

ANSWERS MATH A – August 16th, 2005

- 1) The problem asks which statistical measure separates the top half of a set of data from the bottom half given that the data is arranged from least to greatest. The answer is choice 3 or the **median**. By definition, the median is the middle score. This means that half of the scores will lie above it and half will be below. **ANSWER: (3)**

- 2) This is a problem involving the counting principle. If one thing can be done in **m** number of ways, and the other can be done in **n** number of ways, then both can be done in **m times n** number of ways. In this example multiply $16 \times 3 \times 7$. The answer is **336**. **ANSWER: (2)**

- 3) Use your calculator to find the value of $7!$ and divide the answer by $3!$. Worked out on paper it would look as follows:

$$\frac{(7)(6)(5)(4)(3)(2)(1)}{(3)(2)(1)} = (7)(6)(5)(4) = \mathbf{840}$$

The $(3)(2)(1)$ in the numerator cancels with the $(3)(2)(1)$ in the denominator

ANSWER: (1)

- 4) The **distributive law** states that the element in front of the parentheses is distributed to the elements inside the parentheses. For example $3(x+y)$ becomes $(3x + 3y)$. This is the law being depicted in the example: $*(\Delta + \heartsuit) = *\Delta + *\heartsuit$ **ANSWER: (3)**

- 5) A disjunction, or “or” statement, is false only when both parts of its statement are false. Only choice 4 would make the given statement “ x is divisible by 5 or x is divisible by 4”, false. Choice 4 is 27, and 27 is not divisible by 5 nor by 4. **ANSWER: (4)**

- 6) The diagram shown in the exam depicts a dilation. A dilation is the only one of the given transformations that does not preserve size. **ANSWER: (4)**

- 7) An octagon has 8 sides. Selecting one vertex from which to draw diagonals, 6 triangles can be formed. (Always two triangles less than the number of sides that comprise the polygon.) The sum of the measures of the angles of a triangle equal 180 degrees. In the case of our octagon which contains 6 triangles, that would be $180(6)$ or 1080 degrees. In other words, the sum of the interior angles of a polygon will equal 1080 degrees. Since all angles of a regular polygon are congruent, each angle in the octagon will equal 1080 divided by 8, or 135 degrees. Angle x , the angle in question, is an exterior angle. As such its measure will be $180 - 135$ or **45 degrees**. **ANSWER: (1)**

- 8) All you have to do for this problem is solve the equation when “ t ”, which represents time in seconds, equals 2.

$$h = -16t^2 + 48t$$

Substitute 2 for t .

$$h = -16(2)^2 + 48(2)$$

Complete the indicated operations

$$h = -16(4) + 96$$

Multiply

$$h = -64 + 96$$

Combine

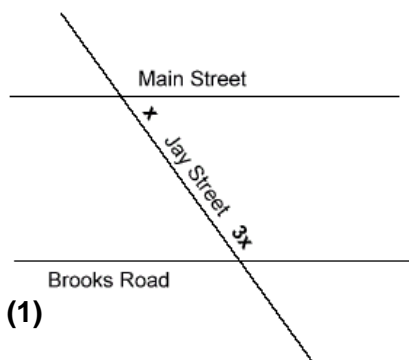
$$\mathbf{h = 32}$$

ANSWER: (2)

- 9) If the sum of their ages is 33, and one age is represented by g , then the other age will be represented by **33-g**. **ANSWER: (1)**

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- 10) The acute angle and the obtuse angle described in the problem are interior angles on the same side of the transversal. As such they are supplementary. You are told that one is 3 times the measure of the other. Let the acute angle equal x and the obtuse angle equal $3x$. At the right is the diagram with the appropriate angles marked x and $3x$.



Set up your equation:

$$\begin{array}{ll} x + 3x = 180 & \text{Combine} \\ 4x = 180 & \text{Divide both sides by 4.} \\ x = 45 & \end{array}$$

ANSWER: (1)

- 11) You are presented with four choices and asked which one equals 0.62×10^3 . First you have to know that 0.62×10^3 requires you to move the decimal point 3 places to the right. 0.62×10^3 therefore equals 620. Now look at **choice 4**. It is 6.2×10^2 . This means that you have to move the decimal point two places to the right. If you do that, you can see that 6.2×10^2 equals 620 as well. **ANSWER: (4)**
- 12) You are asked for the equation that represents the locus of all points 5 units below the x-axis.

Choice 1
 $x = -5$
5 units
left of the y-axis

Choice 2
 $x = 5$
5 units right of the y
right of the y-axis

Choice 3
 $y = -5$
5 units
below the x-axis

Choice 4
 $y = 5$
5 units
above the x-axis

ANSWER: (3)

- 13) You are asked to select the ordered pair that is not in the solution set of $y > 2x + 1$. Let's substitute the given choices for x and y and see which choice is not in the solution set.

Choice 1: (1, 4)
 $y > 2x + 1$
 $4 > 2(1) + 1$
 $4 > 2 + 1$
 $4 > 3$
True

Choice2: (1,6)
 $y > 2x + 1$
 $6 > 2(1) + 1$
 $6 > 2 + 1$
 $6 > 3$
True

Choice3: (3,8)
 $y > 2x + 1$
 $8 > 2(3) + 1$
 $8 > 6 + 1$
 $8 > 7$
True

Choice 4: (2,5)
 $y > 2x + 1$
 $5 > 2(2) + 1$
 $5 > 4 + 1$
 $5 > 5$
FALSE

ANSWER: (4)

- 14) In an operation, the identity element is the element which when operated upon will result in the same element. For example, for addition, the identity element is 0 because any element plus 0 will result in that same element. That is why the identity element for multiplication is 1. Any element times 1 will result in the same element. Look at the accompanying table and you will see that the elements are $r, s, t,$ and u . The column under s repeats the elements in that exact order. This makes s the identity element. Try it. Any element operated on s will result in that same element. For example $t \clubsuit s = t$.

\clubsuit	r	s	t	u
r	t	r	u	s
s	r	s	t	u
t	u	t	s	r
u	s	u	r	t

ANSWER: (2)

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- 15) The midpoint of a line segment can be found by adding the two sets of coordinates and dividing by 2. To find the x coordinate of the midpoint you add the two x values and divide by 2, and to find the y coordinate of the midpoint you add the two y values and divide by 2. The problem you are presented with has a twist. You are given the midpoint and the two endpoints but are missing the y-coordinate of one of the endpoints. Here is what you do. You see that the y-coordinate of the midpoint is 7. This means that when the y-coordinates of the endpoints are added together and then divided by two, the answer will be 7. What will the result be before they are divided by 2? Obviously 14, as 14 divided by 2 is 7. The rest is simple. One y-coordinate is 4. 4 and what will get you 14? **The answer is 10.**
ANSWER: (2)

- 16) You are asked to arrange four numbers from smallest to largest. Here are the numbers: 3.14, $\frac{22}{7}$, **B**, $\sqrt{9.1}$. Perhaps the simplest way to do this problem now is to use your calculator to convert all these number to decimals to at least 4 places.
 $3.14 = 3.1400$ **B** = 3.1415 $\frac{22}{7} = 3.1428$ $\sqrt{9.1} = 3.0166$
 Now, in order from lowest to highest they are: 3.0166 3.1400 3.1415 3.1428 or
 $\sqrt{9.1}$, 3.14, **B**, $\frac{22}{7}$
ANSWER: (4)

- 17) Of the four given choices, all but one can be considered a parallelogram. A parallelogram is a quadrilateral whose opposite sides are parallel. In addition, its opposite sides will also be congruent. The **trapezoid**, choice number 4, is a quadrilateral which has **only one pair of parallel sides. The other two sides do not have to be congruent.** They may be congruent if the trapezoid is isosceles.
ANSWER: (4)

- 18) BIKE 12 24 36 48 60
 CAR 30 60
 The problem states that the **car starts 3 hours later.** At that point the bike will already have traveled 36 miles. This is when the car starts. In 2 more hours the bike will have traveled a total of 60 miles which is the same mileage the car will have traveled in those two hours. In other words, **the bike and car will meet in 2 hours.**
 Here is how you would do this problem algebraically. Recall that Rate X Time = Distance

	RATE	X	TIME	=	DISTANCE	Let x= number of hours traveled
BIKE	12		x + 3		12(x+3)	
CAR	30		x		30x	(the car will have traveled 3 less hours)

Now we want to know when will the car and bike meet. We know that at that point, the distances traveled will be equal. Set up your equation:
 $30x = 12(x+3)$ Use the distributive property.
 $30x = 12x + 36$ Subtract 12x from both sides.
 $18x = 36$ Divide both sides by 18.
 $x = 2$ **The car will have traveled 2 hours.**
ANSWER: (2)

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- 19) The letter T has line symmetry (vertical) but has no point symmetry. If the object looks the same upside down (180 degree rotational symmetry) then it has point symmetry. The remaining three choices demonstrate both line and point symmetry. **ANSWER: (1)**

- 20) The sum of the measures of any two sides of a triangle has to be greater than the measure of the third side.

8, 15, 13 Yes

8, 15, 7 No, because $8 + 7$ is not greater than 15

8, 15, 3 No, because $8 + 3$ is not greater than 15

8, 15, 23 No, because $8 + 15$ is not greater than 23

ANSWER: (1)

- 21) Given the conditional If P then Q ($P \rightarrow Q$), its converse is if Q then P ($Q \rightarrow P$)
Switch the position of the first and second parts (the antecedent and conclusion).

ANSWER: (3)

- 22) Any number raised to a negative exponent is equal to 1 divided by that number raised to the same positive exponent.

$$X^{-a} = \frac{1}{X^a} \text{ Therefore, } 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

ANSWER: (2)

- 23) **B** is irrational. It can not be represented in the form of a over b where a and b are integers and b is not equal to 0.

ANSWER: (4)

- 24) To find the sum of $5\sqrt{7} + 3\sqrt{28}$, first simplify the radicals. Remember you can only combine like radicals.

$$5\sqrt{7} + 3\sqrt{28} \qquad \sqrt{28} = \sqrt{4} \sqrt{7}$$

$$5\sqrt{7} + 3\sqrt{4} \sqrt{7} \qquad \sqrt{4} = 2$$

$$5\sqrt{7} + 3(2)\sqrt{7} \qquad \text{Simplify}$$

$$5\sqrt{7} + 6\sqrt{7} \qquad \text{Combine}$$

$$11\sqrt{7}$$

ANSWER: (2)

- 25) You are asked to find the solution set for the following equation: $x^2 - 5x = 6$

$$x^2 - 5x = 6 \qquad \text{Subtract 6 from both sides.}$$

$$x^2 - 5x - 6 = 0 \qquad \text{Factor}$$

$$(x - 6)(x + 1) = 0 \qquad \text{Set each factor equal to 0.}$$

$$x - 6 = 0 \qquad x + 1 = 0$$

$$x = 6 \qquad x = -1$$

ANSWER: (3)

- 26) You are being asked to simplify $\frac{5x^6y^2}{x^8y}$. You know that $\frac{x^6}{x^8} = \frac{1}{x^2}$ and $\frac{y^2}{y} = y$

$$\text{Therefore, } \frac{5x^6y^2}{x^8y} = \frac{5y}{x^2}$$

ANSWER: (2)

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- 27) ${}_9C_2$ is the method of calculating the number of ways 9 items can be combined when taken 2 at a time. In general it is written as nCr . It can be shown that $nCr = nC(n-r)$. This means that a ${}_9C_2$ would be equivalent to a ${}_9C_{(9-2)}$ or ${}_9C_7$. You can check this on your calculator and see that both a ${}_9C_2$ and ${}_9C_7$ equal 36.

$${}_9C_7 = \frac{9P_7}{7!} = \frac{(9)(8)(7)(6)(5)(4)(3)}{(7)(6)(5)(4)(3)(2)(1)} = \frac{(9)(8)}{(2)(1)} = 36$$

$${}_9C_2 = \frac{9P_2}{2!} = \frac{(9)(8)}{(2)(1)} = 36$$

ANSWER: (3)

- 28) $x^2 + y^2 = r^2$ represents the graph of a circle with a radius of r , whose center is at the origin. $x^2 + y^2 = 4$ can be rewritten as $x^2 + y^2 = 2^2$. It therefore represents a circle with a radius of 2 whose center is at the origin.

ANSWER: (3)

- 29) Two distinct lines will always intersect at exactly one point unless they are parallel lines. By looking at the equation of a line we can determine its slope. Parallel lines will have equal slopes. Therefore, if we see two linear equations with equal slopes, we know that the lines they represent will not intersect as they are parallel. You are presented with 4 sets of linear equations. Let's look at each set. (They are already in the form of $y=mx+b$ where the slope is represented by m).

$y = -x - 20$ $m = -1$	$y = .5x + 30$ $m = .5$	$y = \frac{3}{5}x + 12$ $m = \frac{3}{5}$	$y = -x + 15$ $m = -1$
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$y = x + 7$ $m = +1$	$y = .5x - 30$ $m = .5$	$y = .6x - 19$ $m = .6$	$y = -x + 25$ $m = -1$
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**Unequal slopes
Lines will intersect**

**Equal slopes
parallel lines**

**Equal slopes
parallel lines**

**Equal slopes
parallel lines**

ANSWER: (1)

- 30) You are presented with $\frac{x}{4} - \frac{a}{b} = 0$ and asked to solve for x .

$\frac{x}{4} - \frac{a}{b} = 0$	Add $\frac{a}{b}$ to both sides.
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$\frac{x}{4} = \frac{a}{b}$	Cross multiply.
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$bx = 4a$	Divide both sides by b .
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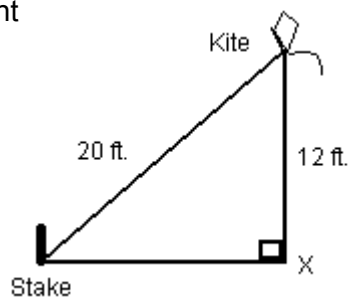
$$x = \frac{4a}{b}$$

ANSWER: (4)

Number 31 begins on the next page

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- 31) You are presented with a diagram which is in essence a right triangle. In addition, you are given the length of one leg as well as the length of the hypotenuse. The triangle is pictured at the right. You are asked for the distance between the stake and point X.



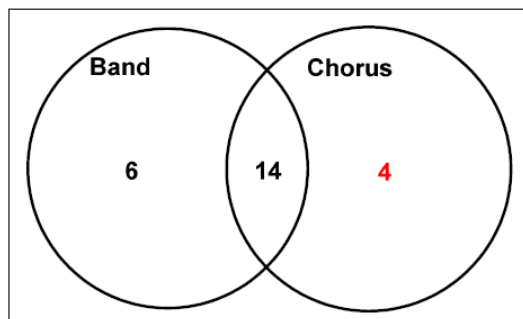
You can use the Pythagorean theory to solve for the third side. $a^2 + b^2 = c^2$ where c is the hypotenuse.

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Substitute the known values.} \\ 12^2 + b^2 &= 20^2 && \text{Simplify.} \\ 144 + b^2 &= 400 && \text{Subtract 144 from both sides.} \\ b^2 &= 256 && \text{Find the square root of both sides.} \\ b &= 16 \end{aligned}$$

You also may have noticed that the given sides fit the pattern of the classic 3, 4, 5 right triangle. The 12 is 4 times 3, the 20 is 4 times 5, leaving the unknown side to be 4 time 4.
ANSWER: The distance between the stake and point X is 16 ft.

- 32) Check the Venn diagram at the right.

You are told that out of 30 students, 24 either play in the school band or sing in the chorus. So all that concerns us in this problem are those 24 students. Out of those 24, you are told that 14 sing in the chorus and also play in the band. That why you see the 14 entered in the intersection of both band and chorus. You are told 6 play only in the band. This information accounts for only 20 students. That leaves 4 students that by default sing in the chorus but do not play in the band. That is shown by the red 4 which is only in the circle marked chorus.



ANSWER: 4 students sing in the chorus but do not play in the band.

- 33) Factor completely:

$$\begin{aligned} 5n^2 - 80 &&& \text{Factor out a 5 from each term.} \\ 5(n^2 - 16) &&& \text{Factor the } n^2 - 16. \\ \mathbf{5(n + 4)(n - 4)} &&& \end{aligned}$$

- 34) You are shown a circle graph which indicates the results of a survey. The results are shown in degree measure rather than percentages. The number of students that favored a partial dress code is indicated by a segment forming a central angle of 25 degrees, while the number of students favoring a full dress code is indicated by a segment with a central angle of 95 degrees. The remaining segment, whose measure is not given indicates students who favor no dress code. These 3 segments complete the circle and equal 360 degrees. The third segment whose measure is not given can now be calculated as follows: $360 - (25+95)$. The angle representing the students who want to dress code is therefore, 240 degrees. Now we can simply set up a proportion to figure out how many students felt that the school should have no dress code:

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$$\frac{240}{360} = \frac{x}{900}$$

Cross multiply

$$360x = 900(240)$$

Simplify

$$360x = 216000$$

Divide both sides by 360

$$x = 600$$

ANSWER: 600 students felt that the school should have no dress code.

- 35) When Seth bought the car, it had been driven 20,000 miles. After 2 years, the mileage on the car was 49,400. This means that in 2 years he put on an additional 49,400-20,000 or 29,400 miles. You are now being asked for the average number of miles he drove each month during those 2 years. Since 1 year is 12 months, divide the 29,400 by 24 which is two years. 29,400 divided by 24 is: 1225.

ANSWER: He drove the car an average of 1,225 miles per month.

- 36) Let X in the diagram at the right represent the height of a tree, 20 represent the length of the tree's shadow, and 66 be the number of degrees in the angle of elevation from the end of the shadow to the top of the tree. You are asked to find the height of the tree, X, to the nearest foot. Let's use the tangent ratio (opposite over adjacent).

$$\tan 66 = \frac{x}{20}$$

Cross multiply

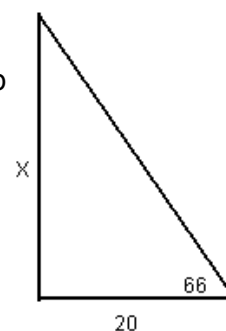
$$x = 20 (\tan 66)$$

Use your calculator to multiply.

$$x = 44.9207$$

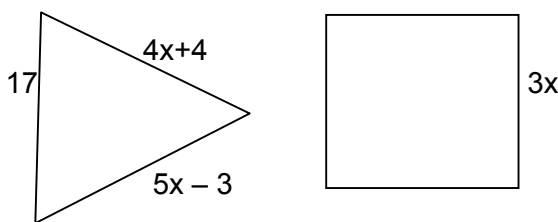
Round to the nearest foot

$$x = 45$$



ANSWER: The height of the tree to the nearest foot is 45 feet.

- 37) Pictured at the right are a triangle and a square. The sides of the triangle are 17, $4x + 4$, and $5x - 3$. And one side of the square is represented by $3x$. In addition, you are told that the perimeters of both figures are equal. You are asked to find the length of a side of the square.



Solution: Find both perimeters and set them equal to each other.

$$17 + 4x + 4 + 5x - 3 = 3x + 3x + 3x + 3x$$

Combine

$$9x + 18 = 12x$$

Subtract 9x from both sides.

$$18 = 3x$$

Divide both sides by 3.

$$6 = x$$

Once you know that $x = 6$, you know that the side of the square which is represented by $3x$ equals $3(6)$ or 18.

ANSWER: The length of the side of the square equals 18.

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38) You are asked to solve the following system of equations:

$$y = x^2 + 4x + 1$$

$$y = 5x + 3$$

Solution: since both right members of the equations equal y , set them equal to each other.

$$x^2 + 4x + 1 = 5x + 3$$

Subtract $5x$ from both sides.

$$x^2 + 4x + 1 - 5x = 3$$

Subtract 3 from both sides.

$$x^2 + 4x + 1 - 5x - 3 = 0$$

Combine like terms.

$$x^2 - x - 2 = 0$$

Factor.

$$(x - 2)(x + 1) = 0$$

Set each factor equal to 0 and solve.

$$x - 2 = 0$$

$$x + 1 = 0$$

$$x = 2$$

$$x = -1$$

You now have 2 values for x . Use each x value to find the corresponding y value. Let's use the linear equation.

When $x = 2$

$$y = 5x + 3$$

Substitute 2 for x .

$$y = 5(2) + 3$$

Simplify

$$y = 13$$

This is one answer: when $x=2$ $y=13$

When $x = -1$

$$y = 5x + 3$$

Substitute -1 for x .

$$y = 5(-1) + 3$$

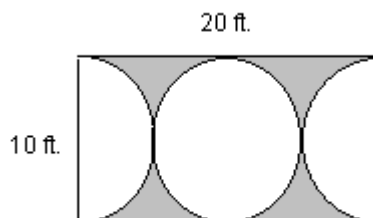
Simplify.

$$y = -2$$

This is the other answer: when $x=-1$ $y=-2$

ANSWER: (2,13) and (-1,-2)

39) The diagram at the right represents a rectangular plot of land. Its length is 20 and its width is 10. Mr. Petri wants to design a flower garden in the shape of a circle with two semi circles at each end of the center circle. He will fill in the shaded area with chips. You are told that one bag of wood chips covers 5 square feet, and are asked how many bags must he buy?



Solution: First find the area of the rectangle. Next find the area of 1 circle and double it. (There are really 2 complete circles in the rectangle). Now subtract the circle area from the rectangle area and you will be left with the shaded area. Finally, divide this shaded area by 5 and you will have your answer.

Area of rectangle = length X width = $20 \times 10 = 200$ square feet.

Area of circle = $\mathbf{B}r^2 = \mathbf{B}(5)(5) = 25\mathbf{B}$ Area of 2 circles = $2(25\mathbf{B}) = 50\mathbf{B}$

Shaded area = $200 - 50\mathbf{B} = 42.92036732$ square feet. Since 1 bag covers 5 sq. ft. Divide by 5.

Number of bags = 42.92036732 divided by 5 = 8.584073464

You cannot buy fractional bags so you will have to buy 9 bags.

ANSWER: He must buy 9 bags.