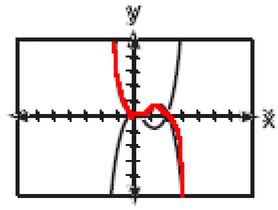
1) You are presented with the graph representing the equation y= f(x)
You are then shown 4 other graphs and asked to select the one that represents g(x) where g(x) = -f(x).
Since f(x)=y, you know that -f(x) will equal -y. In other words, the new graph will be a reflection of the original graph under the x-axis.
It is pictured in red at the right.
(x,y) → (x,-y) represents a reflection under the x-axis.
Choice 1 is a reflection of the original graph under the x-axis.



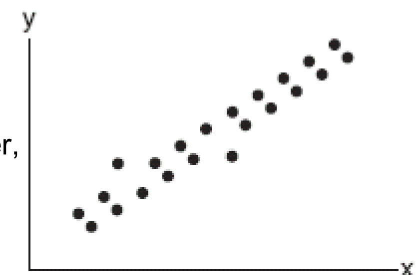
ANSWER: (1)

2) A probability of 5 times out of 7 will translate to $_7C_5$. (Order does not count.)
The probability that the song will be played is given as .38. You want that song played 5 times.
That will translate as $\mathbf{(.38)^5}$.
If the probability that the song will be played is .38, the probability that it won't we played is 1 - .38 or .62. If the song plays 5 times **it will not play 2 times**.
This will translate to $\mathbf{(.62)^2}$.
The final answer will therefore be: $\mathbf{{}_7C_5(.38)^5(.62)^2}$. **ANSWER: (2)**

3) The graph is that of a parabola which means it represents a quadratic equation. Choices 2 & 4 are both quadratic equations. The parabola opens downwards so the numerical coefficient of the x^2 term must be negative. Choice 4 is the ony correct answer. **ANSWER: (4)**

4) One equation for the area of a triangle, included on the formula sheet accompanying the regents, is k = 1/2 ab sin C where a and b are two sides of the triangle, and C is the included angle (the angle between these two sides).
In this problem, the two given sides are 8 and 12, and the included angle is 87.
It is now a matter of simple substitution: 1/2 (8)(12) sin 87. **ANSWER: (4)**

5) The correlation coefficient is a number between -1 and +1.
It denotes how strong a linear relationship exists between two variables. The closer the correlation coefficient is to +1, the more positive the correlation. The closer to -1, the more negative the correlation. If one variable, let's call it x, increases while the other, let's call it y, increases, you have a positive correlation (positive slope). If, however, as x increases y decreases, you have a negative correlation (negative slope). The scatterplot at the right represents a positive slope. It is also pretty much a straight line.



It therefore represents a correlation coefficient close to +1. Choice 4 is your best answer.
ANSWER: (4)

6) The easiest way of doing this problem is to simply use your graphing calculator. Input both equations and graph them. Then either use the **TRACE** and **CALC** function to determine the points of intersection, or use the **TABLE** function to determine the points of intersection. The points of intersection will be the solution sets.

On the next page is an algebraic solution.

$y = -x^2 + 5$
$y = -.5\,x^2 + 3$ Since both equations equal y, set them equal to each other.

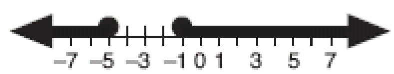
$-x^2 + 5 = -.5\,x^2 + 3$	Add $.5\,x^2$ to both sides. ($-x^2$ is really $-1x^2$)
$-.5\,x^2 + 5 = 3$	Subtract 5 from both sides.
$-.5\,x^2 = -2$	Divide both sides by -.5.
$x^2 = 4$	Find square root of both sides.
$\mathbf{x = \pm 2}$	

Now look through the choices for x=2 or x= -2. **Only choice 3 matches.** It shows x = -2.
(You can ascertain that y will equal 1 in this case).

$y = -x^2 + 5$	Substitute -2 for x	$y = -.5\,x^2 + 3$	Substitute -2 for x.
$y = -(-2)^2 + 5$	First square the -2.	$y = -.5\,(-2)^2 + 3$	Square the -2.
$y = -4 + 5$	Combine	$y = -.5\,(4) + 3$	Multiply.
$y = 1$		$y = -2 + 3$	Combine
		$y = 1$	

ANSWER: (3)

7) The inequality at the right shows x as being less than or equal to -5, **or** x being greater than or equal to -1.



You should immediately recognize that it is the graph of a disjunction (**or**). Choices 1,2, and 4 would result in disjunctions. Choice 1, however, would not include the endpoints. That leaves choices 2 and 4. Let's try them.
Remember that an absolute inequality can be broken down into two inequalities.

$|x + 3| \geq 2$

$x + 3 \geq 2$	$-(x + 3) \geq 2$
$\mathbf{x \geq -1}$	$-x - 3 \geq 2$
	$-x \geq 5$ Divide both sides by -1. Inequality symbol switches.
	$\mathbf{x \leq -5}$ **Choice 2 is the answer**

ANSWER: (2)

8) When presented with a number raised to a fraction, the numerator of the fraction will represent the exponent, while the denominator will represent the root index. For example

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

In our case there is the square root of A to the third. Here are two ways of writing this:
Note that when the root index is 2, it is not inserted.

$\mathbf{0.094\sqrt{A^3}}$ or $\mathbf{0.094\,A^{\frac{3}{2}}}$

ANSWER: (1)

9) This problem requires you to rationalize the expression $\frac{3}{\sqrt{6} - 1}$.

To do this you have to multiply both the numerator and denominator by the conjugate of the denominator. The conjugate of $\sqrt{6}$ -1 is $\sqrt{6}$ +1. To multiply them, use FOIL as in #19.

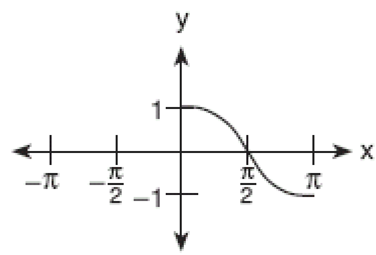
$\frac{3}{\sqrt{6} - 1}\left(\frac{\sqrt{6}+1}{\sqrt{6}+1}\right) = \frac{3\sqrt{6}+3}{6-1} = \mathbf{\frac{3\sqrt{6}+3}{5}}$ **ANSWER: (3)**

10) When a function is reflected in the line y=x, the following rule is observed:

$P(x,y) \rightarrow P'(y,x)$

Using this rule, the points (2,3), (4,7), and (-1,5) will become (3,2), (7,4) and (5,-1). **ANSWER: (1)**

11) To begin, the graph at the right is a cosine curve because at 0° we see a value of 1. The cosine of 0° is 1. The sine of 0° would have been 0.



A cosine curve can be represented as follows:

$y = a \cos bx$

The amplitude is represented by "a." At the right we see the graph with an amplitude of 1.

The period is represented by b. The period of the graph at the right is 1, since it would complete the regular cosine curve (one cycle) in 360 degrees.

The curve at the right is therefore represented by **y = cos x** **ANSWER: (1)**

12) All expressions except for choice 2 can be simplified.

$\frac{x}{x^2} = \frac{x}{x(x)} = \frac{1}{x}$ $\frac{x^2 - 4}{x + 2} = \frac{(x+2)(x-2)}{x+2} = x - 2$ $\frac{x^2 -6x +9}{x^2 -x-6} = \frac{(x-3)(x-3)}{(x-3)(x+2)} = \frac{x-3}{x+2}$

Choice 2 cannot be simplified. It is already is simplest form. **ANSWER: (2)**

13) You are asked to simplify the expression below:

$\frac{\frac{1}{3} - \frac{1}{x}}{\frac{3}{x} - 1}$

To this, the first step involves combining the numerator into one expression.
The second step involves doing the same to the denominator.
The final step will be to divide the numerator by the denominator.

The numerator consists of $\frac{1}{3} - \frac{1}{x}$. The lowest common denominator is 3x. So in order to be able to combine these two terms we have to change their denominators to equal terms--in this case 3x.

$\frac{1}{3}\left(\frac{x}{x}\right) - \frac{1}{x}\left(\frac{3}{3}\right) = \frac{x}{3x} - \frac{3}{3x} = \mathbf{\frac{x - 3}{3x}}$ **This is your new numerator.**

The denominator consists of $\frac{3}{x}$ - 1. Think of it as -1 + $\frac{3}{x}$. Now think back to how you learned to change a mixed numeral to an improper fraction. $2\frac{1}{3}$ which is really 2 + $\frac{1}{3}$ was changed by doing the following: 2 times 3 plus 1, over 3. The equivalent improper fraction was $\frac{7}{3}$.

(Continue on to the next page)

. Now back to our problem. We want to combine the denominator which we have rewritten as $-1 + \frac{3}{x}$

Here is how we do it. -1 times x, +3, over x.

This results is $\frac{-x+3}{x}$ **. This is your new denominator. (Note that -x+3 is the same as 3-x)**

You now want to divide the numerator by the denominator. Remember to change the division to multiplication of the reciprocal of the denominator.

$$\frac{x-3}{3x} \div \frac{3-x}{x} = \frac{x-3}{3x} \bullet \frac{x}{3-x}$$

(the x-3 and 3-x, are opposites, so they cancel to -1. The -1 can be put either in the numerator or denominator)

$= \frac{-1}{3x} \bullet \frac{x}{1} = \frac{-x}{3x}$ (the x's now cancel)$= \frac{-1}{3}$ **or** $-\frac{1}{3}$ **ANSWER: (2)**

14) You are looking for the expression equivalent to $1 + \sqrt{2} + \sqrt[3]{3}$
Let's try each of the choices:

(1) $\sum_{n=1}^{3} \sqrt{n} = \sqrt{1} + \sqrt{2} + \sqrt{3}$

(2) $\sum_{n=0}^{3} n^{n} = 0^0 + 1^1 + 2^2 + 3^3$

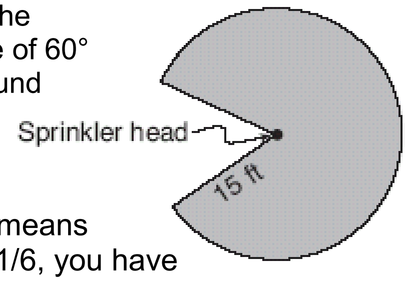
(3) $\sum_{n=1}^{3} n^{-n} = 1^{-1} + 2^{-2} + 3^{-3}$

(4) $\sum_{n=1}^{3} n^{\frac{1}{n}} = 1^{\frac{1}{1}} + 2^{\frac{1}{2}} + 3^{\frac{1}{3}} = 1 + \sqrt{2} + \sqrt[3]{3}$ **ANSWER: (4)**

(Remember problem 8 on this regents. The denominator of a fractional exponent indicates the root)

15) To be considered a function, no two ordered pairs can have the same first element. In our case, an x-value can have only one unique y-value. Now look at choice 2. You immediately see the ordered pairs that disqualify themselves from representing a function. The ordered pairs (3,-2) and (3,-4) cannot define a function. Neither can the pairs (4,-1) and (4,-3). **ANSWER: (2)**

16) You are in essence being asked for the area of the shaded portion of the diagram at the right. The white non-shaded area forms a central angle of 60° since the shaded area forms an angle of 300°. The area of circle is found by multiplying π by r^2. In our case at the right, the radius r equals 15. This means that the area of the complete circle would equal **$(\pi)(15)^2$**.



Remember now that you are missing 60° of this circular area. This means that you are missing 60/360 or 1/6 of the circle. If you are missing 1/6, you have 5/6 of the complete circle. The area in our case would therefore be:

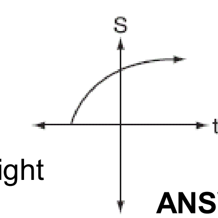
$\left(\frac{5}{6}\right)(\pi)(15)^2$ = 589.0486. To the nearest square foot this is **589**. **ANSWER: (3)**

17) To begin with, you should know that the discriminant, the part of the quadratic formula that is under the radical sign, is called the discriminant because it allows you discriminate or determine the nature of the roots of a quadratic equation.
There are 4 possibilities regarding the roots of a quadratic equation. The roots can be:

1. Real, rational and unequal	Discriminant will be greater than 0 and a perfect square.
2. Real, rational and equal	Discriminant will equal 0.
3. Real, irrational, and unequal	Discriminant will be greater than 0 and not a perfect square.
4. Imaginary.	Discriminant will be less than 0 (negative).

This problem asks you for the discriminant of a quadratic whose roots are **real, unequal, and irrational**. Looking above you see that such a discriminant will be greater than 0 and not a perfect square. Of the four given choices, **7** fits the criteria. **ANSWER: (3)**

18) You are presented with the formula $S = 20\sqrt{t + 273}$ and asked which one of the four given graphs best represents this function. You'll notice that the lowest possible value for t is -273. Anything lower would yield a negative number under the radical resulting in an imaginary answer. When t= -273, S will equal 20 times 0 which equals 0. Choice 2 at the right is the **only graph that shows a 0 value for S when t is negative**. **ANSWER: (2)**



19) When presented with a quadratic equation in the form of $ax^2 + bx + c = 0$, the product of the roots will equal c/a. In other words, if you know the product of the roots, you know the value of c/a. You are presented with the equation $x^2 - 4x + c = 0$. In this case a=1, b=4, and c = ?
Since in this case a=1, the product of the roots will simply equal c. We are told that the roots are **2** + i and **2** - i. Find their product and you will have the value of c.
(**2** + i)(**2** - i) Use FOIL (Keep in mind that $i^2 = -1$)
Firsts = (2)(2)=**4** Outers = (2)(-i)= **-2i** Inners = (i)(2)=**2i** Lasts = (i)(-i)= - i^2 = -(-1)= **+1**

Now combine: **4 + 1 - 2i** + **2i** = **5** **ANSWER: (2)**

20) You are asked to simplify the following expression: $\sin A + \dfrac{\cos^2 A}{\sin A}$

As in problem 13, treat this expression as you would a mixed numeral that you are changing to an improper fraction. You will first multiply sin A by sin A, then add $\cos^2$ A to your answer, and divide your result by sin A.
You will be using the following identities: $\sin^2 A + \cos^2 A = 1$ and $1/\sin A = \csc A$

$\sin A + \dfrac{\cos^2 A}{\sin A} = \dfrac{\sin^2 A + \cos^2 A}{\sin A} = \dfrac{1}{\sin A} = \mathbf{\csc A}$ **ANSWER: (4)**

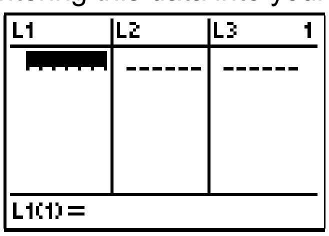
21) You are presented with the following equation: $P(t) = 80\,(0.98)^t$, where t is the number of years since 1998. You are asked to predict the population in the year 2008.
You first subtract to get the value of t. In our case it is 2008 - 1998 or 10.
Now it's simply a matter of using your calculator and substituting 10 for t.

$P(t) = 80\,(0.98)^t$ Substitute 10 for t.
$P(10) = 80\,(0.98)^{10}$ Use your calculator: 80*.98^10 65.36582455
$P(10) = 65$

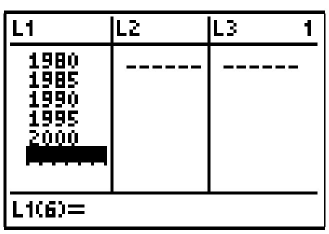
ANSWER: There will be 65 wolves by the year 2008.

22) You are presented with the table at the right and asked to write a linear regression to model its data. The first step consists of entering this data into your calculator.

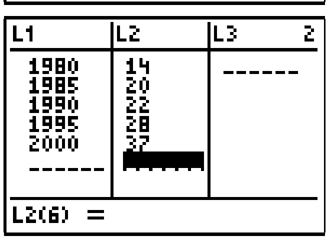
Year (*x*)	Enrollment (*y*)
1980	14
1985	20
1990	22
1995	28
2000	37



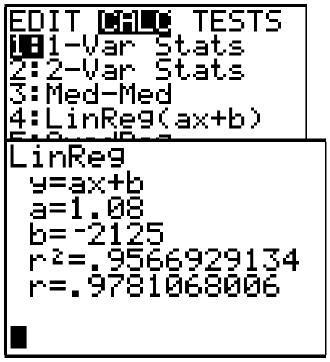
Hit the STAT key followed by the ENTER key and your screen will look like the one at the left.
STAT **ENTER**



Entering the values under L1 and L2 are easy. Let's enter the Year data under L1, and the Enrollment data under L2.
Simply enter each value starting with 1980 and ending with 2000.
Hit **ENTER** after entering each value. Your screen should look like the one on the left that has values in the column labeled L1.
To enter the corresponding Enrollment data, first hit the ▶ (right arrow key) and your cursor will automatically move to the top of the L2 column. At this point simply add the proper values hitting ENTER after each one.



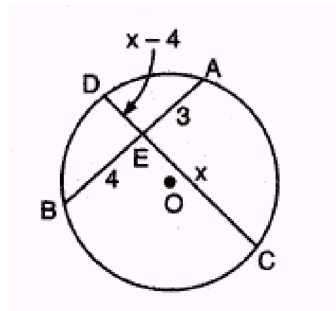
At the left is a screen capture of what your screen should look like after you have entered your data
At this point you are ready to have your calculator calculate the the linear regression equation based on the data you have entered.



Hit the following keys:
STAT ► Your screen will now look like the one at the left with the cursor over the middle column which reads CALC.
Now hit the **4** key, and at the top of your screen you will see:
LinReg(ax+b) Now Hit **ENTER** and your screen will look like the one to the left. Based on your last screen, the equation is in the form ax+b. Substitute the values for a and b that you see on your calculator screen, and your equation becomes: **y = 1.08 - 2125**

ANSWER: y = 1.08 - 2125

23) You are presented with the diagram of the circle at the right. The key to completing this problem is to realize that when two chords intersect in a circle, the product of their segments will be equal. In the circle at the right this means that:



(CE)(DE)= (AE)(BE) Substitute the shown values for the segments.

$x(x - 4) = (3)(4)$	Multiply.
$x^2 - 4x = 12$	Subtract 12 from both sides.
$x^2 - 4x - 12 = 0$	Factor.
$(x - 6)(x + 2) = 0$	Set each factor equal to 0, and solve for x.

$x - 6 = 0$ Add 6 to each side.
x = 6

$x + 2 = 0$ Subtract 2 from each side.
$x = -2$ Reject the negative value, as the length of a segment cannot be negative.

ANSWER: x = 6

24) The volume of a cube can be found by cubing the measurement of one of its sides. Each side being equal, this is the equivalent of multiplying its length by its width by its height. You are told that each side of this cubic box has a length of 10 inches, and that you want to increase it sides so that its volume will equal 2000 cubic inches. In other words, each side will be increased by x. This means that each side will now measure (10 + x), and you want its volume to be 2000.
Set up your equation:

$(10 + x)^3 = 2000$	Take cube root of both sides.
$10 + x = 12.5992105$	Subtract 10 from both sides.
$x = 2.5992105$	

ANSWER: To the nearest tenth x = 2.6 inches

25) $f(x) = \log_2 x$ and $g(x) = 2x^2 + 14$ You are asked to determine $(f \circ g)(5)$, which we can call y to simplify matters. $(f \circ g)(5)$ is read as
f composition g of x, or f following g of x. This means you first evaluate g(x) and use that result to evaluate f(x). Let's begin:

$g(x) = 2x^2 + 14$
$g(5) = 2(5)^2 + 14$
$g(5) = 2(25) + 14$
$g(5) = 50 + 14$
$g(5) = 64$ Now go on to the next page…

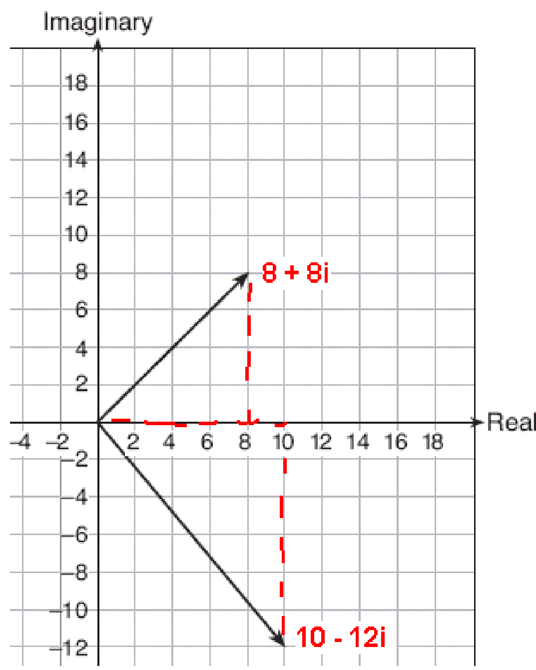
$g(5) = 64$ Now use this value to evaluate f(x)

$y = f(64) = \log_2 64$ Substitute above value for x
$y = \log_2 64$ Rewrite in exponential form..
$2^y = 64$ Rewrite 64 with a base of 2.
$\mathbf{2^6} = 64$

ANSWER: (f ∘g)(5) = 6

26) Part of the graph you are presented with appears at the right. It shows two complex numbers.
If you start at the origin and move 8 units to the right along the Real axis, and then 8 units up along the Imaginary axis, you have represented the complex number **8 + 8i**
If you move 10 units to the right along the Real axis and then 12 units down along the Imaginary axis, you have represented the complex number **10 - 12i**
Expressing their sum is simple. Think of them as combining like terms.
8 +10 = 18 **8i - 12i = -4i**
Therefore:
(8 + 8i) + (10 - 12i) = 18 - 4i

ANSWER: 18 - 4i



#27 begins on the next page

27) You are asked to divide f(x) by g(x). This problem is similar to number 13. You will again be dividing and factoring two expressions that are fractions. Begin by writing out the expressions for f(x) and g(x), and divide.

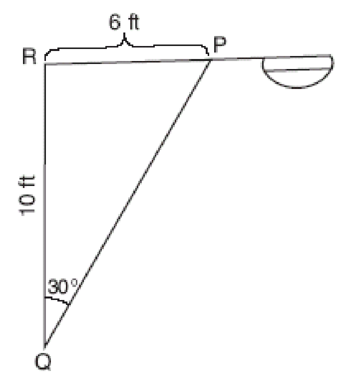
$$\frac{3x^2 - 27}{18x + 30} \div \frac{x^2 - 7x + 12}{3x^2 - 7x - 20} = \frac{3x^2 - 27}{18x + 30} \bullet \frac{3x^2 - 7x - 20}{x^2 - 7x + 12} =$$ Now factor as shown below

$3x^2 - 27 = 3(x^2 - 9) = 3(x+3)(x - 3)$
$18x + 30 = 6(3x + 5)$
$3x^2 - 7x - 20 = (3x + 5)(x - 4)$
$x^2 - 7x + 12 = (x - 4)(x - 3)$

$$\frac{3x^2 - 27}{18x + 30} \bullet \frac{3x^2 - 7x - 20}{x^2 - 7x + 12} = \frac{3(x + 3)(x - 3)}{6(3x + 5)} \bullet \frac{(3x + 5)(x - 4)}{(x - 4)(x - 3)} = \text{cancel} = \frac{\cancel{3}(x + 3)\cancel{(x - 3)}}{{}_2\cancel{6}\cancel{(3x + 5)}} \bullet \frac{\cancel{(3x + 5)}\cancel{(x - 4)}}{\cancel{(x - 4)}\cancel{(x - 3)}}$$

You are left with $\frac{x + 3}{2}$ **ANSWER:** $\frac{x + 3}{2}$

28) You are presented with the diagram at the right and asked to determine the length of PQ. Angle R, you are told, is an obtuse angle. Here is the plan for solving this problem. You will first use the Law of Sines to calculate angle RPQ. Once you know this angle, you will know enough to calculate angle R. (The 3 angles have to add up to 180 degrees). Once you know angle R, you can again use the Law of Sines, this time to determine the length of PQ. The Law of Sines is the relationship in a triangle of its sides to the sines of the angles opposite these sides. In our triangle we can therefore solve first for ∠RPQ.



$\frac{\text{side}}{\text{sine of angle opposite side}}$ $\frac{RP}{\sin 30} = \frac{RQ}{\sin \text{angle } RPQ}$ Now substitute:

$\frac{6}{\sin 30} = \frac{10}{\sin \text{angle } RPQ}$ Cross multiply.

$6(\sin \angle RPQ) = 10 \sin 30$ Multiply (sin 30 = 1/2) (Make sure you are in degree mode.)
$6(\sin \angle RPQ) = 5$ Divide both sides by 6
$\sin \angle RPQ) = \frac{5}{6}$ Use SIN^{-1} to find ∠RPQ

```
sin-1(5/6)
        56.44269024
```

$\angle RPQ = 56.44°$ Now you know the measures of two angles. Let us find the third angle, ∠R, which has PQ as its opposite side. 180 - (30+56.44) = **∠R = 93.56**
Now we can use the Law of Sines again to determine side PQ.

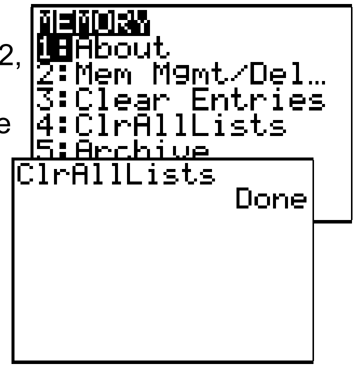
$\frac{6}{\sin 30} = \frac{PQ}{\sin 93.56}$ → (PQ)(sin 30) = 6 (sin 93.56) Divide both sides by sin 30.

$PQ = \frac{6(\sin 93.56)}{\sin 30} = 11.9768$

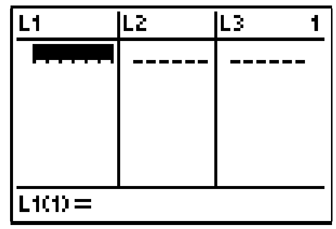
```
(6*sin(93.56))/s
in(30)
        11.97684386
```

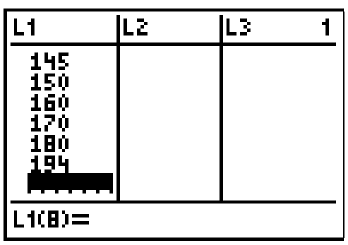
ANSWER: To the nearest foot, PQ = 12

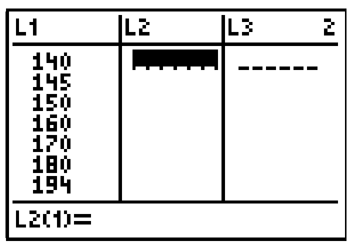
29) For this problem you again have to enter data into lists. You can use L3 and L4 since L1 and L2 still have the information from problem #22, but here is a quick way to clear all lists.
Hit **2nd** followed by **+** the plus key. Remember that after you hit the 2nd key, the next key you will be accessing will be the "yellowish" one above the key you are actually hitting. In this case you will actually be accessing the **MEM** key. At this point select choice 4 and hit **ENTER**. You will see the second screen at the right. The data you entered for L1 and L2 will no longer be there.

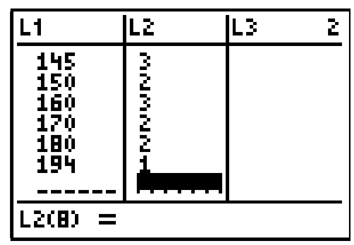


Step number one will involve entering the information into your calculator. Press **STAT** **ENTER** and you will see the screen at the right . It is called the STAT LIST EDITOR screen. This is the screen where you will enter your data. You can enter all the data for Score into L1 (list 1) and all the data for Frequency into L2 (list 2). The simplest way to enter the data is to first enter the Score data into L1. This is done by simply typing each number and then hitting the **ENTER** key. Below, to the left is a screen capture of what your screen will look like after you have entered all the data into L1. You can scroll up and down to make sure you entered all the data properly. Now hit the ► right scroll key once and you will immediately see the see the middle screen pictured below. You are now ready to enter the data for L2. Enter the data the same way you entered it into L1. Enter each number followed by the ENTER key. The final screen capture shows all the data entered into L1 and L2.

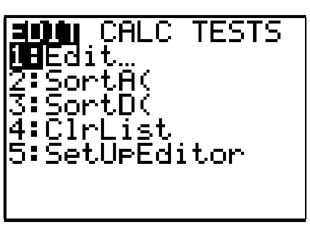


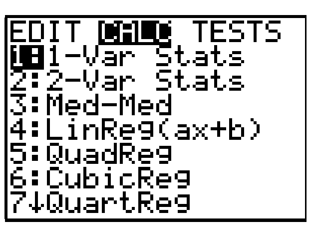






Your first task is to determine mean and standard deviation for this data to the nearest tenth. You will use the mean to answer the second part of this question.
Again, press **STAT** followed by the ► right scroll key and you will access the **CALC** menu. Now the item you want is the first one,1-Var Stats. However, don't forget that you have entered two lists. The second list was the frequency of the data entered in L1. You have to take this into account when determining the mean and standard deviation. Below you see the first two screen captures of what your calculator screen will look like. As soon as you hit **ENTER** or **1** to select 1-Var Stats your screen will look like the third one below. It is now important to enter L1, L2 the way you see below on the final screen capture. This makes sure you are incorporating the frequency of the data.





Hit **ENTER** once.

This problem continues on the next page.

```
1-Var Stats L1,L
2
```

Perhaps I should really explain at least one way of entering the L1, L2 the way you see it in the screen at the left. You will notice on your calculator keyboard that the L1 key and L2 key can be accessed by hitting the 2nd key followed by the 1 and 2 key respectively. Make sure to separate them with a comma.

```
1-Var Stats
 x̄=157
 Σx=2669
 Σx²=423511
 Sx=16.72946502
 σx=16.22996502
↓n=17
```

At the point when you hit **ENTER**, you will have the screen you see at the right, and your answer. Notice, by the way, that your data consists of 17 items. That is indicated by that n on the last line. If you add up the numbers you entered in L2, they should equal 17, your total frequency. At the right you see that your **mean, ü, equals 157** and the **population standard deviation, Fx, equals 16.2**, all to the nearest tenth. (The value following Sx is called the sample standard deviation.)

Now that you know that the mean is 157, you can answer the second question. of how many scores fall within one standard deviation of the mean.
The score **one standard deviation below the mean** would be 157 - 16.2, which equals **140.8**
One standard deviation above the mean would be 157 + 16.2 or **173.2**.

Look at the table below, and count how many scores are included within the range of 140.8 thru 173.2.

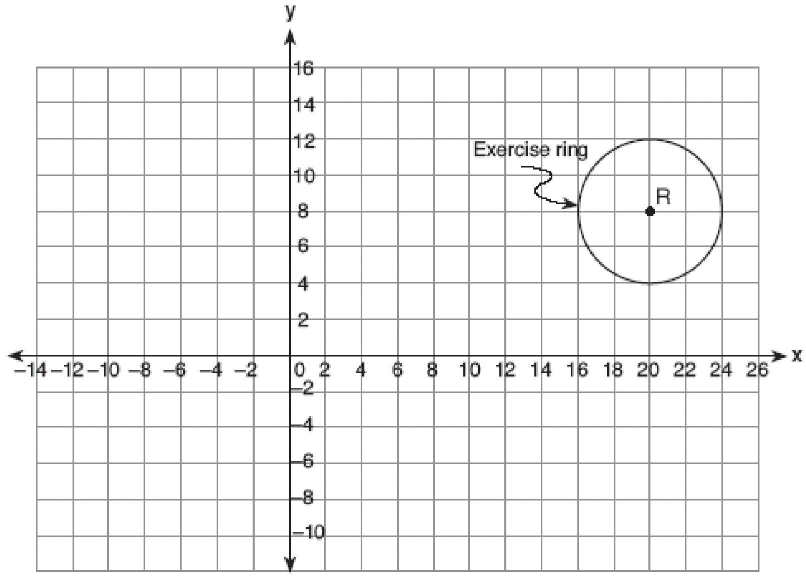
Score (x_i)	Frequency (f_i)
140	4
→ 145	3 ←
→ 150	2 ←
→ 160	3 ←
→ 170	2 ←
180	2
194	1

I have marked with arrows the scores that are within that range. Now simply add up their frequency and you have your answer.
3 + 2 + 3 +2 = **10**

ANSWER: The population standard deviation to the nearest tenth is 16.2.
There are 10 scores within one standard deviation of the mean.

30) You are presented with the diagram at the right. Circle R represents an exercise ring, and you are asked to write its equation. You've already learned in Math A that the equation of a circle with its center at the origin would be: $x^2 + y^2 = r^2$. In general, the equation of a circle is given as:

(x – x-coordinate at center)^2 + (y – y-coordinate at center)^2 = r^2

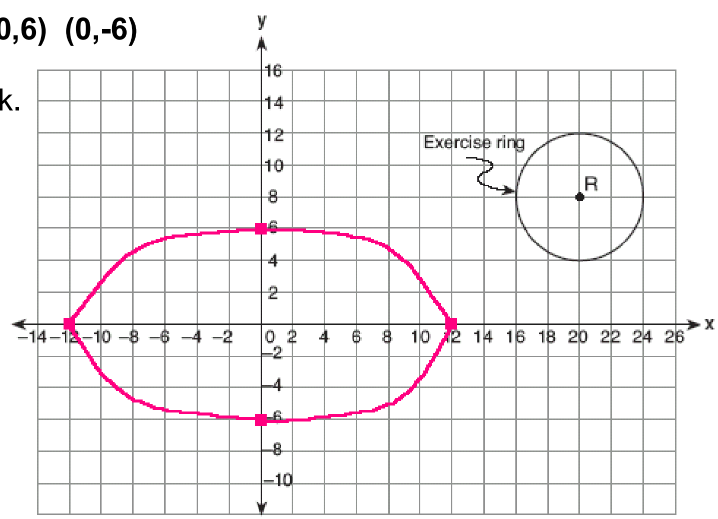


In this problem you are shown that the center of the circle in question is (20,8) with a radius of 2. This means that the equation will be:

$(x - (20))^2 + (y - (8))^2 = 2^2$ or $\mathbf{(x-20)^2 + (y-8)^2 = 4}$,

You are also given the equation of the outside edge of a racetrack, and are asked to sketch it on the graph. The equation is $\frac{x^2}{144} + \frac{y^2}{36} = 1$. You should recognize this as the equation of an ellipse. Let's find the x and y intercepts. If y=0, you are left with $\frac{x^2}{144} = 1$. This means that $x^2 = 144$ and **x = +12** or **x= -12**. To find the y-intercepts, set x equal to 0. That leaves you with $\frac{y^2}{36} = 1$, or y2=36. In this case **y =+ 6** or **y = -6**. Now all you have to do is graph these points and sketch your ellipse. **(12,0) (-12,0) (0,6) (0,-6)**

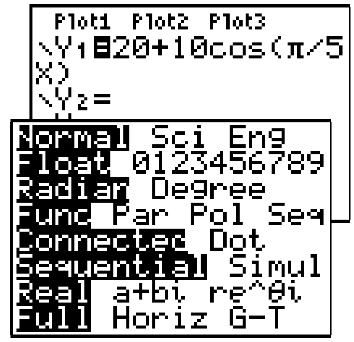
To the right, in red, you see the graph of the ellipse. It is the sketch of the racetrack.



ANSWER: Equation of exercise ring
$\mathbf{(x-20)^2 + (y-8)^2 = 4}$

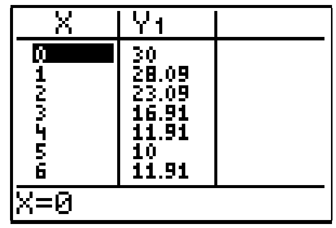
Sketch of racetrack is the red area shown at the right.

31) First let's do this problem simply by using the graphing calculator. Enter the equation into the y= editor the way you see it written. To make life easier use the variable x instead of t, since t is your x-axis, and y will be your S. At the right is a screen capture of how your equation should be entered. Woops! I almost forgot. Put your calculator into radian mode.



Hit the MODE key and you will see the second screen at the right. Move the blinking cursor so that it covers the word Radian and hit **ENTER**. You are doing this because the equation uses pi which is in radian measure.

Now comes the part that will show you the answer to the first question: What is the minimum annual snowfall in inches?



Hit **2nd** followed by the **GRAPH** key. Here at the right is my screen capture. If yours looks different, simply use the up and down scroll keys to get the x value to equal 0. The reason you want x at 0 is because you know that the domain (the x values) you are interested in will go from 0 to 30. The reason you know this is because the problem states that t, in our case x, will represent the years since 1970. You are then asked a question regarding the years from 1970 thru the year 2000. This means that an x-value of 0 represents 1970, a value of 1 will represent 1971, all the way to a value of 30 which will represent the year 2000. As you scroll down from 0 (increasing your x-value) you can see that **the minimum annual snowfall will be 10 inches.**

The next question is in which years did this occur?

Again, simply look at the table and note for which x-values from 0 thru 30 do you have a y-value of 10. You can see that y-values of 10 will occur when x equals 5, 15, and 25. Since 0 represents 1970, these y-values represent 1970+5, 1970+15, 1970+25 or **the years 1975, 1985, and 1995**.

Here is a bit more of an explanation. Your original equation rewritten slightly differently is:

$S(t)= 10\cos\left(\frac{pi}{5}t\right) + 20$ In radian measure, pi/5 equals 36 degrees. The standard cosine curve, y=cos x, has a maximum value (amplitude) of 1, and a minimum value of -1. The curve above will have a maximum value of 10 plus a vertical shift of 20 which will total a maximum of 30. Now follow this. Had the +20 not been there, the low value would have been -10 and the high value +10. Now, the vertical shift of 20, adds 20 to the maximum of 10 causing it to have a maximum of 30, and at the same time it adds +20 to the low value of -10, causing the new low to be +10. This was the answer to the first part of the question.

Now for the second part: cos 0 = 1 cos 90=0 cos 180= -1 cos 270= 0 cos 360= 1

As I mentioned earlier, pi/5=36 degrees. This is then multiplied by t.

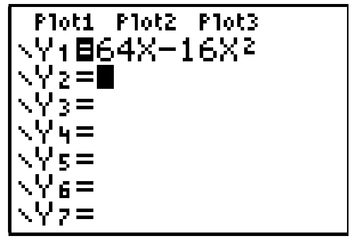
When t=0, your equation actually becomes 10 cos 36(0) + 20 or 10 cos 0 + 20 or 10(1) + 20 which equals 30, your maximum value.

When t=5, your equation actually becomes 10 cos 36(5) +20 which equals 10 cos 180 + 20 which equals 10(-1) + 20 or +10, your lowest value.

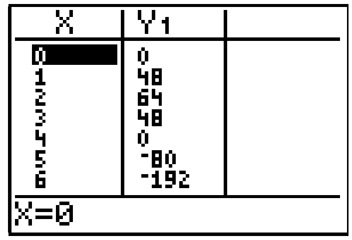
When t=15, (pi/5)(15) will equal 36(15) or or 540, which is a multiple of 180. Again accounting for the low value. The same happens again when t=25. as 36(25)=900 which again is a multiple of 180.

ANSWER: Minimum value is 10 inches.
This minimum value occurred in 1975, 1985, and 1995.

32) You are given the following equation: $s(t) = 64t - 16t^2$, and are told it represents the path of a fired rocket. Since it is a quadratic equation you know its graph will be a parabola. Furthermore, since the numerical coefficient of the squared term is negative, it will be a parabola opening towards the bottom. You are again asked two questions--the first being to find the maximum height of the rocket. As in the previous problem, type the equation into the y= editor, using x for the t variable. Make sure to **CLEAR** the previous equation you had entered. Above is a screen copy of what your equation should look like as you enter it.



Again, as in the last problem, you want to see the table generated by this equation in order to find the maximum height. Hit the **2nd** key followed by **GRAPH** to access the **TABLE** screen. For the last problem I told you how to get the screen at the x-value of 0 via scrolling if it is not already there. Here is another way. Hit the **2nd** key followed by the **WINDOW** key to access the **TBLSET** screen.

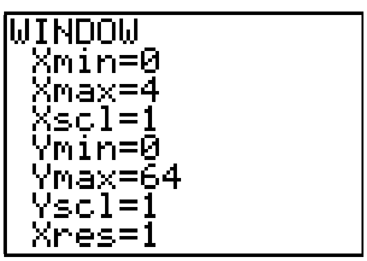


To the right is a screen capture of your window. Whatever number you enter where you see TblStart=, will be the number that appears when you first get to the **TABLE** screen. In addition, whatever number you enter at the ∆Tbl =, will determine the x intervals. Make sure you have it set the way you see it at the right for this problem. Now for the reason of starting at 0 for this problem. You know that the x-value where the rocket hits the ground is 0, therefore we are starting with 0. The equation of the above graph will be actually be at 0 at two points as seen on the table.

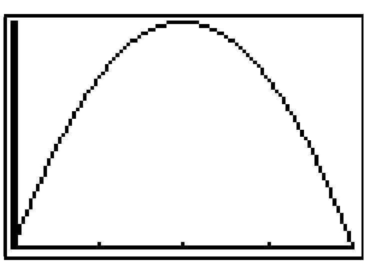
Back to the problem…What will be the maximum height attained by the rocket? **64 feet,** as is clearly shown on the table generated by your calculator.

The next question is in how many seconds will it hit the ground? **4 seconds.** You see this also clearly on the table where when x, the number of seconds, equals 4, the height is 0. A height of 0 means it is on the ground. (The first 0 you see on the table is before firing),

You can easily graph this equation if you want to, using the following **WINDOW**.



The screen at the left is generated by hitting the **WINDOW** key. Based on the information you saw on the table, you set the Xmin (minimum x-value) at 0, and its maximum at 4. An Xscl of 1 indicates that the scale interval is in units of 1. The minimum y-value is 0, and the maximum is 64. Once you have this information entered, you can hit the GRAPH key to generate a graph of the equation.



At the left is your graph. Remember, though, all that was necessary for this problem was to generate the **TABLE** screen and read the required values.

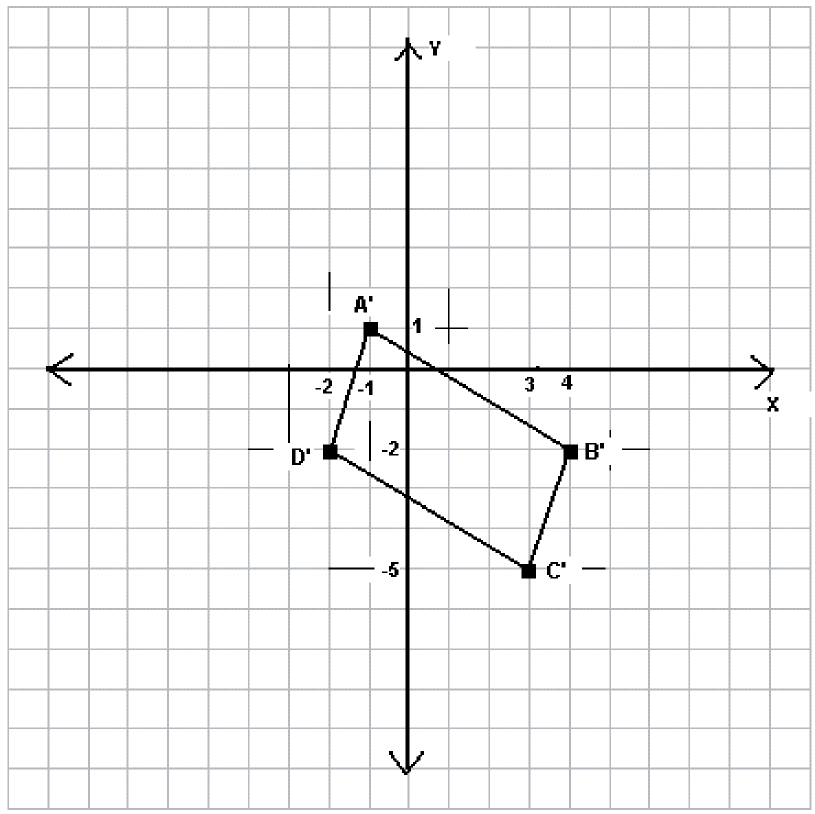
ANSWER: Maximum height is 64 feet.
It will hit the ground in 4 seconds.

33) The first part of the problem is easy. It asks for a dilation. A transformation that is a dilation will either reduce or stretch a figure by a constant. In our case, the constant is given as 1/2. This means that each x-coordinate will be halved, as will each y-coordinate. Each will be multiplied by 1/2.

$D_{\frac{1}{2}}$ (The symbol to the left indicates a dilation of one-half)

A(-2,2) → A'(-1,1)
B(8,-4) → B'(4,-2)
C(6,-10) → C'(3,-5)
D(-4,-4) → D'(-2,-2)

The new coordinates are A'(-1,1), B'(4,-2), C'(3,-5), D'(-2,-2)



At the right you can see the graph of quadrilateral A'B'C'D', The graph is not really required but it will make the next part easier to complete. You are asked to prove the quadrilateral a parallelogram. One easy way to do this is when you can see the graph of the quadrilateral.
Here is one of many proofs:
A parallelogram is a quadrilateral whose opposite sides are parallel. The slopes of parallel lines are equal.

Slope A'D' = -3/-1 or 3/1
Slope B'C' = -3/-1 or 3/1
A'D' and B'C', one pair of opposite sides of quadrilateral A'B'C'D', have equal slopes and are therefore parallel.

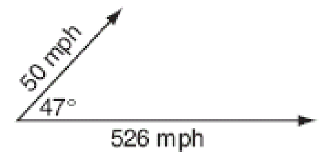
Slope A'B' = -3/5 or -(3/5)
Slope D'C' = -3/5 or -(3/5)
A'B' and D'C', the other pair of opposite sides of quadrilateral A'B'C'D', have equal slopes and are therefore parallel.

Quadrilateral A'B'C'D' is therefore a parallelogram because it is a quadrilateral whose opposite sides are parallel.

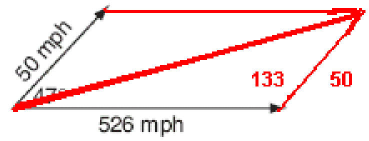
(To find the slope of a line when looking at the plotted points, simply count the units, down or up in the y-direction, from one endpoint until you are level with the other endpoint. Then count either right or left along the x, until you reach the other endpoint. The number of units you count first will be the numerator. It is the difference in y-units. The second number of units right or left will be your denominator since it is the difference in x-units.)
Without looking at a graph, here is one example of how to calculate the slope of A'B'.

A'(-1,1) B'(4,-2) Slope A'B' = $\frac{y_1-y_2}{x_1-x_2} = \frac{1-(-2)}{-1-4} = \frac{3}{-5} = -\frac{3}{5}$

34) At the right you see the diagram that accompanied this problem. The speed of a jet is 526mph, and the wind is blowing at 50mph at an angle of 47° with the direction of the jet. You are asked to find the resultant speed of the jet to the nearest tenth of a mile.



Whenever you see a problem of this type where you are given two sides and the included angle, you will either end up using the Law of Sines, the Law of Cosines or a combination of both. Look at the next diagram at the right. It is a completed parallelogram based on the original information. Opposite sides are equal. That accounts for red side of 50. Consecutive angles are supplementary. The sums of their measures equal 180. That is why that red angle measures 133°, (133+47=180). The resultant, whose measure you are trying to determine is represented by the diagonal. The resultant can be determined by using the Law of Cosines. You know two sides and the included angle and want to determine the side opposite the included angle. **(Make sure your calculator is back in degree mode!)**



$a^2 = b^2 + c^2 - 2bc \cos A$ a, b, and c will always be 3 sides of a triangle, and A will be the angle between sides b and c. We want to figure out side a, so we can set up the equation as follows:

$a^2 = (526)^2 + (50)^2 - 2(526)(50)\cos 133$ Complete the indicated operations.

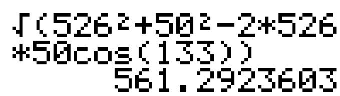
$a^2 = 276{,}676 + 2500 - 52{,}600\ (-.68199)$ Multiply first.

$a^2 = 276{,}676 + 2500 + 35872.674$ Simplify.

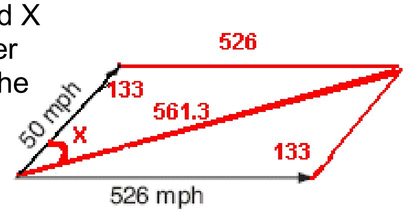
$a^2 = 315048.874$ Find square root of both sides (reject negative)

$a = 561.2921$

RESULTANT = 561.3 mph to nearest tenth



The next part of the question wants you to find the measure of the angle between the resultant force and the wind vector. That is the angle I have marked X in the figure at the right. You will be working with the upper triangle. Its sides are 50, 561.3, and 526. You also know the the measure of the 133 degree angle which is opposite the side of 561.3. You are looking for the measure of angle X which is opposite the side of 526. To find this angle, you use the Law of Sines.



This law states that in a triangle we can set up proportion between the sides of triangles and the sines of the angles opposite these sides. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Using the final diagram above, you can set up the following proportion:

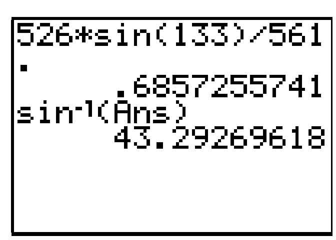
$\frac{561.3}{\sin 133} = \frac{526}{\sin X}$ Cross multiply.

$561.3(\sin X) = 526(\sin 133)$ Divide both sides by 561.3.

$\sin X = \frac{526(\sin 133)}{561.3}$ Use your calculator.

$\sin X = .6857255741$ Use $\sin^{-1}$ key to solve for angle X

X = 43.3 degrees to the nearest tenth.



ANSWER: The resultant speed to the nearest tenth is 561.3 mph.
The measure of the angle between the resultant and the wind force is 43.3 to the nearest degree.