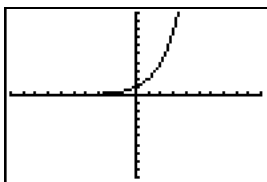
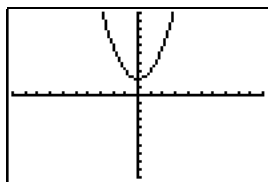


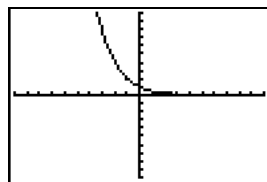
- 1) One way to do this problem is to enter each equation into your calculator's Y= editor and see the resulting graph. The answer will be **choice 3**.



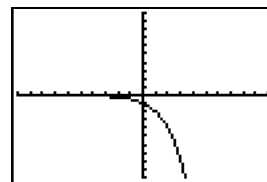
$Y = 2^x$



$Y = x^2 + 2$



$Y = 2^{-x}$



$Y = -2^x$

**ANSWER: (3)**

- 2) Use the Law of Sines. In this case we will use ratios of the Sine of an angle over the side opposite the angle. The side opposite the 65° angle is x, and the side opposite the 75° angle is 32. Set up your proportion:

$$\frac{\sin 65}{x} = \frac{\sin 75}{32}$$

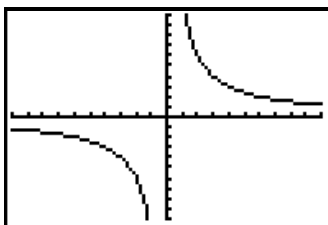
**ANSWER: (2)**

- 3) Solve for x:

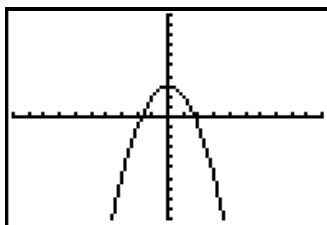
$$\begin{aligned} \sqrt{x-a} &= b && \text{Square both sides.} \\ x - a &= b^2 && \text{Add "a" to both sides.} \\ \mathbf{x} &= \mathbf{b^2 + a} \end{aligned}$$

**ANSWER: (2)**

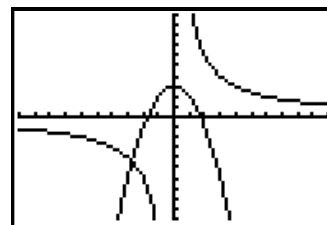
- 4) The graph of  $xy=12$  is a hyperbola in quadrants 1 and 3. The graph of  $y=-x^2 + 3$  is a parabola opening downwards, whose axis of symmetry is the y-axis, and whose maximum point is (0,3). Enter both of these equations into your graphing calculator and you will see that they intersect in only one point. When entering  $xy=12$  you first have to solve for y in terms of x. Your result will be  $y= 12/x$ . Input this equation into your calculator.



Graph of  $xy=12$



Graph of  $y=-x^2 + 3$



Both graphs on one axis

**ANSWER: (1)**

- 5) You are asked to simplify  $i^{25}$ .

$$i^0=1 \quad i^1=i \quad i^2=(\sqrt{-1})(\sqrt{-1})=-1 \quad i^3=(i^2)(i)=-i \quad i^4=(i^2)(i^2)=1$$

Now, in order to simplify powers of  $i$ , simply divide by 4 and **keep the remainder**. In our case, divide 25 by 4. Your answer will be 6 and **the remainder will be 1**.

$i^{25}$  is therefore equivalent to  $i^1$  or  $i$ .

**ANSWER: (3)**

- 6) You are presented with a complex fraction. Let us first simplify the numerator:

$$\frac{1}{3} + \frac{1}{3x} \quad \text{The LCD is } 3x. \text{ Multiply the } \frac{1}{3} \text{ by } \frac{x}{x}.$$

$$\frac{1}{3} \left(\frac{x}{x}\right) + \frac{1}{3x} = \frac{x}{3x} + \frac{1}{3x} \quad \text{Add.}$$

$$\frac{x+1}{3x} \quad \text{This is your new numerator.}$$

Now let us simplify the denominator:

$$\frac{1}{x} + \frac{1}{3} \quad \text{The LCD is again } 3x. \text{ Multiply 1}^{\text{st}} \text{ fraction by } \frac{3}{3} \text{ and the second by } \frac{x}{x}.$$

$$\frac{1}{x} \left(\frac{3}{3}\right) + \frac{1}{3} \left(\frac{x}{x}\right) = \frac{3}{3x} + \frac{x}{3x} \quad \text{Add.}$$

$$\frac{x+3}{3x} \quad \text{This is your new denominator.}$$

Now divide the numerator by the denominator:

$$\frac{x+1}{3x} \div \frac{x+3}{3x} = \frac{x+1}{3x} \cdot \frac{3x}{x+3} = \frac{x+1}{x+3} \quad (\text{the } 3x \text{ in the numerator and denominator cancel})$$

**ANSWER: (1)**

- 7) You are presented with snowfall amounts and asked to find the mean and population standard deviation to the nearest hundredth. To find the standard deviation using your calculator, first enter the data into **L1** as follows:

**STAT** **ENTER** Now continue to enter the data, hitting enter after each one. When completed, your screen will look like the one at the right.

L1	L2	L3	1
8.4			
7			
11.5			
14.1			
9.5			
8.6			
L1(19) =			

To find the standard deviation for the data in **L1** continue with the following key strokes:

**STAT** **▶** **ENTER** **ENTER**

Your screen will now look as follows:  
**The mean** rounded to the nearest hundredth is **9.46**. **The population standard deviation** is **3.74**.

1-Var Stats
$\bar{x}=9.455555556$
$\Sigma x=170.2$
$\Sigma x^2=1861.58$
$Sx=3.852000584$
$\sigma x=3.743471684$
$\downarrow n=18$

**ANSWER: (1)**

- 8) You are asked to find the equivalent of  $\frac{4}{5-\sqrt{13}}$ . To arrive at your answer, you have to simplify or rather rationalize the denominator. To do this you have to multiply both the numerator and denominator by the conjugate of the denominator. The denominator is  $5-\sqrt{13}$  and its **conjugate** is  $5+\sqrt{13}$

$$\frac{4}{5-\sqrt{13}} \left( \frac{5+\sqrt{13}}{5+\sqrt{13}} \right) = \frac{20+4\sqrt{13}}{25-13} = \frac{20+4\sqrt{13}}{12} \quad \text{Divide each term by 4 and you get: } \frac{5+\sqrt{13}}{3}.$$

**ANSWER: (1)**

- 9) What is the value of  $b$  in the following equation:  $4^{2b-3} = 8^{1-b}$ .  
 The first step involves getting a common base.  $4 = 2^2$  and  $8 = 2^3$ .  
 The equation can now be rewritten as:  $(2^2)^{2b-3} = (2^3)^{1-b}$   
 When a power is raised to a power you actually multiply the powers. Your equation will now become:  $2^{4b-6} = 2^{3-3b}$

Now since the bases are equal, you can set the powers equal to each other and solve:

$$4b - 6 = 3 - 3b \quad \text{Add } 3b \text{ to both sides.}$$

$$7b - 6 = 3 \quad \text{Add 6 to both sides.}$$

$$7b = 9 \quad \text{Divide both sides by 7.}$$

$$\mathbf{b = 9/7}$$

**ANSWER: (3)**

- 10) You are asked to solve the following absolute inequality:

$|2x-1| < 9$  What this really means is that  $2x-1$  and  $-(2x-1)$  will both be less than 9.

Let us solve both inequalities:

$$2x-1 < 9 \quad \text{Add 1 to both sides}$$

$$2x < 10 \quad \text{Divide both sides by 2}$$

$$\mathbf{x < 5}$$

$$-(2x-1) < 9 \quad \text{Distribute the negative sign.}$$

$$-2x + 1 < 9 \quad \text{Subtract 1 from both sides}$$

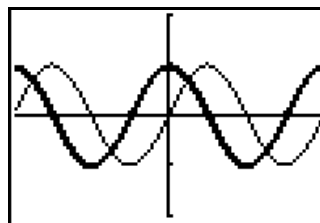
$$-2x < 8 \quad \text{Divide both sides by } -2 \text{ (symbol will switch)}$$

$$\mathbf{x > -4}$$

Your answer is that  $x > -4$  and  $x < 5$ . In other words,  $x$  will be a number between  $-4$  and  $5$ . (Not including the  $-4$  or  $5$ )

**ANSWER: (1)**

- 11) At the right you see the basic sine and cosine curves. The one in bold is the cosine curve. The lighter one is the basic sine curve. You can see that a translation of 90 degrees on either curve will make them coincide.

**ANSWER: (1)**

- 12) The range is defined by your set of  $y$  values. Looking at the given graph, you can see that the lowest  $y$ -value is 0, while the greatest is 100.

**ANSWER: (3)**

- 13) This problem is in essence asking you to determine the nature of the roots of the equation  $y = x^2 - 7x - 60$ . In other words when  $y$  will equal 0, what can you know about  $x$ . To answer this question without solving the equation requires you to determine the value of the discriminant. The discriminant is represented by the expression  $b^2 - 4ac$ . The standard form of a quadratic equation is  $ax^2 + bx + c = 0$ .

In our equation,  $a=1$ ,  $b=-7$ , and  $c=-60$ . Let us substitute these values for  $b^2 - 4ac$ .

$$b^2 - 4ac \quad \text{Substitute.}$$

$$(-7)^2 - 4(1)(-60) \quad \text{Simplify.}$$

$$49 - 4(-60) \quad \text{Continue simplifying}$$

$$49 + 240 \quad \text{Continue simplifying}$$

$$289$$

**289 is a positive and a perfect square. This means that the roots are real rational and unequal. This is what choice 4 is saying.**

Had the result of the discriminant been negative then the roots of the given equation would have been imaginary, and the answer would have been choice 2. Had the result been 0, then the roots would have been real, rational and equal. This would have meant that the parabola would be tangent to the  $x$ -axis (touching it at only one point).

**ANSWER: (4)**

- 14) You are told that the roots of an equation are  $3+i$  and  $3-i$ . You are asked to determine the equation. Here is one way of solving this problem. The **sum of the roots** is  $(3+i)+(3-i)$  or **6**.

The **product of the roots** is  $(3+i)(3-i) = 9 - i^2 = 9 - (-1) = 9+1=10$

The general form of a quadratic equation is  $ax^2 + bx + c = 0$ . The sum of the roots is represented by  $-b/a$ . The product of the roots is represented by  $c/a$ .

You now have enough information to solve for  $a$ ,  $b$ , and  $c$  for the equation whose roots will be  $3+i$  and  $3-i$ .

The sum of the roots as stated earlier is 6, so we know that  $\frac{-b}{a} = \frac{6}{1}$

This tells us that  $a = 1$  and  $-b = 6$  which makes  $b = -6$ .

The product of the roots is 10, which tells us that  $\frac{c}{a} = \frac{10}{1}$ . We now know that  $c = 10$ .

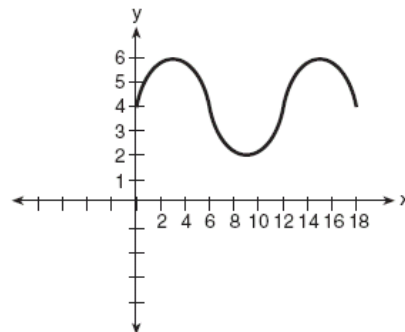
Now substitute the values of  $a$ ,  $b$ , and  $c$  into  $ax^2 + bx + c = 0$ .  **$1x^2 - 6x + 10 = 0$**

**ANSWER: (3)**

- 15) You are presented with the graph at the right and asked to determine its amplitude. The amplitude will be equal to one-half the difference of the greatest  $y$ -value and the lowest  $y$ -value. The greatest  $y$  in the case at the right is 6. The lowest  $y$  is 2. The amplitude is therefore:

$$\frac{1}{2}(6 - 2) = \frac{1}{2}(4) = 2$$

**ANSWER: (2)**



- 16) When presented with the graph of a circle, it is easy to determine its equation if you know the coordinates of the circle's center and its radius. The equation of a circle whose center is at the origin with a radius of  $r$  will be  $x^2 + y^2 = r^2$ .  
 If its center is not the origin then its equation would be  $(x - X_c)^2 + (y - Y_c)^2 = r^2$ , where  $X_c$  is the x-coordinate of the circle's center, and  $Y_c$  is the y-coordinate of the circle's center, and  $r$  is still the radius. In other words, if the center of a circle is  $(2,5)$  and its radius is 4, then its equation would be  $(x - 2)^2 + (y - 5)^2 = 4^2$   
 If its center would instead be  $(-2,5)$  then its equation would be  $(x + 2)^2 + (y - 5)^2 = 4^2$ .  
 In this problem, you are presented with a circle whose **center** is  $(1,-2)$ . To determine its radius, simply count along the x-axis from the center to the point given on the circumference.  
 Its **radius** is **3**.  
 Its equation will therefore be:  $(x - 1)^2 + (y + 2)^2 = 3^2$  or  $(x - 1)^2 + (y + 2)^2 = 9$       **ANSWER: (2)**
- 17) In order to complete this problem you have to know some basic rules regarding logarithms. The one that we will use first in this problem is the **Product Rule for Logarithms**. That is  $\log [(a)(b)] = \log a + \log b$ .  
 This problem presents us with the equation:  $R = \frac{2GM}{c^2}$ .  
 This means that  $\log R = \log \frac{2GM}{c^2}$   
 At this point you can use the Product Rule of Logarithms:  $\log [(a)(b)] = \log a + \log b$  and rewrite the **numerator** as  **$\log 2 + \log G + \log M$**   
**The Quotient Rule for Logarithms** states  $\log \left(\frac{a}{b}\right) = \log a - \log b$ .  
 this means that you will subtract the log of denominator from the log of the numerator or  **$(\log 2 + \log G + \log M) - \log c^2$** .  
**The Power Rule for Logarithms** states  $\log (c^2) = 2 \log c$   
 Putting all of this together we get:  $\log R = (\log 2 + \log G + \log M) - 2 \log c$       **ANSWER: (3)**
- 18) You are presented with a unit circle, whose terminal side forms an angle of 30 degrees in the 3<sup>rd</sup> quadrant. In such a circle, the sine of the angle is represented by the y-coordinate and the cosine of the angle is represented by the x-coordinate.  $\cos 30^\circ = \frac{\sqrt{3}}{2}$  However, in the 3<sup>rd</sup> quadrant, cosine is negative so its value here would be  $-\frac{\sqrt{3}}{2}$ . In addition, it would also be the x-coordinate.  
 $\sin 30^\circ = .5$  Sine in the 3<sup>rd</sup> quadrant is also negative, so its value here would be  $-.5$ . In addition, it would also be the y-coordinate. The answer therefore is.  $(-\frac{\sqrt{3}}{2}, -.5)$       **ANSWER: (1)**
- 19) Choice 4 represents a dilation of 1/2. Each coordinate is multiplied by  $\frac{1}{2}$  to obtain its image.  
 $8(1/2) = 4$      $4(1/2) = 2$  hence the image  $(4,2)$       **ANSWER: (4)**

- 20) This problem is just another way of phrasing a question dealing with the ambiguous case. The usual way this problem has been posed in the past was how many triangles are possible given certain information. The information given for this type of problem always involves two sides and an angle opposite one of the given sides. You then have to determine the missing angle. Whenever a triangle problem involves two sides and the angles opposite these sides you can

generally use the Law of Sines:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ .

In our case here you are told that the measure of  $\angle A = 30$ ,  $a = 14$ , and  $b = 20$ . Set up your proportion to determine the measure of angle B.

$$\begin{aligned} \frac{14}{\sin 30} &= \frac{20}{\sin B} && \text{Cross multiply.} \\ 14 \sin B &= 20 \sin 30 && \text{Simplify.} \\ 14 \sin B &= 20(.5) && \text{Simplify} \\ 14 \sin B &= 10 && \text{Divide both sides by 14.} \\ \sin B &= \frac{10}{14} && \text{Now use the SIN}^{-1} \text{ key to find angle B.} \end{aligned}$$

**The approximate measure of  $\angle B$  is 46 degrees**

Based on the above information you can now determine angle C.

The three angles of a triangle always add up to 180 degrees. So far we know that in our triangle, angles A and B add up to  $30 + 46$  or 76. This leaves  $180 - 76$  or 104 for angle C. **In this case, angle B is an acute angle.**

However there is another angle whose sin will also be  $10/14$ . It will be an angle in the second quadrant or  $180 - 46$  which equals 134.  $\sin 46$  and  $\sin 134$  will be equal. Now if angle  $A = 30$ , and **angle B = 134**, then angle C would equal  $180 - (30 + 134)$  or 16 degrees. This means there are actually two triangles possible with the given information. In one, angle B would be 46 and acute, in the other it would be 134 and obtuse. **ANSWER: (4)**

- 21) You are presented with the diagram at the right and told that arcs AC and BC are in a ratio of 7 to 2. Since AOB is a diameter, you know that arc ACB will equal 180 degrees. It is a semicircle. You now have enough information to determine the measures of arc AC and BC. Together they equal 180. Set up your equation.

$$\begin{aligned} 7x + 2x &= 180 && \text{Combine like terms.} \\ 9x &= 180 && \text{Divide both sides by 9} \\ x &= 20 \end{aligned}$$

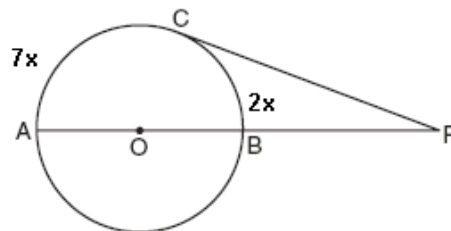
Arc AC therefore equals  $7x$  or  $7(20)$  which equals 140.

Arc BC equals  $2x$  or  $2(20)$  which equals 40.

Angle P intercepts both of these arcs and will equal to one-half their difference.

$$\text{Angle P} = \frac{1}{2} (\text{AC} - \text{BC}) = \frac{1}{2} (140 - 40) = \frac{1}{2} (100) = 50$$

**ANSWER:  $\angle CPA = 50^\circ$**



- 22) At the right is your diagram. The diagram indicates that the three sections are equal. That is why I entered 120 as a central angle. Since 3 sections are equal we can divide 360 by 3 and determine that each central angles is 120. Now it's simply a matter of using the law of cosines to solve for "w".

$$a^2 = b^2 + c^2 - 2bc \cos C$$

In our case at the right it ends up being:

$$w^2 = 4^2 + 4^2 - 2(4)(4) \cos 120 \quad \cos 120 = -.5$$

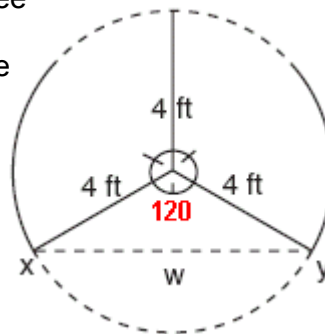
$$w^2 = 4^2 + 4^2 - 2(4)(4)(-.5) \quad \text{Simplify.}$$

$$w^2 = 16 + 16 - 32(-.5) \quad \text{Continue simplifying.}$$

$$w^2 = 16 + 16 + 16 \quad \text{Continue simplifying.}$$

$$w^2 = 48 \quad \text{Find square root of both sides (reject negative value)}$$

$$w = \sqrt{48} = 6.9 \text{ to the nearest tenth.}$$



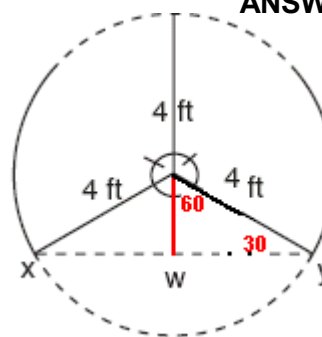
ANSWER: w = 6.9

Alternate method: Since the central angle is 120°, angles x and y will equal 30 degrees apiece for a total of 180°. If you drop an altitude, you end up bisecting the central angle and also side w. You therefore end up with a 30-60-right triangle. In such a triangle the following relationships exist:

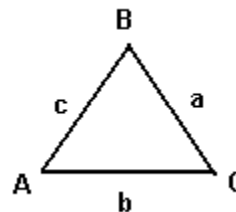
The side opposite the 30° angle will be equal in measure to half the hypotenuse. The side we are concerned with, the one opposite the 60° angle will equal half the hypotenuse times  $\sqrt{3}$ .

In the little triangle we formed at the right, the hypotenuse equals 4. The side opposite the 60° angle (which in this case will equal half of w) is one-half the hypotenuse times  $\sqrt{3}$ , or  $2\sqrt{3}$ .

The complete side w will equal twice this or  $4\sqrt{3}$ . Use your calculator and you will see that  $4\sqrt{3}$  equals 6.9 to the nearest tenth.



- 23) You are given information regarding triangle ABC. In general when labeling a triangle do it the way you see at the right. Angles are labeled using capital letters, while the sides are labeled with lower case letters. In addition, an angle and its opposite side are the same letter the only difference being that one is uppercase and one is lower-case.



One formula for finding the area of a triangle is  $K = \frac{1}{2} ab \sin C$ .

In essence to use this formula you are required to first know the measures of two sides and the included angle. You are given the measures of BC, AC and  $\cos C$ . Your first step is to determine  $\sin C$ .

$\cos C$  is 1/2. This means that **angle C is 60 degrees**. If you did not already know this you can figure it out using the  $\cos^{-1}$  to determine which angle has a cosine of one-half.

Next step is to use the area formula given above.

You know that the two sides measure 18, and 10, and that the angle is 60 degrees.

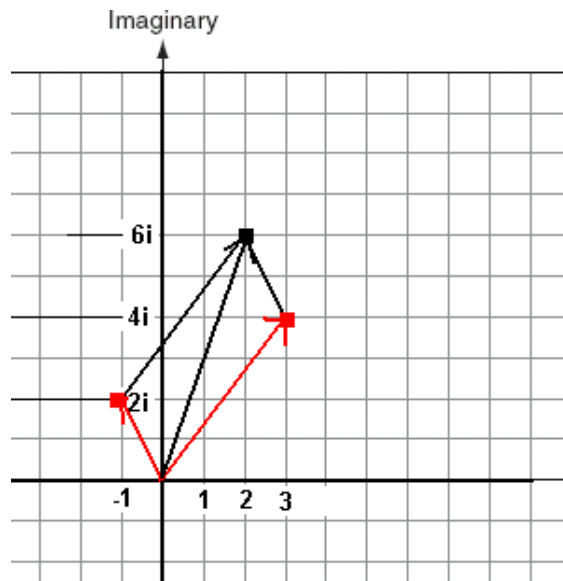
$$K = \frac{1}{2} (18)(10)(\sin 60) \quad \text{Use your calculator.}$$

$$K = 77.9 \text{ to the nearest tenth.}$$

ANSWER: 77.9 square units

- 24) Step one involves the graphing of the two points on the axis. As indicated, the x-axis is your real axis, and the y-axis is your imaginary axis. Graph the two given points. You see them in red at the right. Draw an arrow from the origin to each point. Next, complete a parallelogram. You see that in black at the right. Finally, draw a diagonal from the origin to the opposite vertex. This diagonal is known as the resultant and is the sum of the original two complex numbers. As you can see their sum is **2+6i**.

**ANSWER: 2+6i**



- 25) What you should understand about radian measure and its relationship with an intersected arc is that one radian measure means that the length of the intersected arc is equal in measure to the radius of the circle in question. Looking at the right, had the arc been equal to 4cm then the angle would have been 1 radian measure. However, the arc does not equal 4 it is a fractional part of 4. Had it been 2, the angle would have been 2/4 or 1/2 a radian.

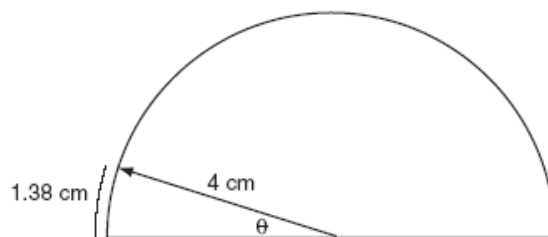
In our case the angle will be  $\frac{1.38}{4}$  or **.345 radians**.

The formula for the angle in radian measure is presented as follows in your textbook:

$$\theta = \frac{s}{r}$$

What this means is that the ratio of the arc length, s, to the radius, r, will be the radian measure of the central angle,  $\theta$ , that arc subtends.

**ANSWER: .345 radians**



- 26) This is a problem using the Binomial Theorem. Here is a brief explanation of the theorem. When raising a binomial to a power you will have numerical coefficients. If you are raising a binomial to the 3rd power, your coefficients will be  ${}_3C_0, {}_3C_1, {}_3C_2, {}_3C_3$ . This shows you that you will always have one more term than the number of the power. In the case above when raising to the 3<sup>rd</sup> power you will end up with 4 terms. If you are raising a binomial to the 5<sup>th</sup> power, as in our problem, you will have 6 terms, and their coefficients will be  ${}_5C_0, {}_5C_1, {}_5C_2, {}_5C_3, {}_5C_4, {}_5C_5$ .

Each term in the expansion will contain a numerical coefficient, an “x” factor, and a “y” factor. The sum of the powers of the “x” and “y” factors will always be the exponent to which you are raising the binomial. In our problem, the X-term is  $2x$  and the Y-term is  $-y$ , and the sum of their powers will always equal 5.

The exponent of the y-term will always match the subscript of the numerical coefficient. And once you know the exponent of the y-term you will automatically know the exponent of the x-term because both exponents have to add up to the exponent to which you are raising the binomial.

Let's begin. You are asked for the fourth term in the expansion. The fourth term will have a numerical coefficient of  ${}_5C_3 \dots {}_5C_0, {}_5C_1, {}_5C_2, {}_5C_3$

The exponent of the y-term will match this subscript of 3 and therefore be  $(-y)^3$  or  $-y^3$ .

The exponent of the x-term will be 2, since the x and y-exponents have to add up to 5 in our case.

The x-term will therefore be  $(2x)^2$  or  $4x^2$ .

Put all this together and you will have the final answer:

$${}_5C_3(4x^2)(-y^3) = 10(4x^2)(-y^3) = -40x^2y^3 \quad {}_5C_3 = 10$$

**ANSWER:  $-40x^2y^3$**

- 27) The first step requires you to factor the equation. (If you have difficulties, you can always use the quadratic formula.)

$$8 \cos^2 \theta - 2 \cos \theta - 1 = 0$$

$$(4 \cos \theta + 1)(2 \cos \theta - 1) = 0 \quad \text{Set each factor equal to 0 and solve for } \theta .$$

$$4 \cos \theta + 1 = 0 \quad \text{Subtract 1 from both sides.} \quad 2 \cos \theta - 1 = 0 \quad \text{Add 1 to both sides.}$$

$$4 \cos \theta = -1 \quad \text{Divide both sides by 4.} \quad 2 \cos \theta = 1 \quad \text{Divide both sides by 2.}$$

$$\cos \theta = -1/4 \quad \cos \theta = 1/2$$

What remains now is to determine the values of  $\theta$  for which cos will equal  $-1/4$  and  $1/2$ . Remember you want all the values between 0 and 180, including 0 and 180.

Use your  $\cos^{-1}$  key to determine when cosine will equal  $-1/4$ .  $\cos^{-1}(-1/4)$   
 To the nearest degree it is an angle of  $104^\circ$ . (Cosine will also be  $104.4775122$   
 negative in the third quadrant but that will be an angle greater than 180, so we reject it.

Now you can use the same method to determine the angle whose cosine is  $1/2$

That angle is  $60^\circ$ . Cosine will also be positive in the fourth quadrant, but that would be an angle greater than 180.

**ANSWER:  $60^\circ, 104^\circ$**

- 28) This problem deals with an exponential function where the initial amount increases by a constant percent of its previous value. **The initial value is 720,500.** That is the population of the city in 1980. It is also the answer to the first question asked. We know that this amount is increasing by a certain percentage each year. To determine that percentage is quite simple. In our problem, 1.022 is that growth factor. The growth factor always represents the original amount plus a constant percentage of that original amount. In the 1.022, the 1 represents the original amount. This leaves the .022 to represent the percent increase. Changed to a percent, .022 is 2.2%. **The 1.022 therefore represents a growth of 2.2% each year.**

The final question is when will the population of the city reach 1,548,800?  
Let's use the equation and solve for x using logarithms.

$$y = 720,500 (1.022)^x$$

$$1,548,800 = 720,500 (1.022)^x$$

$$\log (1,548,800) = \log (720,500 (1.022)^x)$$

$$\log 1,548,800 = \log 720,500 + x \log 1.022$$

$$\log 1,548,800 - \log 720,500 = x \log 1.022$$

$$\frac{\log 1548800 - \log 720500}{\log 1.022} = x$$

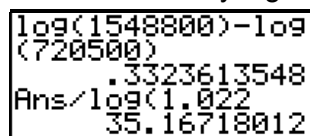
Substitute 1,548,800 for y

Use logarithms.

Use the laws of logarithms.

Subtract log 720,500 from both sides.

Divide both sides by log 1.022.



```

log(1548800)-log
(720500)
.3323613548
Ans/log(1.022
35.16718012
  
```

X, which represents the number of years in question, equals 35.16718012. Now add this number to 1980 and your answer will be the year the population will hit the required number.

$$1980 + 35 = \mathbf{2015}$$

**ANSWER:**    **720,500** represents the initial population in the year 1980.  
**1.022** represents a 2.2% growth rate.  
**2015** is the year the population will reach 1,548,800.

- 29) The patio in question measures 9 by 12 so its area is 9(12) or 108 square feet. Matt wants the new area to be twice what it is now. This means he wants the new area to be 108(2) or 216 square feet. He wants to increase the length and width by the same amount "x" to obtain this new area. You are asked to determine the value of x.

The new dimensions will be x+9 and x + 12. These are the two sides to be used to calculate the new area of 216. Set up your equation.

$$(x+9)(x+12) = 216 \quad \text{Multiply}$$

$$x^2 + 21x + 108 = 216 \quad \text{Subtract 216 from both sides setting the equation equal to 0.}$$

$$x^2 + 21x - 108 = 0 \quad \text{This is a quadratic formula in the form of } ax^2 + bx + c = 0$$

You can not factor the above equation using FOIL. Although factors of 108 are 9 and 12 which can get you the middle term of 21x, they will not yield a negative 108 when multiplied.

Use the quadratic formula. **In our equation, a = 1 b = 21 c = -108**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Substitute the values for a, b, and c}$$

$$x = \frac{-21 \pm \sqrt{21^2 - 4(1)(-108)}}{2(1)} = \frac{-21 \pm \sqrt{441 + 432}}{2} = \frac{-21 \pm \sqrt{873}}{2} \quad \text{Now let's use the calculator.}$$

It looks like there will be two values for x.

$$x = \frac{-21 + \sqrt{873}}{2} \quad \text{and} \quad x = \frac{-21 - \sqrt{873}}{2} \quad \text{Let's solve for the first value.}$$

$$x = \frac{-21 + \sqrt{873}}{2} = 4.27 \text{ to the nearest hundredth. } \left( \frac{-21 + \sqrt{(873)}}{2} \right) = 4.273286703$$

The second value will be

$$x = \frac{-21 - \sqrt{873}}{2} = -25.27 \quad \text{Reject this value as it will } \left( \frac{-21 - \sqrt{(873)}}{2} \right) = -25.2732867$$

result in negative dimensions.

**ANSWER: 4.27 feet**


- 30) You are asked for the power regression equation based on the data presented in the table, rounding all values to thousandths (three decimal places). Your first step involves the entering of the data into two tables.

Begin by hitting **STAT** **ENTER** Your screen should look like the one below to the left.

L1	L2	L3	1
-----	-----	-----	
L1(1) =			


Make sure there is no information in any of the columns. The next part is easy. Enter into the **L1** column the numbers 1,2,and3. Simply hit the first number 1 followed by ENTER. Then 2 followed by ENTER, and so on until you have entered all the data from 1 thru 3. These numbers represent the dates 2000, 2001, and 2002. Your screen will now look like the second one to the left.

L1	L2	L3	1
1	-----	-----	
2	-----	-----	
3	-----	-----	
L1(4) =			

Now use your cursor key  to move into the **L2** column. Enter the data found in the New Cases (y) column of the table. Begin with 457 followed by ENTER, and so on, until you have entered all three. Don't forget to hit ENTER. You will now have completed the L1 and L2 column's with the data that was presented in the table. For each L1 you should have an accompanying L2. Check the screen below to the left.

L1	L2	L3	2
1	457	-----	
2	369	-----	
3	353	-----	
L2(4) =			

You now need to find the power regression equation for this data. Hit the following keys:

**STAT**  Now scroll down the menu until you see the choice that reads PwrReg. It will be choice A Now hit: **ENTER**

PwrReg			
y=a*x^b			
a=451.4309303			
b= -.2429561915			

The problem states that coefficient and the base should be rounded to the nearest thousandth. **a=451.431 b= -.243**

**ANSWER:  $y=451.431x^{-.243}$**

$$451.431 * 8^{-.243} = 272.3582357$$

The last part of the problem asks you to use this equation to find the number of new cases, to the nearest whole number, for the year 2007. The main thing to realize here is that for the year 2007, x will be 8. Enter the problem into your calculator the way you see at the left, hit the ENTER key, and you will have solved for y. Your answer to the nearest whole number is 272.

**ANSWER:  $y=451.431x^{-.243}$  and 272**

- 31) You are told that the probability of a student being **on the team** is **.39**. This means that the probability of a student **not being on the team** is  $1 - .39$  or **.61**.

The first question is what is the probability that **at least 4** of the 5 students will be on the team. To determine this answer you have to calculate the probability of 4 students being on the team, then the probability of all 5 students being on the team (both of these scenarios satisfy the condition of at least 4), and then add their probabilities together for the final answer of at least 4 students being on the team.

$$\begin{aligned}
 & \text{P of 5 students (4 on team, 1 not on team)} \quad {}^5_1 nCr \quad 4 * .39^4 * .6 \\
 & {}^5C_4 \text{ P(4 on team)} \cdot \text{P(1 not on team)} \quad \quad \quad .0705599505 \\
 & {}^5C_4 (.39)^4 (.61)^1 \\
 & \mathbf{.0705599505}
 \end{aligned}$$

$$\begin{aligned}
 & \text{P of 5 students (5 on team, 0 not on team)} \quad {}^5_1 nCr \quad 5 * .39^5 * .6 \\
 & {}^5C_5 \text{ P(5 on team)} \cdot \text{P(0 not on team)} \quad \quad \quad .0090224199 \\
 & {}^5C_5 (.39)^5 (.61)^0 \\
 & \mathbf{.0090224199}
 \end{aligned}$$

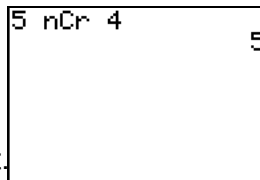
Now add together these two probabilities and round to the nearest hundredth.  
**.0705599505 + .0090224199 = .08** to the nearest hundredth.

The next question is what is the probability that **exactly one** student will **not** be on the team. This question can really be rephrased as what is the probability that **4** students will be **on** the team. This question has already been answered above. The answer is **.0705599505**. To the nearest hundredth it is **.07**.

**Answer: .08 and .07**

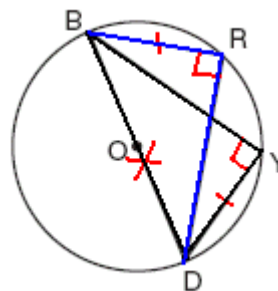
**PLEASE NOTE:**

Here is how you can do  ${}^5C_4$  on your calculator.



In our probability problem don't hit the enter key yet. Just continue until the whole calculation is complete, and then hit enter to obtain the final answer.

- 32) At the right is your diagram with some modifications that show how to answer it. You are asked to prove that the two overlapping triangles,  $\triangle RBD$  and  $\triangle YDB$  are congruent.



One way of proving this is by using the method of hypotenuse leg. You will show that the two triangles are right triangles, and that the hypotenuse and leg of one triangle are congruent to the hypotenuse and corresponding leg of the other triangle. This makes the two triangles congruent.

Begin by stating the given:

Given: Arc BR=70, Arc YD=70 Line segment BOD is the diameter of circle O.  
 Prove:  $\triangle RBD \cong \triangle YDB$

STATEMENTS	REASONS
1) Arc BR=70 and Arc YD=7	1) Given
2) BOD is a diameter	2) Given
3) $\angle R$ and $\angle Y$ are right angles	3) Angles inscribed in a semicircle are right angles.
4) $\triangle RBD$ and $\triangle YDB$ are right $\triangle$ 's	4) Definition
5) Chord BR (leg of $\triangle RBD$ ) $\cong$ chord YD (leg of $\triangle YDB$ ) leg $\cong$ leg	5) In a circle, congruent arcs have congruent chords.
6) $BD \cong BD$ hyp $\cong$ hyp	6) Reflexive property
7) $\triangle RBD \cong \triangle YDB$	7) Hypotenuse leg $\cong$ Hypotenuse Leg

- 33) This problem requires you to know how to factor and also to know that when dividing fractions you multiply by the reciprocal of the divisor (the quantity following the division symbol. Let's begin:

$$\frac{x^2 - 9}{x^2 - 5x} \cdot \frac{5x - x^2}{x^2 - x - 12} \div \frac{x - 4}{x^2 - 8x + 16} = \frac{x^2 - 9}{x^2 - 5x} \cdot \frac{5x - x^2}{x^2 - x - 12} \cdot \frac{x^2 - 8x + 16}{x - 4}$$

Now let's factor...

$$= \frac{(x + 3)(x - 3)}{x(x - 5)} \cdot \frac{x(5 - x)}{(x - 4)(x + 3)} \cdot \frac{(x - 4)(x - 4)}{x - 4}$$

On the next line you will see the canceling

$$= \frac{\cancel{(x + 3)}(x - 3)}{\cancel{x}(x - 5)} \cdot \frac{x\cancel{(5 - x)}}{\cancel{(x - 4)}(x + 3)} \cdot \frac{\cancel{(x - 4)}(x - 4)}{\cancel{x - 4}}$$

-1

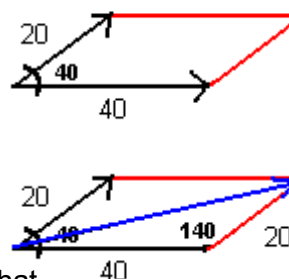
$$= \frac{x - 3}{-1} = -(x - 3) \text{ or } -x + 3 \text{ or } 3 - x$$

Cancel any numerator and denominator that are equal. That is why the  $x+3$ ,  $x$ , and  $x-4$ 's are crossed out. The  $x - 5$  and  $5 - x$  reduce to  $-1$ . You can put the  $-1$  on top or bottom. Wherever everything is canceled it is as if there is a 1 in that spot.

Rewrite what is left and simplify.

**ANSWER:  $-(x - 3)$  or  $-x + 3$  or  $3 - x$**

34) The diagram that I drew at the right, can more or less represent the information given in this problem. A force of 20 pounds and 40 pounds are acting on an object. The angle between the two forces is 40°. You are asked to find the magnitude of the resultant to the nearest tenth of a pound. The second diagram depicts the resultant in blue. I almost forgot to mention the fact that in order to solve this problem you first complete a parallelogram. By doing that you can now use the Law of Cosines to solve the magnitude of the resultant which happens to be the longer diagonal in this parallelogram. You also know that the larger angle will be 140°, as consecutive angles of a parallelogram are supplementary. Now whenever you are given two sides and an included angle, you can solve for the third side--the side opposite the given angle. The Law of Cosines states:



$$a^2 = b^2 + c^2 - 2bc \cos A$$

a, b, and c will always be 3 sides of a triangle, and A will be the angle between sides b and c. We want to figure out side A, so we can set up the equation as follows:

$$a^2 = 40^2 + 20^2 - 2(40)(20) \cos 140$$

$$a^2 = 1600 + 400 - 1600 \cos 140$$

$$a^2 = 2000 - 1600 \cos 140$$

$$a^2 = 3225.671109$$

$$a = 56.8 \text{ to the nearest tenth}$$

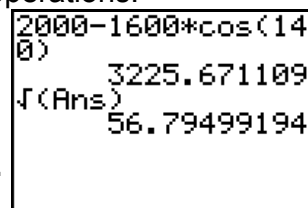
**The magnitude of the resultant to the nearest tenth is 56.8.**

Complete the indicated operations.

Continue simplifying.

Use your calculator.

Find square root.



With this information, you can now figure out the angle I have marked with an x in the diagram at the right. It is the angle between the resultant and the larger force. We can use the Law of Sines. This law states that in a triangle we can set up proportion between the sides of triangles and the sine of the

angles opposite these sides.  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Using the final diagram you see at the right, we can set up the following proportion:

$$\frac{20}{\sin X} = \frac{56.8}{\sin 140}$$

cross multiply

$$56.8 (\sin X) = 20 (\sin 140)$$

Divide both sides by 56.8.

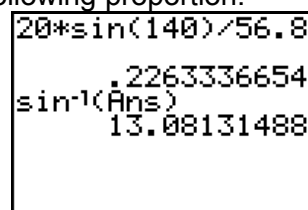
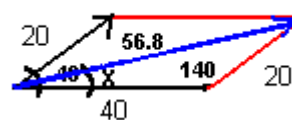
$$\sin X = \frac{20(\sin 140)}{56.8}$$

Use calculator:

$$\sin X = .226336654$$

Use  $\sin^{-1}$  key to solve for angle X.

$$X = 13^\circ$$



**ANSWER: The magnitude of the resultant to the nearest tenth is 56.8**

**The measure of the angle between the resultant and the larger force is 13, to the nearest degree.**