inches, a width of 16 inches, and a height of 15 inches.

$V = l\ w\ h$ Volume equals length times width times height.

$V = (24)(16)(15)$

$V = 5{,}760$ cubic inches Divide this by 500 cubic inches – the rate the water is flowing

$5760 \div 500 = 11.52$ from the hose per minute.

The answer to the nearest minute is 12.

38) You are told that in triangle ΔABC, the measure of ∠B is 21 less than four times the measure of ∠A. This means that if ∠A is represented by x, then ∠B would be represented by 4x – 21. You are also told that the measure of ∠ C is 1 more than five times the measure of ∠ A. We would therefore represent the measure of ∠C by 5x + 1.

Here is the algebraic solution to this problem:

Let ∠ A= x

∠ B= 4x – 21

∠ C= 5x + 1 The sum of the angle measures of a Δ is 180º.

$x + 4x – 21 + 5x + 1 = 180$ Combine like terms

$10x – 20 = 180$ Add 20 to both sides.

$10x = 200$ Divide both sides by 10.

$x = 20$

∠A = 20º ∠B= 4(20)-21= 59º ∠C= 5(20) + 1 = 101º

ANSWER : ∠A = 20º ∠B= 59º ∠C= 101º

39) You are given the following: Tickets for a dance recital are $5.00 for adults and $2.00 for children. The total number of tickets sold was 295, and the total amount collected was $1,220. **How many adult tickets were sold?**

Here is one way of doing this problem.

Let x = number of children tickets

Let y = number of adult tickets

x + y = 295 the total number of tickets that were sold

2x = dollar amount of children's tickets $2.00 per ticket

5y = dollar amount of adult's tickets $5.00 per ticket

2x + 5y = 1220 total amount of money collected.

2x + 5y = 1220		2x + 5y = 1220	
x + y = 295	multiply each term by –2	-2x –2y = -590	Combine both equations
		3y = 630	Divide both sides by 3
		y = 210	

ANSWER: 210 adult tickets were sold.