

## ANSWERS MATH A – August 17<sup>th</sup> 2004

- 1) If a diagram is folded over its line of symmetry then each point will lie on its corresponding point. It's as if you would be stacking two congruent pieces one on top of the other without having to rotate the pieces. Diagrams 1, 2, and 3 show lines of symmetry. Diagram 4 is not a line of symmetry. If diagram 4 were folded at the dotted line you would still end up with two separate curves—they would not lie one directly on top of the other. **ANSWER: (4)**

- 2) Find the average of Rosario's scores by adding up the 5 test scores and dividing by 5:

$$(78+77+64+86+70)/5 = 75$$

Now find Enrique's average in the same manner:

$$(90+61+79+73+87)/5 = 78$$

$$78 - 75 = 3$$

**ANSWER: (3)**

- 3) Diagram 1 is a cube whose volume is equal to its edge cubed:  $4^3=64$ .

Intuitively one can see that the sphere as well as the other pictured diagrams would fit in the cube with room to spare. **The cube has the greatest volume.**

The volume of a sphere happens to be:

$$V = \frac{4}{3} \mathbf{A}^3 \quad \text{The radius of this particular sphere is 2 since its diameter is 4.}$$

$$\text{Therefore, } \frac{4}{3} \mathbf{A}^3 = \frac{4}{3} (3.14)(2)^3 = \frac{4}{3} (3.14)(8) \approx \mathbf{33.493}$$

The volume of the cylinder:

$$V = \mathbf{A}^2 h = (3.14)(2^2)(4) \approx \mathbf{50.24}$$

The volume of the cone:

$$V = \frac{1}{3} \mathbf{A}^2 h = \frac{1}{3} (3.14)(2^2)(4) \approx \mathbf{16.746}$$

**ANSWER: (1)**

- 4) This is a simple example using the counting principle.

If one thing can be done m number of ways, and the other thing can be done in n number of ways, then both can be done in mn number of ways.

$$(5)(4)(3) = 60$$

**ANSWER: (4)**

- 5)  $-9x^5 \div -3x^3?$   $-9/-3=3$   $x^5 \div x^3 = x^2$  Final answer:  $3x^2$

**ANSWER: (2)**

- 6) Solve for n:  $0.6(n+10) = 3.6$  Multiply each term in the parenthesis by .6

$$.6n + 6 = 3.6$$

Subtract 6 from both sides.

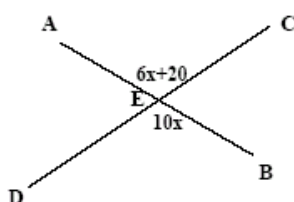
$$.6n = -2.4$$

Divide each side by .6

$$n = -4$$

**ANSWER: (3)**

- 7) Here is a diagram of the given information:



The two angles pictured, are vertical angles. Vertical angles are congruent. Therefore:

$$10x = 6x + 20$$

Subtract 6x from both sides.

$$4x = 20$$

Divide both sides by 4

$$x = 5$$

**ANSWER: (2)**

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8) If  $x = -4$  and  $y = 3$ , what is the value of  $x - 3y^2$ ?

$(-4) - 3(3)^2$

$(-4) - 3(9)$

$-4 - 27$

**-31**

**ANSWER: (3)**

9) A translation of  $(x-3, y+6)$  means that 3 is subtracted from the x coordinate, and 6 is added to the y coordinate. You are given the point P whose coordinates are  $(-4, 0)$ .

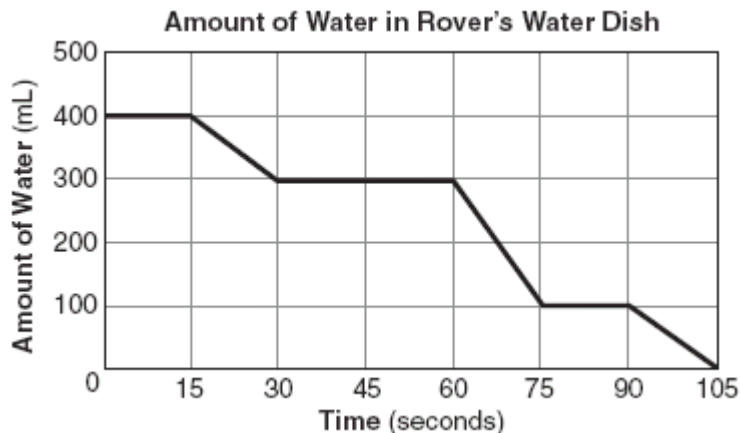
So the x coordinate is  $-4$  and the new x will be  $-4 - 3$  or  $-7$ .

The y coordinate is 0, and the new y will be  $0+6$  or **6**.

**The coordinates of P' will be  $(-7, 6)$**

**ANSWER: (1)**

10) The diagram below shows that Rover's dish started out with 400mL of water. After 15 seconds it still contained the same amount of water.



However between 15 and 30 seconds the water level fell to 300mL of water. This indicates that at 15 seconds Rover had his first drink. You also see that from 30 seconds to 60 seconds, the water level does not change. At 60 seconds it begins to change. That is when Rover began his second drink. This means that **Rover waited 30 seconds** to begin his second drink of water: from 30 sec. until 60 seconds.

**ANSWER: (2)**

11) The line graph shows the  $-3$  unshaded and the shaded part is to the right.



This means that  $x$  is greater than  $-3$  or  $x > -3$ . It also shows the 4 as being shaded and it continues being shaded to the left this means that  $x$  is less than or

equal to 4 or  $x \leq 4$ . Combining both, you get:  **$-3 < x \leq 4$**

**ANSWER: (4)**

12) Their bills are in a ratio of 7:5. Let Tariq's bill be represented by  $7x$ , Pria's bill by  $5x$ . You are also told that Tariq's bill was \$14 more than Pria's. So you now can set up the following equation:

$7x$  (Tariq's bill) =  $5x$  (Pria's bill) + 14

$7x = 5x + 14$       Subtract  $5x$  from both sides.

$2x = 14$       Divide both sides by 2.

$x = 7$

You are asked for Tariq's bill. As long as you know that  $x=7$ , Tariq's bill is  $7x$  or simply  $(7)(7)$  or **\$49**.

**ANSWER: (4)**

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- 13) Choice #2  $6(3a+4b)=18a + 24b$  illustrates the distributive property of multiplication over addition. The 6 is being distributed to terms in the parenthesis. Both the 3a and 4b are being multiplied by 6. **ANSWER: (2)**

- 14) The cosine ratio is represented by the adjacent side divided by the hypotenuse. The side adjacent angle A is 5, and the hypotenuse is 13.  
 $\cos A$  therefore equals  $5/13$  **ANSWER: (1)**

- 15) Set up a proportion of  $\frac{\text{miles}}{\text{minutes}} \quad \frac{640}{60} = \frac{384}{x}$   
 The product of the means equals the product of the extremes. Therefore:  
 $640x = (384)(60)$   
 $640x = 23040$  Divide both sides by 640  
 $x = 36$  minutes **ANSWER: (1)**

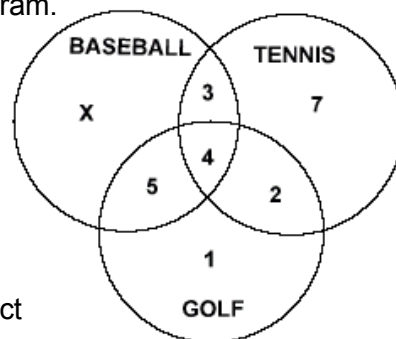
- 16) To find the inverse of a biconditional, negate each part.  
 The given statement begins... If I do not buy a ticket. This becomes...If I do buy a ticket.  
 The statement concludes with...then I do not go to the concert. This becomes ...then I do go to the concert. Put them together and you have...  
 If I buy a ticket, then I go to the concert. **ANSWER: (2)**

- 17) Choice #1—a horizontal line. In an equation where the graph is horizontal, the y remains constant as the x increases.  
 Choice #2—a vertical line. In an equation where the graph is a vertical line, the x will remain the same as the y increases.  
 Choice #3—a line with a negative slope. In an equation whose graph has a negative slope, as the x increases, the y will decrease.  
 Choice #4—a line with a positive slope. In an equation whose graph has a positive slope, as the x increases, the y will also increase. **ANSWER: (4)**

- 18) Under a reflection in the origin, the point (x,y) becomes (-x,-y). You negate the x and y. In this problem you are given the point (-3,-1). Reflected through the origin this point will become (3,1) **ANSWER: (1)**

- 19) This problem is easily pictured and done using a Venn diagram.  
 The diagram at the right pictures the given information.  
 You can easily see the following:

4 students play all three sports, 5 play baseball and golf, 2 play tennis and golf, 3 play baseball and tennis, 7 play only tennis, and 1 plays only golf. You are asked to figure out how many play baseball only. This is indicated by the x in the diagram. You are also told that the number of students involved in the three sports total 78.  
 All you have to do now is add up all the numbers, and subtract that sum from 78.  $4+5+2+3+7+1= 22$   
 $78-22= 56$



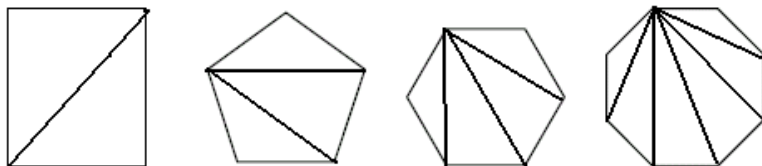
**ANSWER: (3)**

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- 20) The first line in the table shows that when  $c=0$ ,  $d=20.00$   
Choice 1 is the equation  $d=1.50c$   $1.50(0) \neq 20.00$   
Choice 2 is the equation  $d=1.50c+20.00$   $1.50(0)+20.00 = 20.00$  **GOOD**  
The above equation will work for all values of  $c$  given in the table to yield the given value for  $d$  and is therefore the answer. **ANSWER: (2)**
- 21) The  $30^\circ$  angle and angle B can be considered corresponding angles. As such they are congruent. **ANSWER: (1)**
- 22) You are presented with the expression  $\frac{x-7}{x+2}$   
A fraction undefined when its denominator, in this case  $x+2$ , equals 0.  
 $x+2=0$  Subtract 2 from both sides.  
 $x = -2$  **ANSWER: (1)**
- 23)  $(3x^2+2xy+7)-(6x^2-4xy+3)=?$  Change the sign of all the terms in the second parenthesis and combine:  $(3x^2+2xy+7)+(-6x^2+4xy-3) = -3x^2+6xy+4$  **ANSWER: (3)**
- 24)  $1.56 \times 10^{-2}=?$  Because the exponent is a negative 2, you have to move the decimal 2 places to the left. Because there is only 1 number before the decimal place, you will have to add 1 zero in order to move it 2 places. Here is your answer:  
**.0156** **ANSWER: (3)**
- 25) The sum of any two sides of a triangle have to always be greater than the third side. Therefore, a triangle can not have sides of 5,5, and 11. This is because  $5+5=10$ , which is less than the third side which is given as 11. **ANSWER: (2)**
- 26) The locus of points 4 units from the origin represents a circle with a radius of 4, whose center is at the origin. The equation of such a circle would be  $x^2+y^2=r^2$   
In our case the radius is 4, therefore the equation will be:  
 $x^2+y^2=4^2$  or  $x^2+y^2=16$  **ANSWER: (4)**
- 27) To find the contrapositive of a statement, find the converse of its inverse.  
The statement is: If I study, then I pass the test.  
Its inverse is: If I do not study, then I do not pass the test.  
The converse of that inverse is: **If I do not pass the test, then I do not study.**  
You can also find the contrapositive by finding the inverse of its converse.  
Remember that a contrapositive and its statement are logically equivalent. **ANSWER: (3)**

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- 28) Look what happens when you select a vertex of a polygon and then draw lines connecting that vertex to each other vertex.



If you count the number of triangles formed you will see that there will always be two less triangles than the number of sides or vertices of the polygon. A 4-sided figure will result in two triangles, a 5-sided figure in 3, and a 6-sided figure will result in 4 triangles. Using this pattern, an 8-sided figure (octagon) will result in  $8-2$  or 6 triangles. You know that the sum of the measures of the angles of any triangle adds up to 180 degrees, therefore all you now have to do is multiply 6 by 180 and you have your answer!  $180 \times 6 = 1,080$

**ANSWER: (2)**

- 29) You can substitute each answer into both equations to see which one will be the solution set, but let's solve it using simultaneous equations:

$$\begin{array}{l} 2x - y = 3 \\ x + y = 3 \end{array} \quad \text{Add both equations.}$$

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$$\begin{array}{l} 3x = 6 \\ x = 2 \end{array} \quad \text{Divide both sides by 3.}$$

You can now select any one of the two equations and substitute 2 for  $x$ , however if you look at the answers given you see that only choice 1 has the  $x$  coordinate as being 2. The answer is (2,1).

**ANSWER: (1)**

- 30) This problem, like number 4 earlier, again uses the counting principle. You are told the probability of Selena getting a hit is  $\frac{8}{20}$ , and the probability of Tracey getting a hit is  $\frac{6}{16}$ . The probability that both will get a hit the next time at bat is simply

$$\text{The product of } \frac{8}{20} \text{ and } \frac{6}{16}: \frac{8}{20} \times \frac{6}{16} = \frac{48}{320}$$

**ANSWER: (4)**

- 31) Complementary angles are two angles whose sum of angle measurements add up to  $90^\circ$ . For this problem:

let  $x$  = the measure of one angle.

$5x$  = measure of the other angle. The sum of the measures of the two is equal to 90.

$$x + 5x = 90$$

$$6x = 90 \quad \text{Divide both sides by 6.}$$

$$x = 15$$

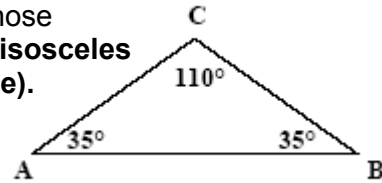
The question asks for the larger angle. The larger angle is  $5x$  or  $5(15) = 75$  degrees.

**ANSWER: 75 degrees**

- 32)  $\sqrt{196}$  is a rational number. It can be represented in the form of  $\frac{a}{b}$  where  $a$  and  $b$  are both integers, and  $b$  is not equal to 0.  $\sqrt{196} = 14$ , which can be represented as  $\frac{14}{1}$ . Both 14 and 1 are integers.

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- 33) An isosceles triangle is a triangle whose base angles will be congruent. An acute triangle is a triangle **each** of whose angles is acute (less than  $90^\circ$ ). **Pictured at the right is an isosceles triangle that contains an angle of  $110^\circ$  (an obtuse angle).** It is therefore an example of an isosceles triangle that is not acute! Dylan's statement is therefore not true.



- 34) Factor completely:  $3ax^2 - 27a$       First factor out a 3 and an a.  
 $3a(x^2 - 9)$       Factor the  $x^2 - 9$   
 **$3a(x+3)(x-3)$**

**ANSWER:  $3a(x+3)(x-3)$**

- 35) The circle graph shows that the area with a  $60^\circ$  central angle indicates the number of students whose favorite color is red. An angle of  $60^\circ$  is  $60/360$  of the complete circle as a circle contains  $360^\circ$ .  $60/360 = 1/6$  of the complete circle. The total number of students is given as 300 students. All you now have to calculate is  $1/6$  of 300.  $1/6$  of 300 = 50.

**ANSWER: 50 students chose red as their favorite color.**

- 36) Walter's daily wages is \$50. On Tuesday he earned \$170. The difference of  $(170-50)$  or \$120 must have been made up in the tips he earned. The problem states that his tips are equal to 15% of the total cost of the dinners he serves. For this problem, then, this means that 15% of the total cost of dinners has to equal \$120.

15% of what equals 120?

Let  $x$  = the unknown amount

15% means  $15/100$

Set up a proportion: 15 out 100 equals 120 out of  $x$

$$\frac{15}{100} = \frac{120}{x} \quad \text{Cross multiply}$$

$$15x = 12000 \quad \text{Divide both sides by 15}$$

$$x = 800$$

**ANSWER: Total cost of dinners on Tuesday was \$800.**

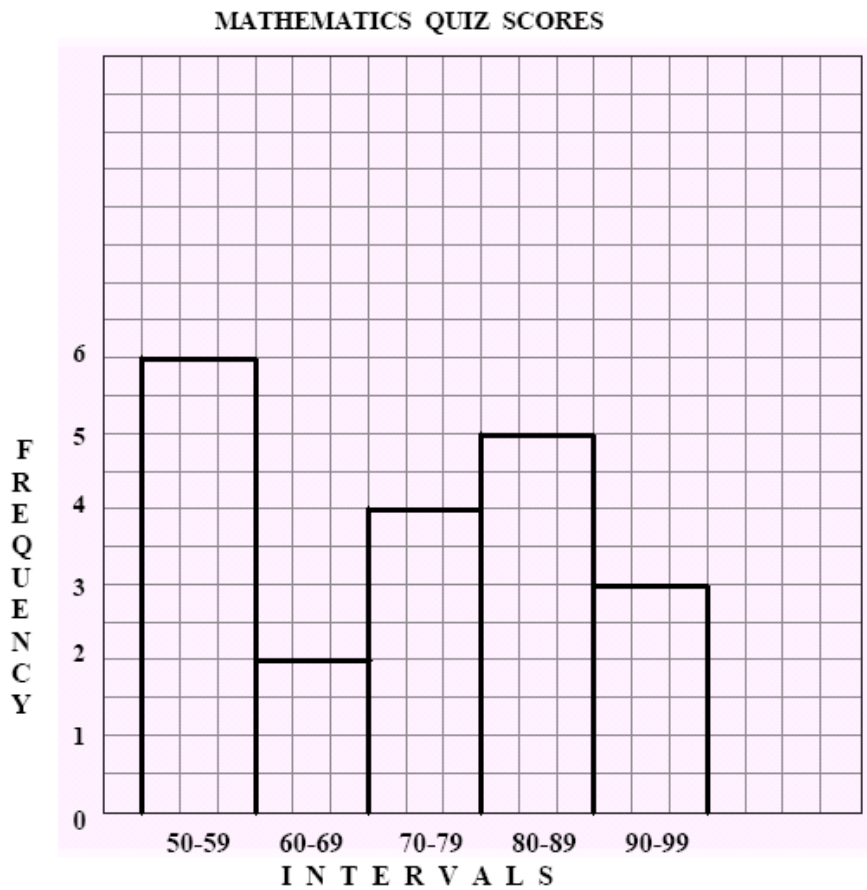
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37) Here is the completed frequency table based on the given data:

58, 79, 81, 99, 68, 92, 76, 84, 53, 57,  
81, 91, 77, 50, 65, 57, 51, 72, 84, 89

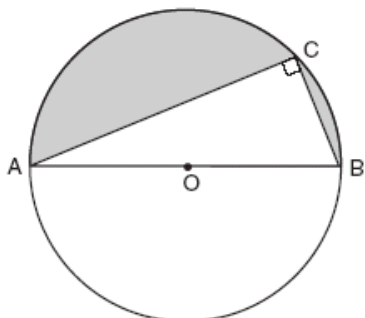
Interval	Tally	Frequency
50-59	//// /	6
60-69	//	2
70-79	////	4
80-89	//// /	5
90-99	///	3

And below is the frequency histogram for these scores:



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38)



Given the diagram at the left with  $AB=26$  and  $CB=10$ , You are asked to find the area of the shaded region to the nearest square unit. In order to find the area of the shaded region, you can first find the area of the semicircle, and subtract from that the part that the area of triangle ABC.

**Step 1.** The formula for the area of a circle is:  $A = \pi r^2$ .  $AB$ , the diameter of this circle is 26. This makes the radius equal to 13. The area of the semicircle will be:  $A = \frac{1}{2} (\pi)(13)(13) = 265.4645792$  (use the pi key on your calculator).

**Step 2** involves finding the area of triangle ABC. As indicated in the diagram, it is a right triangle. The area of any triangle can be found by multiplying  $\frac{1}{2}$  its base times its height. In our case  $A = \frac{1}{2} (CB) (AC)$ . We know that  $CB$  is 10. We can now use the pythagorean theorem to find  $AC$ . (You may also notice that it is a 5, 12, 13 triangle).

$$(AB)^2 = (AC)^2 + (CB)^2$$

$$(26)^2 = (AC)^2 + (10)^2$$

$$676 = (AC)^2 + 100 \quad \text{Subtract 100 from both sides.}$$

$$576 = (AC)^2 \quad \text{Find square root of both sides.}$$

$$AC = \sqrt{576} = 24$$

Now, the area of the triangle equals  $\frac{1}{2} (CB) (AC)$  or  $\frac{1}{2} (10)(24) = 120$

**Step 3** subtract the 120 fro the area of the semicircle and you will be left with the area of the shaded region.  $265.4645792 - 120 = 145.4645792$

**To the nearest square unit the answer is 145**

39) You are asked to solve for all values of  $x$  that satisfy the following equation:

$$\frac{x}{x+3} = \frac{5}{x+7}$$

Cross multiply. In other words, the product of the means equals the the product of the extremes.

$$x(x+7) = 5(x+3)$$

Use the distributive law.

$$x^2 + 7x = 5x + 15$$

Bring everything to the left side setting the quadratic equal to 0.

$$x^2 + 7x - 5x - 15 = 0$$

Combine like term.

$$x^2 + 2x - 15 = 0$$

Factor

$$(x+5)(x-3) = 0$$

Set both factors equal to 0

$$x+5 = 0 \quad x-3 = 0$$

Solve the 2 equations.

$$x = -5 \quad x = 3$$