ANSWERS MATH B – January 27th, 2006

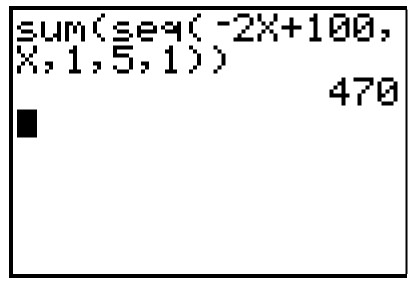
1) This problem asks you to find the value of $\sum_{n=1}^{5}(-2n + 100)$. This summation problem requires you to evaluate the sum of all the values of (-2n + 100) as the value of n goes from 1 thru 5.

$-2(1) + 100 = -2 + 100 = 98$
$-2(2) + 100 = -4 + 100 = 96$
$-2(3) + 100 = -6 + 100 = 94$
$-2(4) + 100 = -8 + 100 = 92$
$-2(5) + 100 = -10+ 100= 90$

The sum of all these values would be 98+96+94+92+90 = **470**

Therefore, $\sum_{n=1}^{5}(-2n + 100) = 470$ **ANSWER: (3)**

You can actually do summation problems using your TI-83 graphing calculator. Here is a screen capture of this problem and the answer. What it does is get the sum of the sequence that you enter. Here are the instructions to enter some of those key strokes.

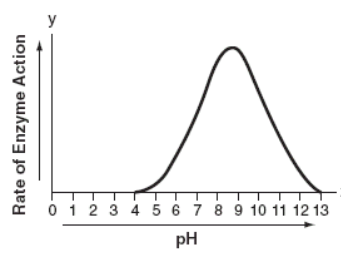


To enter the **sum** function: Hit 2nd STAT This accesses the LIST menu. Your screen will show the following drop-down menus: NAMES OPS MATH
Move your cursor over MATH and then hit the 5 key.
The 5 key will put the **sum(** function on your screen.
To enter the **seq** function: Follow the same instructions as above but this time don't select the MATH drop-down menu. Move your cursor over the OPS selection and then hit the 5 key. This will enter the **seq(** function on your screen. It is easier to enter the letter x on your calculator screen than using the ALPHA key to enter the letter N, so I used X above. To enter an X simply hit the X,T, θ,n key , and to enter a -2 remember to use the **(-)** key.

As you see on the screen capture, the answer is 470.

(Remember that after you've hit the 2nd function key you are really accessing the function of what is in yellow above the key you will hit next. What this means, for example, is that when you hit 2nd followed by STAT, you are really hitting the LIST key.)

2) You are presented with the graph at the right and asked for the domain of its function. Simply stated, the domain is all possible x-values, while the range is all possible y-values. As you can see on the graph, the lowest x-coordinate is 4 while the highest is 13. The answer is therefore: **$4 \le x \le 13$** **ANSWER: (1)**



3) An inverse variation defines a relationship where as one variable increases the other decreases proportionately so that their product is unchanged. An example would be Distance = Rate X Time. Assuming that Distance is the constant (product), then Rate and Time would have to vary inversely. As your rate (speed) would increase, the Time it takes to travel the constant Distance would decrease, and if your rate would decrease then the Time traveled would increase. The graph pictured for choice 1 is the only one that shows an inverse relationship. As Soil Permeability increases, Runoff decreases. **ANSWER: (1)**

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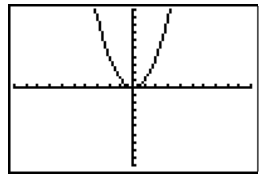
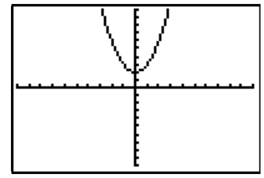
4) You are told that on a standardized test, a score of 86 falls exactly 1.5 standard deviations below the mean. In addition, you are told that the standard deviation for this test is 2. You are asked for the mean score for this test. Since the standard deviation in this problem is given as 2, 1.5 standard deviations is 2 plus half of 2 which totals 3. **Add 3 to the 86 and you arrive at a mean of 89.** Subtract 3 from the mean of 89 and you will be back at 86 which is 1.5 standard deviations below the mean. **ANSWER: (4)**

5) The first screen capture at the right is the graph of $y = x^2$. The one to its right is the graph of $y = x^2 + 2$. You immediately see that the second graph has moved 2 units up in the "y" direction. This is indicated as $T_{0,2}$ which represents a translation that involves a move of 0 units in the x direction and +2 units in the y-direction. In general, a function f(x) + c will move the function f(x) up "c" units-- in other words, "c" units the +y direction. **ANSWER: (2)**





6) You are asked for the period of the curve modeled by the equation y = 3 sin 4x. Given the equation y = a sin bx you should know the following. **lal** (absolute value of "a") represents the amplitude, **lbl** represents the frequency, while 2**B**/b will represent the period of the curve. In our problem, b= 4 so the period will be 2**B** divided by 4. In this case, 2**B** means 2**B** radians which equals 2(180) degrees or 360 degrees. So, to continue, 2**B** divided by 4 will be 360 divided by 4 which equals 90 degrees. Keep in mind that when dealing with radian measure, **B** radians equals 180 degrees. Choice 2 which is **B**/2, equals 90 degrees and is therefore the answer. Another way of thinking about this is as follows. The period for the basic sine and cosine curve is 360 degrees. This means that the curve completes itself in 360 degrees and then begins repeating itself. The 4 in the equation y = 3 sin 4x tells you that the curve, in this case the sine curve, will repeat itself 4 times in 360 degrees. This means that it will complete itself once in 360/4 degrees or 90 degrees. **ANSWER: (2)**

7) You are given the equation $\sqrt{2x-1} + 2 = 5$ and asked to solve for x.

$\sqrt{2x-1} + 2 = 5$	Subtract 2 from both sides.
$\sqrt{2x-1} = 3$	Square both sides.
$2x - 1 = 9$	Add 1 to both sides.
$2x = 10$	Divide both sides by 2.
x = 5	

ANSWER: (3)

8) First multiply using any method you are familiar with.
(1 + cos x)(1 - cos x)
Using FOIL: The firsts are (1)(1) or **1**. The outers are (1)(-cos x) or **-cos x**. The inners are (cos x)(1) or **cos x**. The lasts are (cos x)(-cos x) or **$-\cos^2 x$**. So you now have:

$1 - \cos x + \cos x - \cos^2 x$ Simplify.

$1 - \cos^2 x$ One of the Pythagorean identities you are required to know is:
$\sin^2 x + \cos^2 x = 1$ If you now subtract $\cos^2 x$ from both sides, you have:
$\sin^2 x =$ **$1 - \cos^2 x$** This means that **$\sin^2 x$** can be substituted for **$1 - \cos^2 x$**

The answer is therefore **$\sin^2 x$.** **ANSWER: (3)**

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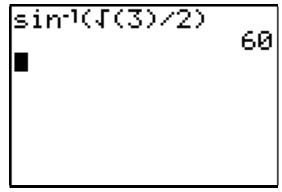
9) You are given that θ is a positive acute angle. In addition you are told that $\sin 2\theta = \frac{\sqrt{3}}{2}$. At this point you have enough information the determine the measure of θ. What you are now trying to figure out is "the sine of what angle will equal $\frac{\sqrt{3}}{2}$." Use the sin-1 key on your calculator to determine this value of θ. At the right is a screen capture of the TI-83 Plus showing the answer. (Make sure that you put the parentheses in the proper spots or you will end up with a wrong answer.)



You now know that in our problem, the measure of 2θ is 60 degrees. Which means that θ equals 30 degrees What remains is to substitute this value in the expression you are given:

$(\cos\theta + \sin\theta)^2$ Substitute

$(\cos 30 + \sin 30)^2$ Simplify By now you are expected to know that $\cos 30 = \frac{\sqrt{3}}{2}$ and that $\sin 30 = \frac{1}{2}$ or .5, which is actually what you determined using your calculator.

$\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)^2 = \left(\frac{\sqrt{3}+1}{2}\right)^2$ To arrive at your answer, you now have to square the numerator and the denominator. First lets square the numerator $\sqrt{3}+1$. Treat it the same way as multiplying a binomial by a binomial. As in the previous problem, we can use FOIL again. $(\sqrt{3}+1)(\sqrt{3}+1) = 3 + \sqrt{3} + \sqrt{3} + 1 =$ **$4+2\sqrt{3}$**. This will be your numerator. The denominator is simply 2 squared or **4**. What you now have is:

$\frac{4+2\sqrt{3}}{4} = \mathbf{1 + \frac{\sqrt{3}}{2}}$ (4/4 = 1 and 2/4= 1/2) **ANSWER: (2)**

10) You are asked for the solution of the following inequality $|y + 8| > 3$. This inequality is really stating the following: $y + 8 > 3$ and $-(y + 8)$ will also be greater than 3. Let us solve both of these inequalities.

$y + 8 > 3$ Subtract 8 from both sides.
$y > -5$

$-(y+8) > 3$ Distribute the negative sign.
$-y - 8 > 3$ Add 8 to both sides.
$-y > 11$ Divide by -1(symbol changes direction
$y < -11$

Your answer is that **$y > -5$ or $y < -11$** **ANSWER: (1)**

11) In order to complete this problem you have to know some basic rules regarding logarithms. The one that we will use first in this problem is the Product Rule for Logarithms. That is $\log[(a)(b)] = \log a + \log b$. This problem presents us with the equation: $v = 1087\sqrt{\frac{T}{273}}$. At this point you can use the Product Rule of Logarithms and rewrite the equation as :

$\log v = \log 1087 + \log\sqrt{\frac{T}{273}}$. Now we can use the Quotient Rule for Logarithms which states $\log\left(\frac{a}{b}\right) = \log a - \log b$. Getting back to our problem, this means that $\log\frac{T}{273} = \log T - \log 273$.

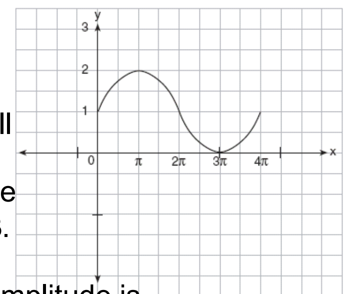
But wait! Our problem does not state $\frac{T}{273}$ but $\sqrt{\frac{T}{273}}$. Don't worry! There is another rule to help us out. But first let's rewrite $\sqrt{\frac{T}{273}}$ as $\left(\frac{T}{273}\right)^{\frac{1}{2}}$. Now we can use the Power Rule for Logarithms which states $\log a^b = b \log a$. Using this rule (combined with the Quotient Rule),

$\log \left(\frac{T}{273}\right)^{\frac{1}{2}} = \frac{1}{2} \log \frac{T}{273} = \frac{1}{2} (\log T - \log 273)$. Now we can answer the question as to which expression is equivalent to log v.

$v = 1087 \sqrt{\frac{T}{273}}$ Take the log of both sides using the rules for logarithms.

$\log v = \log 1087 + \frac{1}{2} (\log T - \log 273) =$ $\mathbf{\log 1087 + \frac{1}{2} \log T - \frac{1}{2} \log 273}$ **ANSWER: (3)**

12) You should definitely recognize the pattern at the right as being that of a sine curve. You should also immediately notice that it is above the x-axis. Instead of having a point going through the origin, it has a point 1 unit above the origin. If you still recall problem number 5 on this regents you will understand that this curve is a sine function +1, and has therefore been translated 1 unit up. The usual period for the sine curve is 2**B**, but this curve completes itself in 4**B**. This means that it completes only half of itself in 2**B**. This would account for an equation like y=a sin 1/2 x. Only choice 1 matches this. The amplitude of this curve, by the way, is not 2 but 1. The amplitude is found by taking the absolute value of the average of the highest point and the lowest point. Regarding our curve this would be the absolute value of 1/2 (2-0) which equals 1.



$\mathbf{y = \sin\left(\frac{1}{2}x\right) + 1}$ **ANSWER: (1)**

13) To simplify the expression $\frac{5}{\sqrt{5}-1}$, multiply its numerator and denominator by $\sqrt{5}$ +1, the conjugate of the denominator. **(Remember that $\sqrt{5} \cdot \sqrt{5} = 5$)**

$\frac{5}{\sqrt{5}-1} \cdot \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{5(\sqrt{5}+1)}{5-1} = \mathbf{\frac{5\sqrt{5}+5}{4}}$ **ANSWER: (2)**

14) This problem is asking you to determine the nature of the roots of the equation $2x^2 - 5 = 0$. To answer this question without solving the equation requires you to determine the value of the discriminant. The discriminant is represented by the expression $b^2 – 4ac$. The standard form of a quadratic equation is $ax^2 + bx + c = 0$.

In our equation, a=2, b=0 (there is no x term), and c= -5. Let us substitute these values for $b^2 – 4ac$.

$b^2 – 4ac$ Substitute.

$0^2 - 4 (2) (-5)$ Simplify.

0 + 40

40

40 is a positive integer and is not a perfect square. Had the result of the discriminant been negative then the roots of the given equation would have been imaginary. Had the result been 0, then the roots would have been real, rational and equal. Had the result been a perfect square then the roots would have been real, rational, and unequal. Since 40 is positive and not a perfect square, the roots of the given equation will be real and irrational. **ANSWER: (4)**

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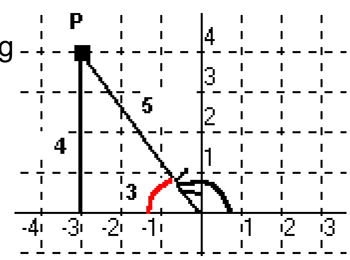
15) At 2:00 p.m. the minute hand of the clock would be on the 12, while the hour hand would be on the 2. A hands of a clock move in a circle, completing 360° each complete revolution. There are 12 divisions (numbers) on a clock. 360 divided by 12 is 30. This means that a movement by a clocks hand from one number to the next forms an angle of 30°. Given our problem, the angle formed by one hand on the 12 and the other on the 2, the angle formed would be 2(30) or 60°.
Now to convert from degrees to radian measure, divide the degree measure by 180 and tag on the pi symbol. $\frac{60}{180} = \frac{1}{3}$ This means that $60^\circ = \frac{1}{3}$**B** or as generally written, **B**/3. **ANSWER: (2)**

16) The diagram pictured at the right shows point P(-3,4). The arrow starting in quadrant I shows the angle drawn in standard position and terminating in quadrant II. The angle shown in quadrant II is that angle coterminal to the angle in question. You are asked to determine the value of sin θ.
$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$. In this case it is: $\mathbf{\frac{4}{5}}$ **ANSWER: (3)**

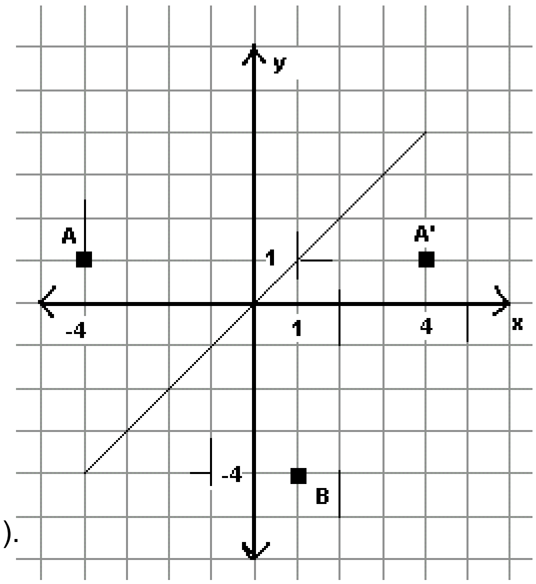


17) To begin this problem you first have to understand fractional exponents. You should know that $x^{\frac{1}{3}} = \sqrt[3]{x}$ and $x^{\frac{2}{3}} = \sqrt[3]{x^2}$ So whenever you see a fractional exponent, remember that the top number is the power and the bottom number is the root. In this problem, one of the factors is $\sqrt[3]{m^4}$ This can be written using a fractional exponent as $m^{\frac{4}{3}}$. The problem can now be rewritten as $\left(m^{\frac{4}{3}}\right)\left(m^{-\frac{1}{2}}\right)$ Now recall that when multiplying variables with exponents, the exponents are added and the base remains the same. 4/3 + (-1/2) = 8/6 - 3/6 = 5/6 Therefore:

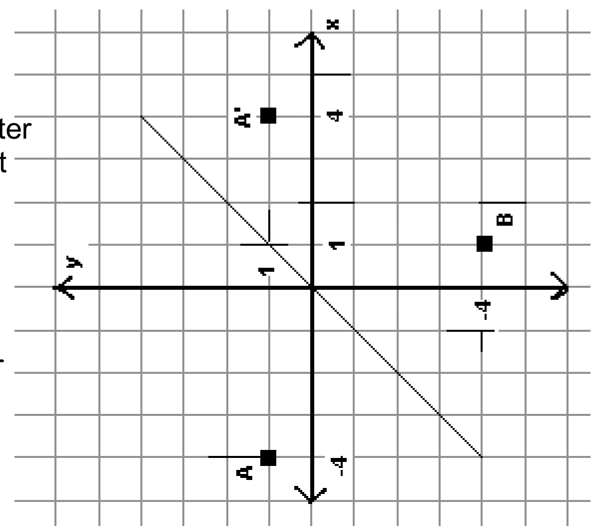
$\left(m^{\frac{4}{3}}\right)\left(m^{-\frac{1}{2}}\right) = m^{\frac{5}{6}}$ This converts to choice 4 which is: $\sqrt[6]{m^5}$ **ANSWER: (4)**

Problem 18 begins on the next page.

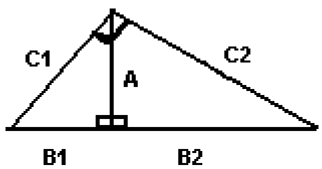
18) This problem asks you to do two transformations-- one following another. This is known as a composition of transformations. In this problem you are presented with the composite transformation $R_{90^\circ} \circ r_{y=x}$.
The above requires you to first reflect the given point A'(-4,1) through the line y=x, and then follow that image with a 90° rotation about the origin.
At the right you see point A. You also see a diagonal line running through the origin. That line would be where y=x. (At any point along that line the x and y-coordinates would be equal.) It is easy to see that point A reflected through that line would be the point I labeled B. This means that the image of point A after a reflection through the line y=x is B. You now have to rotate this point B 90° counter clockwise about the origin. Point B will end up where I indicated the point A'. That point is (4,1).



In general, when a point is reflected through the line y=x simply switch its x and y-coordinates. In our case you see the reflection of A(-4,1) through the line y=x as being the point B(1,-4).
Now see if you can follow the following method on how to determine the image of a point after a rotation about the origin. In our case you were required to do a 90° rotation, which is a 1/4 turn counter clockwise Take your sheet of graph paper and rotate it the required number of degrees. In this case, you can see what a 90° rotation would look like at the diagram at the right. Look at the first graph above and see where point B is, and see where it ends up in the second diagram after the rotation. Now imagine that the coordinate axis has been relabeled and the former y-axis is now the x-axis. The coordinate of point B would then be (4,1). Move the graph paper back to its original position and plot the point (4,1). That is the new image after the rotation. **ANSWER: (4)**

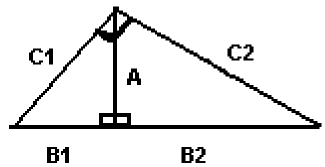


19) In order to understand this problem you should be aware of the three, actually two, possible proportions that can be set up when you are presented with a right triangle that has an altitude drawn to its hypotenuse.



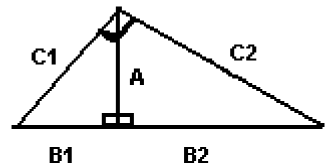
$$\frac{B1}{A} = \frac{A}{B2}$$

The altitude A is the mean proportional between the two segments of the hypotenuse.



$$\frac{(B1+B2)}{C1} = \frac{C1}{B1}$$

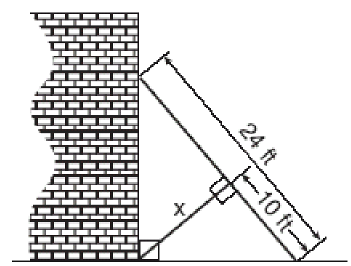
The leg, in this case C1, is the mean proportional between its projection on the hypotenuse, and the complete hypotenuse.



$$\frac{(B1+B2)}{C2} = \frac{C2}{B2}$$

The leg, in this case C2, is the mean proportional between its projection on the hypotenuse and the complete hypotenuse.

In this problem you are presented with the diagram at the right. You can immediately see that it is that of a right triangle that has an altitude drawn to its hypotenuse. In addition, you are given no information about the legs. That means that you will be using the first proportion above-- that the **altitude is the mean proportional** between the two segments of the hypotenuse. Since the complete hypotenuse is 24 and **one segment is 10**, the **other segment is 14.** The answer is therefore choice (1) which shows the altitude as being the mean proportional between the two segments.



ANSWER: (1)

20) The equation of a circle whose center is at the origin is given as $x^2 + y^2 = r^2$, where r is the radius. $(x - Xc)^2 + (y - Yc)^2 = r^2$, where Xc is the x-coordinate of the circle's center, and Yc is the y-coordinate of the circle's center, and r is still the radius. In other words, if the center of a circle is (2,5) and its radius is 4, then its equation would be $(x - 2)^2 + (y - 5)^2 = 4^2$
If its center would instead be (-2,5) then its equation would be $(x + 2)^2 + (x - 5)^2 = 4^2$. In this problem, you are presented with the equation $(x-2)^2 + (y+3)^2 = 100$. **Its center would be (2,-3).** (Its radius incidentally would be 10.) The point (2,-3) is in quadrant IV. **ANSWER: (4)**

21) This problem falls under the topic of composition of functions. You are presented with two functions. $f(x)= 5x^2 - 1$ and $g(x) = 3x - 1$ You are asked to find g(f(1)) One important thing to remember is to work from right to left. This means you are to first find f(1), and then use that answer on function g.
Let's find f(1)

$f(x)= 5x^2 - 1$ Substitute the value of 1 for x.

$f(1)= 5(1)^2 - 1 = 5(1) - 1 = 5 - 1 = 4$

Now you use your answer of 4 to find g(4)

$g(x) = 3x - 1$ Substitute 4 for x.

$g(4) = 3(4) - 1 = 12 - 1 = 11$ **ANSWER: 11**

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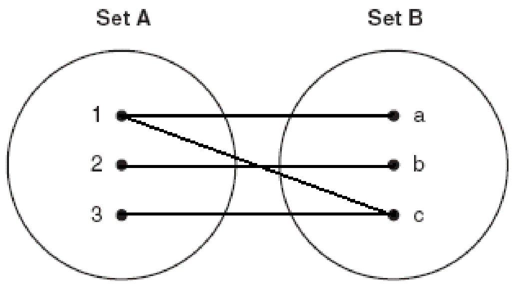
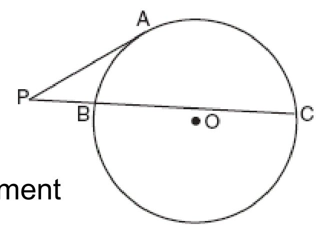
22) You are asked to draw a mapping of a relation from set A to set B that is not a function. The set you are starting from is your domain, and the set you are going to is your range.



The diagram to the left shows a relation that is not a function. **In order for a relation to be a function, each member in the domain can yield only one member in the range. To the left you see that 1 maps on "a" and also on "c". It is therefore not a function.**

23) You are presented with the diagram at the right and asked to find PA. PA is the tangent to the given circle. To solve this problem you have to know the theorem that states that when a tangent and secant segment intersect at a point outside of a circle (in our case, point P), then the square of the length of the tangent $(PA)^2$, will equal the product of the length of the secant segments (PBC) and the external secant segment (PB). In other words, $(PA)^2 = (PBC)(PB)$



PA is unknown so we will call it X. PB = 4. PBC=PB+BC=4+12= 16

$(PA)^2 = (PBC)(PB)$ Substitute

$X^2 = (16)(4)$ Simplify

$X^2 = 64$ Find square root of both sides.

$X = \pm 8$ Lengths cannot be negative so we use only the positive answer.

PA = 8

24) When two variables vary inversely, their products will always equal a constant. In our case that constant will be (4)(55) or 220. This is actually the formula you once learned that Rate times Time equals Distance or RT=D. In this problem, by multiplying the rate and time you figure out the distance 220 which is the constant. It is called a constant because it remains the same. Now the question is what happens if you are driving 50 miles an hour. The distance you will be traveling will remain that constant of 220 miles, but obviously the time will change because you are driving slower. What you have to figure out now is 50 times what will equal 220, or:

$50x = 220$ Divide both sides by 50.

$x = 4.4$ **It will take 4.4 hours.**

25) The probability that students **DO** drink coffee is given as **2/3**. This means that the probability that a student **DOES NOT** drink coffee is **1/3**. If **six students** are selected, what is the probability that **exactly two** of them drink coffee? The first part to solving this question requires the answer to ${}_6C_2$, because you are selecting 2 students out of a possible 6 students. The answer is then multiplied by the probability of a student drinking coffee to the second power, because we are looking for the probability of 2 students drinking coffee. This in turn will be multiplied by the probability of a student not drinking coffee raised to the 4^{th} power because out of the six students selected 4 will not be drinking coffee (exactly 2 will). You now have:

${}_6C_2$ (P of students drinking coffee)^2 (P of student not drinking coffee)^4.

$({}_6C_2)\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4$ This problem continues on the next page…..

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$_6C_2 = \frac{6P2}{2!} = \frac{(6)(5)}{(2)(1)} = \frac{30}{2} = 15$ Using your calculator, these are the key strokes for $_6C_2$.

[6] [MATH] ◀ [3] [2] [ENTER]

$(_6C_2)\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 = 15 \cdot \frac{4}{9} \cdot \frac{1}{81}$ = .0823045267 **To the nearest four places: .0823**

As a fraction: $\mathbf{\frac{20}{243}}$

The first screen capture is how to get your decimal answer. Or to use the calculator completely look at the second screen capture. To convert a decimal to a fraction use the following key strokes:
[MATH] [ENTER] [ENTER]
The final screen capture shows what it would look like on your screen.

```
15*(4/9)*(1/81)
     .0823045267
```

```
6 nCr 2*(2/3)^2*
(1/3)^4
     .0823045267
```

```
6 nCr 2*(2/3)^2*
(1/3)^4
     .0823045267
Ans▸Frac
          20/243
```

26) Solve algebraically for x, given the equation:

$8^{2x} = 4^6$	Change to the common bases. ($8=2^3$ and $4=2^2$)
$(2^3)^{2x} = (2^2)^6$	Raising a power to a power…actually multiply the exponents.
$2^{6x} = 2^{12}$	Now, set the exponents equal to each other
$6x = 12$	Divide both sides by 6.
$\mathbf{x = 2}$	

27) You are presented with $f(x) = x^2 + 2x + 7$, and asked to solve the equation when $f(x) = 0$.
It is simply a matter of using the quadratic formula properly.

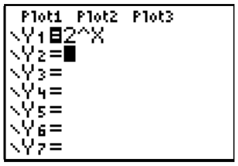
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ In our case, a=1 b=2 c=7

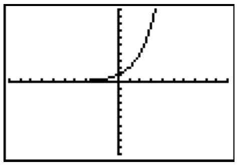
Substitute these values into the quadratic formula.

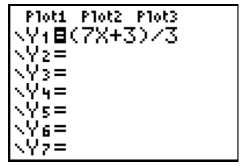
$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(7)}}{2(1)} = \frac{-2 \pm \sqrt{4 - 28}}{2} = \frac{-2 \pm \sqrt{-24}}{2} = \frac{-2 \pm \sqrt{-1}\sqrt{4}\sqrt{6}}{2} = \frac{-2 \pm 2i\sqrt{6}}{2} = \mathbf{-1 \pm i\sqrt{6}}$$

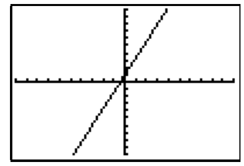
28) You are presented with two equations and asked to sketch their graphs on the same coordinate axis, as x increases from -3 to 4. In addition you are to identify and state the coordinates of all points of intersection. The equations are $y = 2^x$ and $3y = 7x + 3$. These equations are easy to input into your calculator so that you can have an idea of what their graphs will look like. In order to enter $3y = 7x + 3$ into y=EDITOR, by pressing the Y= key of your TI-83 Plus calculator, you will first have to transpose it so that it is $y = (7x + 3)/3$. You will also be able to use your calculator to assist you in graphing certain points by looking up their values using the TABLE key. The first screen capture below shows what your screen will look like after entering the equation $y = 2^x$. The second screen capture is what the graph will look like after you hit the GRAPH key. Your screen may look slightly different depending on how you have

your window set. The final two screen captures are what your screen will look like after entering the linear equation and its graph.

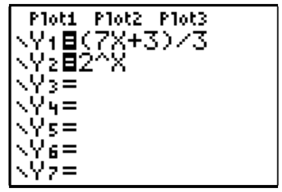


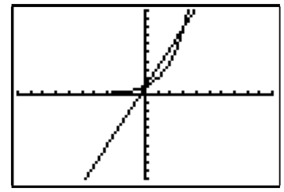




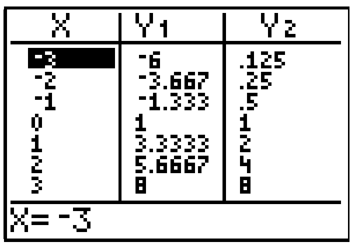


Here they are together on one axis You can see that they will intersect at two points. What you want to do now is generate a table of values so that you can sketch their graph on the given grid. In order to do that lets first use the TABLSET key to set up the table. Hit the **2nd** key followed by the **WINDOW** key



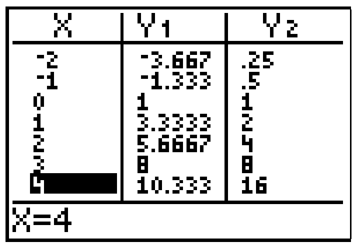


to access the **TBLSET** key. Below to the left is what your screen should like like after you've entered a -3 as the point where your table should start. Beneath that line, the **ΔTbl** indicates that the variables will be incrementing in units of one. Now you can hit the **2nd** key followed by the **GRAPH** key to generate a table of values that you can use to sketch your graph.



X	Y1	Y2
-3	-6	.125
-2	-3.667	.25
-1	-1.333	.5
0	1	1
1	3.3333	2
2	5.6667	4
3	8	8

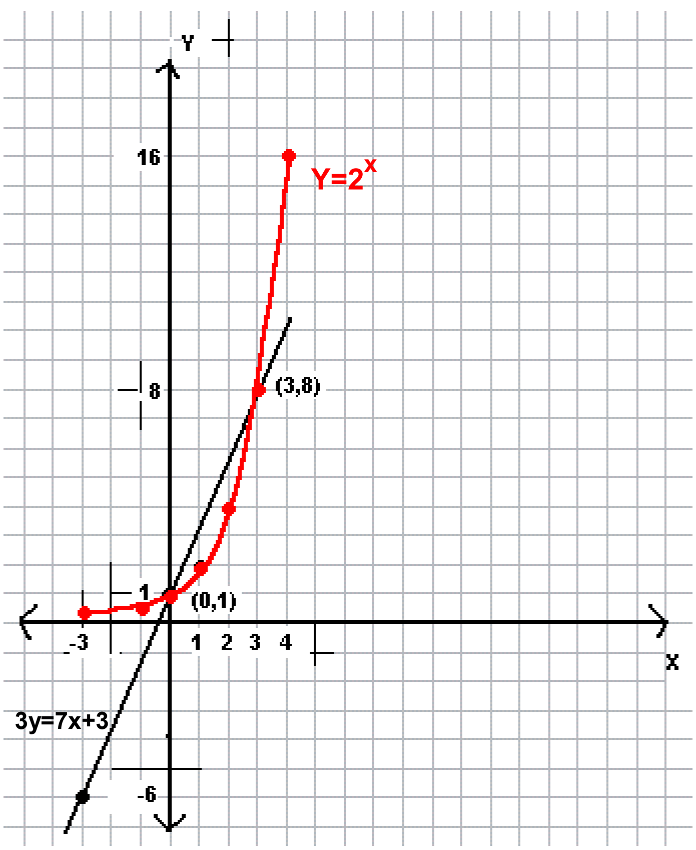
X= -3



X	Y1	Y2
-2	-3.667	.25
-1	-1.333	.5
0	1	1
1	3.3333	2
2	5.6667	4
3	8	8
4	10.333	16

X=4

The first screen capture shows the values of Y1 and Y2 from -3 to 3, so you have to scroll down the screen to see the values at 4. The Y1 values are the y-values for the equation 3y=7x+3 as x moves from -3 to 4, because that is the first equation typed into the y= editor. Y2 represents the y-values of the exponential equation as x moves from -3 to 4.



To the left is the sketch of the two graphs. The straight black line is the graph of 3y=7x+3. The curved red line is the graph representing 2^x. **Their points of intersection are indicated. They are the points (0,1) and (3,8)**

ANSWERS MATH B – January 27th, 2006

29) You are asked to simplify $\dfrac{\frac{1-m}{m}}{m-\frac{1}{m}}$ In simple English, one fraction on top of another fraction is a complex fraction. All you have to do is change the above into something that looks a bit more familiar: $\frac{1-m}{m} \div m-\frac{1}{m}$ Next, treat the $m-\frac{1}{m}$ as a mixed numeral. For example here is one way of changing a mixed numeral into an improper fraction. Imagine the mixed numeral $5\frac{1}{5}$. It is really $5+\frac{1}{5}$. To change it to an improper fraction all we do is 5 times 5, plus 1, over the current denominator. This results in $5+\frac{1}{5}$ being equal to $\frac{26}{5}$. Imagine having $5-\frac{1}{5}$. This can be treated in the same manner, but don't forget the minus: 5 time 5, minus 1, over the current denominator or $\frac{24}{5}$. Now back to our problem we have $m-\frac{1}{m}$. We can treat it the same way:

m times m, minus 1, over the current denominator, or $\frac{m^2-1}{m}$. So we now go back to our problem.

$$\frac{1-m}{m} \div m-\frac{1}{m} = \frac{1-m}{m} \div \frac{m^2-1}{m} = \frac{1-m}{m} \bullet \frac{m}{m^2-1} = \frac{1-m}{m} \bullet \frac{m}{(m+1)(m-1)} = \frac{-1}{m} \bullet \frac{m}{m+1} = \mathbf{-\frac{1}{m+1}}$$

A brief explanation as to where the -1 came from on the line above. (a-b) will cancel with another (a-b) as long as one is a numerator and the other is a denominator, but what about an (a-b) and a (b-a)? They are opposites, and their quotient will be -1. That -1 can then be placed either in the numerator or the denominator. That's what happened to the (1-m) and (m-1) above. The final answer is: $\mathbf{-\frac{1}{m+1}}$ **or** $\mathbf{\frac{-1}{m+1}}$ **or** $\mathbf{\frac{1}{-m-1}}$

30) You are told that the entrance to a tunnel is modeled by the function f(x) = 8 sin x +2. Let's do the second part of the problem first. You are asked for the maximum height of the entrance of the tunnel. f(x) = sin x is your basic sine curve that has an amplitude of 1. The curve represented by f(x) = 8 sine x has an amplitude of 8. The curve represented by our problem f(x) = 8 sin x +2 will, in addition to having an amplitude of 8, be translated in the y-direction a distance of two units. This will account for a **maximum height of 10.**

The first part of the problem asks you solve algebraically for all values of x in the interval $0 \le x \le$ **B**, where the height of the opening is 6. Set up your equation:

8 sin x +2 = 6 Subtract 2 from both sides.
8 sin x = 4 Divide both sides by 8.
sin x = 4/8 or .5

By now you should know that the sine of a 30° angle is .5. If you don't, then you can use the $\sin^{-1}$ on your calculator to compute the angle when you know its sine is .5. Now you are looking for the values of x in the interval 0 through 180. This means that you will have more than the one answer of 30°. The sine of an angle in the second quadrant is also positive. A reference angle in the second quadrant is found by subtracting from 180: 180-30=150. An angle of 150° will therefore also have a sine of .5. The problem asks you to express your answer in terms of **B**.

$\frac{30}{180} = \frac{1}{6}$ $\frac{150}{180} = \frac{5}{6}$ Your answers in terms of **B** are therefore: $\mathbf{\frac{\pi}{6}}$ and $\mathbf{\frac{5\pi}{6}}$.

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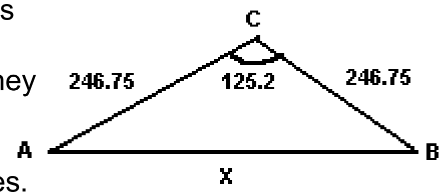
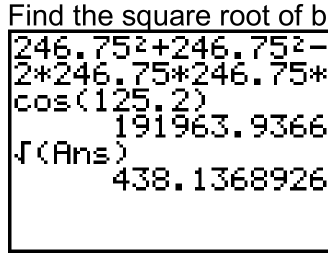
31) Imagine the diagram at the right as being a top view of the two walls that meet at an angle of 125.2°. Let one wall be AC and the other wall BC. You are told that the length of the two walls are equal. They are both 246.75 feet long. You are asked for the distance , to the nearest foot, between the ends of the walls that do not meet. Let's call that distance X. This problem involves the Law of Cosines.



$x^2 = (AC)^2 + (BC)^2 – 2(AC)(BC) \cos C$ In general, when you are presented with 2 sides of a triangle and the included angle and are asked to find the opposite side, or all 3 sides of a triangle and are asked to find one of the angles, you will use the Law of Cosines.

$x^2 = (AC)^2 + (BC)^2 – 2(AC)(BC) \cos C$ Substitute the given values.
$x^2 = (246.75)^2 + (246.75)^2 - 2 (246.75)(246.75) \cos 125.2$ Simplify.
$x^2 = 60885.5625 + 60885.5625 - 121771.125 \cos 125.2$ Continue simplifying.
$x^2 = 121771.125 - 121771.125 \cos 125.2$ Continue simplifying.
$x^2 = 121771.125 - 121771.125 (-.5764323162)$ Continue
$x^2 = 121771.125 + 79192.81163$ Continue
$x^2 = 191963.9366$ Find the square root of both sides.
$x = 438.1368925$
438 ft. to the nearest foot.

To the right is a screen capture of all the above work using your calculator. It makes life somewhat easier.



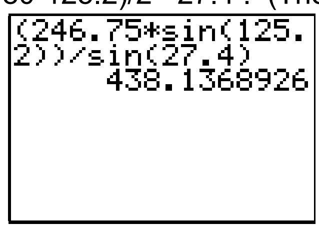
Alternate solution and easier using the Law of Sines:

$\frac{246.75}{\sin A} = \frac{X}{\sin 125.2}$ Cross multiply.

$(\sin A)(x) = (246.75) (\sin 125.2)$ Divide both sides by sin x.

$x = \frac{(246.75)(\sin 125.2)}{\sin A}$ Angle A will equal (180-125.2)/2 =27.4°. (The Δ is isosceles)

$x = \frac{(246.75)(\sin 125.2)}{\sin 27.4}$ Use your calculator:



x = 438 ft. to the nearest foot

32) You are presented with the following formula: $P = A(1.3)^{-0.234t}$

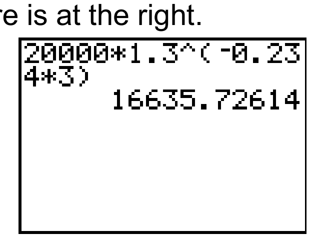
The first part of the problem requires you to substitute properly the given values. You are told that the current population is 20,000. This means that A which is equal to the initial population is 20,000. Your equation now looks like this: $P = 20,000(1.3)^{-0.234t}$

P is your final population which is what this formula calculates. You are told that you want to know the final population in 3 years. Since t = time in years, in this case we will use the value of 3. Your formula now looks like this: $P = 20,000(1.3)^{-0.234(3)}$

Now it's simply a matter of using your calculator. A screen capture is at the right.

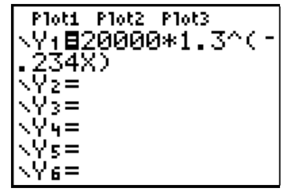
To the nearest hundred, the population 3 years from now will be 16,600 people.

The next part of the problem continues on the next page.

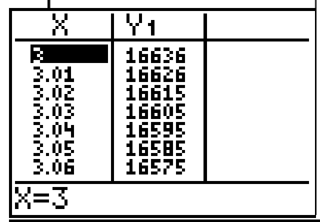


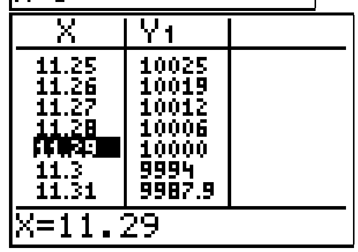
One way of doing this part is also through the use of your calculator. Imagine the formula as being:

$Y = A(1.3)^{-0.234x}$



Input this formula into your calculator using the Y= editor. Remember that A is 20,000. The screen capture for this is at the right. Now recall that you are looking to find for what value of X, in reality t, will Y, in reality P equal 10,000. In addition you will have to round your answer to the nearest tenth. Now let's use the **TBLSET** key to access the TABLE SETUP window to set up a table. Hit **2nd** followed by **WINDOW**. Just for the fun of it set your table start at 3 and since you will be rounding to the nearest tenth let the table increment by hundredths by setting the **ΔTbl** to .01. At the right is what your screen should look like. Now hit **2nd** followed by **GRAPH** to access the **TABLE** key. Look to right to see what it will look like. There's proof to your answer to the first part of the question. You see that when X=3 the population will be 16636, which we rounded to 16600. Now you are looking for a value for X where your Y1 column will read 10,000, which is half of the current population of 20,000. Use the ▼ key to scroll downwards where you see the X column increasing. Keep your finger on that downward scroll key until you see the number 10,000 appear in the Y1 column. You will notice that it will take quite a while. What you can do to speed things up is keep going back and forth to the TABLE SETUP window and keep changing the TblStart value until you see the Y1 column approaching 10,000. **At the right you see that when X=11.29 Y1=10,000. Rounding 11.29 to the nearest tenth gets you 11.3 which is the answer you are looking for.**





Alternate solution using logarithms:

You are asked, to the nearest tenth of a year, in how many years will it take for the population to reach half the present population. The present population is 20,000 so half of the population is 10,000. Set up your formula. For this problem, t, time, will be your unknown.

$P = A(1.3)^{-0.234t}$

$10{,}000 = 20{,}000(1.3)^{-0.234t}$ We can use logarithms to solve for t.

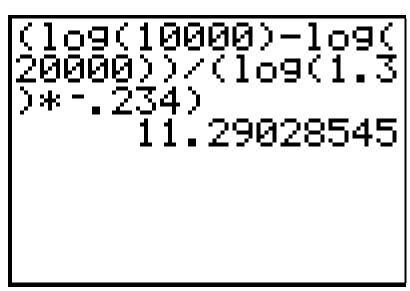
Look back at number 11 on this regents to refresh your memory regarding the logarithm rules.

$\log 10{,}000 = \log 20{,}000 + (-0.234t)(\log 1.3)$ Subtract log 20,000 from both sides.

$\log 10{,}000 - \log 20{,}000 = (-0.234t)(\log 1.3)$ Divide both sides by log 1.3.

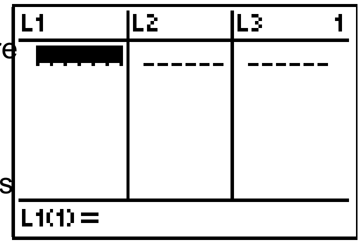
$\frac{\log 10000 - \log 20000}{\log 1.3} = -0.234t$ Divide by -.234 This is the same as multiplying by $\frac{1}{-.234}$

$\frac{\log 10000 - \log 20000}{\log 1.3(-.234)} = t$



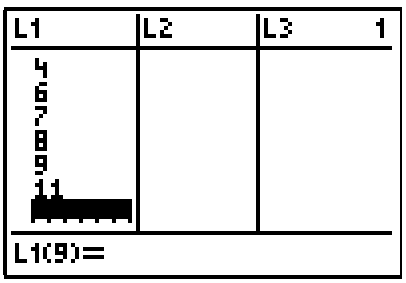
To the nearest tenth, t = 11.3 It will take 11.3 years. Be careful to include parentheses when entering data into your calculator. Also do not mix up the negative key which is on the bottom row of your calculator, enclosed in parentheses, and the minus key which is along the right column of your calculator.

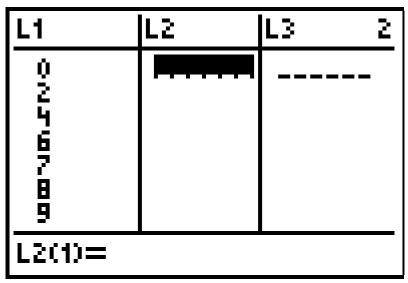
33) You are given a set of data and asked to find the linear regression for the set, rounding to four decimal places. Step number one involves the entering of this data into your calculator. Press **STAT** **ENTER** and you will see the following screen.

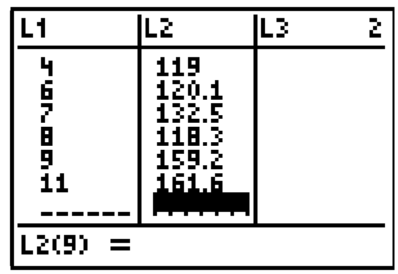


It is called the STAT LIST EDITOR screen. This is the screen where you will enter your data. You can enter all the data for **Number of Years** into L1 (list 1), and all the data for **Fireworks Usage** into L2 (list 2). (As indicated in the problem, the data representing the number of years will be your t, while the data representing fireworks usage will be your p.(Keep in mind that as far as your calculator is concerned, t will be represented by x, while p will be represented by y.

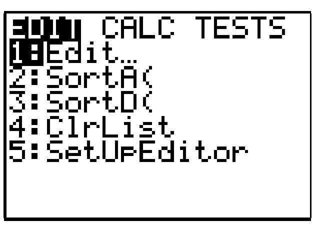
The simplest way to enter the data is to first enter the data for the number of years into L1. This is done by simply typing each number and then hitting the **ENTER** key. Below, to the left is a screen capture of what your screen will look like after you have entered all the data into L1. You can scroll up and down to make sure you entered all the data properly. Now hit the ▶ right scroll key once and you will immediately see the see the middle screen pictured below. You are now ready to enter the data for L2. Enter the data the same way you entered it into L1. Enter each number followed by the ENTER key. The final screen capture shows all the data entered into L1 and L2.

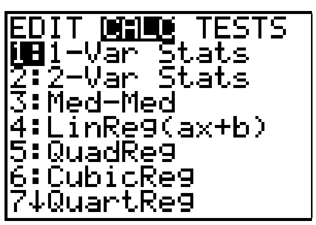


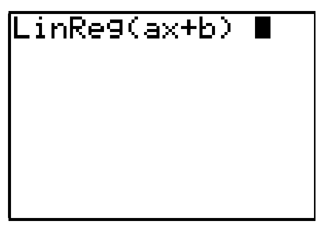


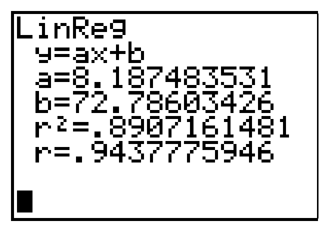


Once your data is entered, hit the following keys: **STAT** ▶ **4** **ENTER** Below is a screen capture for each one of the keys that you will be hitting:









The final screen above shows your answer, after you have hit the ENTER key. A linear equation is in the form of y= ax + b. You are shown the values for a and b and are expected to round them off to the four decimal places. In addition remember that the y shown on the screen is really your p and x is your t. **ANSWER: Your equation is: p= 8.1875t + 72.7860**

The next part of the question asks you to use the above equation to determine in what year fireworks usage would have reached 99 million pounds. Let's do some simple substitution. In the above equation, p represents pounds, so let us substitute 99 for p.

$99 = 8.1875t + 72.7860$	Subtract 72.7860 from both sides.
$26.214 = 8.1875t$	Divide both sides by 8.1875
$3.201709924 = t$	

t is a bit more than 3 which ends up being 1993

Whatever t equals is added to 1990 to determine the year in question. Therefore when t=3 the year is 1993.

The final part of the question can be calculated in the same manner as this one. Now you are asked to determine how many millions of pounds of fireworks would be used in the year 2008? First determine the value of t for the year 2008. Remember that 0 is1990, and 2 is 1992. Therefore, 8 would be 1998. To find t for 2008 we can do a bit of subtraction: 2008-1990 = 18. In essence then you are being asked to determine the value of p when t is 18

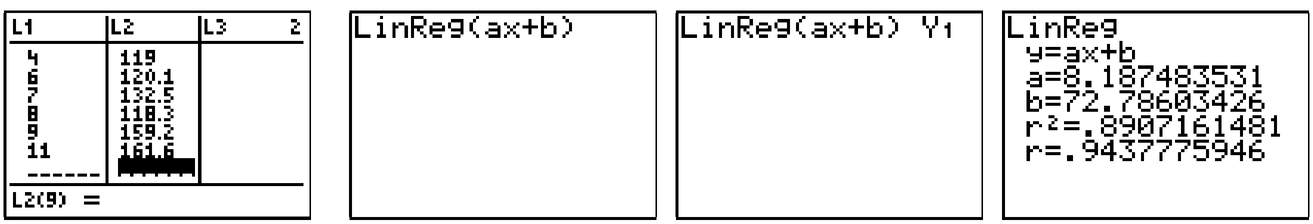
$p= 8.1875t + 72.7860$	Substitute 18 for t.
$p = 8.1875\ (18) + 72.7860$	Simplify
$p = 147.375 + 72.7860$	Simplify
$p = 220.161$	Round to nearest tenth.

ANSWER: 220.2 pounds

ALTERNATE METHOD USING YOUR CALCULATOR:

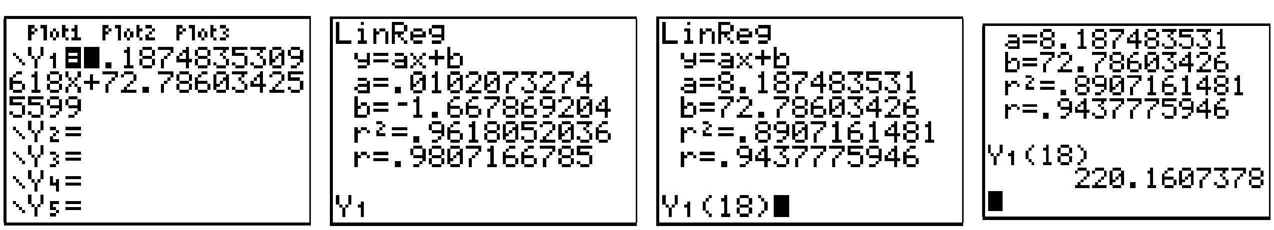
Here is an explanation of how you can do this question completely using your calculator. You have already typed in the data into L1 and L2. Now before you actually find the linear regression equation, it can automatically be entered into your y-editor as an equation. Here is how it is done. As mentioned before, we begin at the point after you have entered the data into L1 and L2. The first screen capture below to the right is what your screen looks like at that point. Now follow these keystrokes:

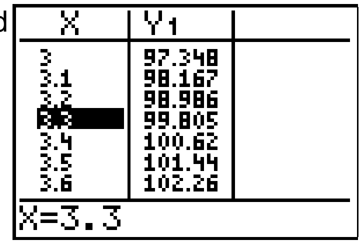
Press STAT ► 4 and **don't** hit ENTER. At this point your screen will look like the second screen below. Now continue with the following key strokes which will store the regression equation, in this case the linear equation, into the y-editor's y_1 VARS ► 1 1 . At this point your screen will look like the third screen below. The regression equation has been entered into y_1. Now hit ENTER and you will see the final screen.



If you were to now hit the **Y=** key, you would see the first screen below to your left. That is your answer to the first part of the problem. You would have to round the decimals to four places, though. Now here is the nice part about finding the second part of the question. (First exit the y-editor by hitting **2nd** followed by the **MODE** key. You will be back at the last screen above). Now, what would be the value of y when x is 18? (In our case it translates as what would be the value of "p" when "t" is 18?)

Hit the following keys. VARS ► ENTER ENTER . At this point you will see the second screen below. Now type in parenthesis your x value of 18. That is the third screen you see. Now hit **ENTER** and you will have your answer! (**Don't forget to round it to the nearest tenth: 220.2**).



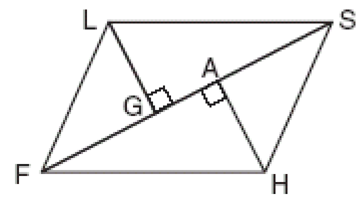


X	Y1
3	97.348
3.1	98.167
3.2	98.986
3.3	99.805
3.4	100.62
3.5	101.44
3.6	102.26

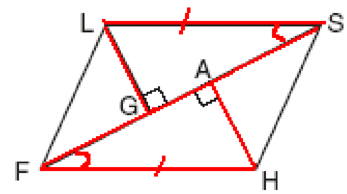
X=3.3

This was really the answer to the last part of the question. The second part was to determine in what year the fireworks usage would have reached 99 million pounds. Since the equation is already entered into the y= editor, you can use the **TABLE** key which automatically generates a table of values. This key is accessed by hitting **2nd** followed by **GRAPH**. You can then scroll either up or down until you see the Y1 values approaching 99. You can see on the screen at the right that Y1 hits 99 when X has passed 3. Add 3 to 1990.
The answer is therefore 1993.

34) You are presented with the diagram at the right and given the following: parallelogram $FLSH$, diagonal $\overline{FGAS}$, $\overline{LG} \perp \overline{FS}$, $\overline{HA} \perp \overline{FS}$
You are asked to prove ΔLGS ≅ ΔHAF



Below that diagram is our plan for proving triangles LGS and HAF congruent. You can see the two triangles in question. They are outlined in red. The plan is to prove them congruent by AAS. This is the theorem that : If two angles and the non-included side of one triangle are congruent to the corresponding parts of another triangle, the two triangles are congruent. As seen at the right, the two triangles in questions both contain right angles. In addition, ∠LSF and ∠HFA are alternate interior angles, which make them congruent. And finally, $\overline{LS}$ which is one side of the parallelogram is congruent to $\overline{FH}$ which is its opposite in the parallelogram. This results in AAS ≅ AAS.



STATEMENTS	REASONS
1. FLSH is a parallelogram	1. Given
2. $\overline{LS}$≅$\overline{HF}$ (S. ≅ S.)	2. Opposite sides of a parallelogram are congruent.
3. LS is parallel to HF	3. Opposite sides of a parallelogram are parallel.
4. ∠LSF and ∠HFA are alternate interior angles.	4. Definition of alternate interior angles.
5. ∠LSF ≅ ∠HFA (A. ≅ A.)	5. When two parallel lines are cut by a transversal, the alternate interior angles formed are congruent.
6. $\overline{LG} \perp \overline{FS}$, $\overline{HA} \perp \overline{FS}$	6. Given
7. ∠LGA and ∠HAF are right angles.	7. Perpendicular lines form right angles.
8. ∠ LGA ≅ ∠HAF (A. ≅ A.)	8. All right angles are congruent.
9. ΔLGS ≅ ΔHAF	9. AAS ≅ AAS