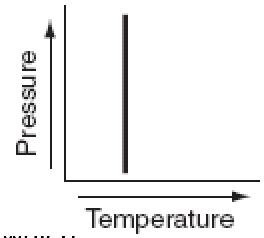
1) You are presented with four graphs and asked to identify the one that does not represent a function. **Choice 3** shown at the right would **not** be **a function** as long as the line shown is truly vertical. **In order for a relation to be a function, each member in the domain can correspond to only one member in the range.** The x-values constitute the domain while the y-values constitute the range. We see at the right that there exists a specific temperature for which there are numerous pressure values, all represented by that solid vertical line. Therefore this graph cannot represent a function.



ANSWER: (3)

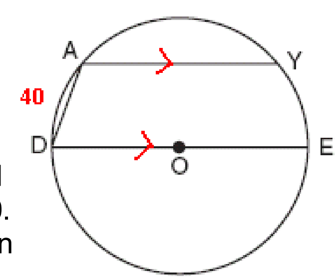
2) In order to determine f $(\frac{1}{4})$ you are required to substitute 1/4 for the value of x in the given equation. First, recall that raising to a negative power is the equivalent of raising the reciprocal to the positive of that power. In other words, $(X)^{-3} = \left(\frac{1}{X}\right)^{3}$. In our case, $f(x) = x^{-\frac{3}{2}}$ $f(x)=\left(\frac{1}{X}\right)^{\frac{3}{2}}$.

Now, we have to substitute 1/4 for x. This will result in 1 divided by 1/4 or simply 4. So we now have: 4 raised to the 3/2. The denominator is the root while the numerator is the power. In our case we now have to find the square root of 4 and raise that to the 3rd power:

$(4)^{\frac{3}{2}} = (\sqrt{4})^{3} = 2^{3} =$ **8**

ANSWER: (1)

3) At the right is the diagram for this problem. I've indicated in red some of the givens. AY is parallel to diameter DE. Therefore, **arcs AD and YE will be congruent**. You are given that the measure of arc AD=40. You are asked to determine the measure of angle DAY. Notice that angle DAY intercepts arc DEY You know that the measure of semicircle DE= 180, and arc YE = arc AD = 40. The measure of arc DEY is therefore 180+40 or 220. The angle you are being asked for is an inscribed angle which intercepts an arc of 220. **An inscribed angle is equal in measure to one-half its intercepted arc.** One-half of 220 = **110**



ANSWER: (2)

4) You are told that x is a positive acute angle, and that sin x = 1/2. You should immediately know that x is a 30° angle. (If you are not sure then you can always use your calculator to determine which angle has a sine of 1/2. All you have to do is use the $\mathbf{sin^{-1}}$ key followed by **(1÷2)**)
Now the problem is asking you for sin 2x or sin 2(30) --the sine of a 60° angle. Again, you should already know that **sin 60 =** $\frac{\sqrt{3}}{2}$. If you don't know this then you can use your calculator but it will be a bit tricky. Your calculator will give you a decimal answer. What you have to do next is to use your calculator to determine which of the four given choices is its equivalent. You will see that **choice 4 is correct.** **ANSWER: (4)**

5) You are presented with two equations. One is used for determining temperature based on moving parts, and the other is used to determine resistance based on temperature. What you will first have to do is use the first equation to determine the temperature based on the number of parts.
The first equation is **$t = 0.3m^2$**. You are told that the circuit has 4 moving parts and that m represents the number of moving parts. So let us substitute 4 for m.

$t = 0.3m^2$ Substitute 4 for m.
$t = 0.3(4)^2$ Raise 4 to the second power.
$t = 0.3(16)$ Multiply.
$t = 4.8$ **Now use this value of r in the equation**

$r = 150 + 5t$ Use the value 4.8 for t, obtained from the previous equation.
$r = 150 + 5(4.8)$ Multiply.
$r = 150 + 24$ Add
$r = 174$ **ANSWER: (3)**

6) You are given the equation $x^2 - kx - 36 = 0$, and told that one of its roots is 12. You are to determine the value of k. If one of the roots is 12 then the other root will have to be a -3, because their product would then yield the -36 you see in the above equation. Using what you know about FOIL would now let you know that the factors of the above equation would be (x - 12) (x +3). (Just a reminder, if x - 12 is set equal to 0 we obtain 12 as a root, and if x+3 is set equal to 0 we obtain -3 as the other root.)
Now let's complete the multiplication using FOIL:

$(x - 12)(x + 3)$ Multiply the Firsts, Outers, Inners, Lasts
$x^2 + 3x - 12x - 36$ Combine like terms.
$x^2 - 9x - 36$ The 9 is in the same spot that the k is in the original equation.
k therefore equals 9

OR: Substitute 12 for x in the equation:

$x^2 - kx - 36 = 0$ Substitute 12 for x
$12^2 - k(12) - 36 = 0$ Simplify.
$144 - 12k - 36 = 0$ Combine like terms.
$108 = 12k$ Divide both sides by 12.
$9 = k$ **ANSWER: (1)**

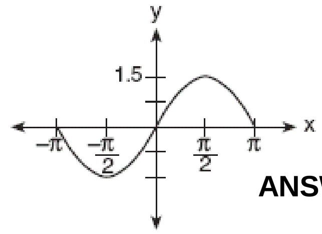
7) To answer this question, first determine f(x) at 1 and then at 4.

$f(x) = 80(0.5)^x$ Determine f(1)
$f(1) = 80(0.5)^1$ Raise .5 to the 1st power.
$f(1) = 80(.5)$ Multiply
$f(1) = 40$

$f(x) = 80(0.5)^x$ Determine f(4)
$f(4) = 80(0.5)^4$ Raise .5 to the 4th power.
$f(4) = 80(.0625)$ Multiply.
$f(4) = 5$

You see that the first bounce, f(1), obtains a height of 40, while the fourth bounce, f(4), obtains a height of 5. **The first bounce is 8 times higher than the fourth bounce.** **ANSWER: (1)**

8) The curve at the right is your by now familiar sine curve with one modification. Its amplitude is not 1, it is 1.5. Therefore it is not represented by y = sin x , but by **y = 1.5 sin x**



ANSWER: (2)

9) You are told that csc θ < 0. This lets you know that cosecant is negative. Recall that cosecant and sine are reciprocal functions and that cosecant will be positive when sine is positive and negative when sine is negative. Sine is negative in quadrants 2 and 3. At this point you therefore know that θ is in quadrant 2 or 3. You are also told that tan θ is positive. Tangent is positive in quadrants 1 and 3. So you now have your answer. You are looking for a quadrant where while tangent is positive, cosecant is negative. **The answer is quadrant 3.** **ANSWER: (3)**

10) You are asked to simplify $\frac{1-\cos^2 x}{\sin^2 x}$

One of the identities you are expected to know is $\sin^2\theta + \cos^2\theta = 1$
If you now subtract $\cos^2\theta$ from both sides you have the identity that will be used for this problem: $\mathbf{\sin^2\theta = 1 - \cos^2\theta}$
This means that wherever you see a $1-\cos^2\theta$, you can simply substitute $\sin^2\theta$.
Therefore:

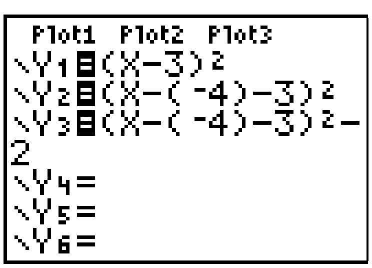
$$\frac{1-\cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x} = 1$$

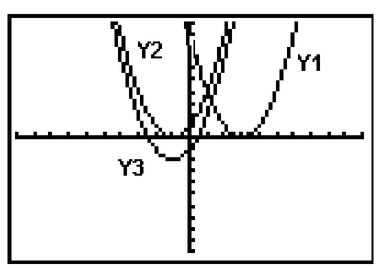
ANSWER: (1)

11) You are presented with the equation $y = (x-3)^2$, and asked to determine the axis of symmetry of its graph after a translation of 4 units to the left and 2 units down. There are many ways of doing this problem--some easier and some more difficult. Here is the way I like best for this particular problem.
You immediately realize that the graph of this equation would be a parabola.
To find the point(s) of intersection of the parabola and the x-axis, set y=0. In other words, the equation $y = (x-3)^2$, becomes $0= (x-3)^2$ or $0 = (x-3)(x-3)$
You set the factors equal to zero, x-3 = 0, and solve. You end up with two roots both equal to 3.
In this case it is apparent that the parabola will hit the x-axis in only one point--where x = 3. That 3 will therefore be the turning point of the parabola, and lies on its axis of symmetry. So you now know that the original equation will have as its axis of symmetry x=3.
Now you are told to move the graph 4 units to the left. Three units to the left would put you at the origin. One more unit to the left would put you at **x=-1**. That would be the axis of symmetry of your new translated graph. It would not make a difference how many units up or down you are moving as the axis of symmetry is a vertical line and would only be affected by movements right or left.

ANSWER: (2)

(For your information, below you can see a screen capture of 3 equations. The one entered first as Y1 is the original equation you are given. The one entered as Y2 is the same equation translated 4 units to the left. Entered as Y3 you see that same first equation moved 4 units to the left and two units down.)



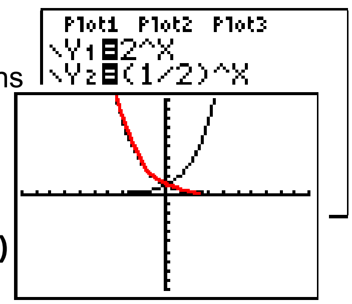


12) You are asked for the solution set of the following equation: $2^{x^2+2x} = 2^{-1}$
You notice right away that the bases are equal. That means that the only way for both sides of the equation to equal each other would be if their exponents are equal as well. In other words:

$x^2 + 2x = -1$		Add 1 to both sides.
$x^2 + 2x + 1 = 0$		Factor.
$(x + 1)(x + 1) = 0$		Set both factors equal to 0.
$x+1= 0$	$x+1=0$	Subtract 1 from both sides.
$x = -1$	**$x = -1$**	**Both roots are equal to -1**

ANSWER: (2)

13) Use your calculator for this one. Enter both equations as you see them using your **Y=** editor. Hit the **GRAPH** key notice how the two graphs compare. To the right you see both equations as they have been entered. The second screen capture is that of the graphs of both equations. I've traced the second one in red. You can clearly see that the graphs are reflections of each other over the y-axis. In other words, the y-axis serves as a line of symmetry. **ANSWER: (4)**



14) In order to find the multiplicative inverse of an element, you first have to know its identity. The multiplicative identity is 1 because any element multiplied by 1 yields the same element. Therefore, to find the multiplicative identity of an element, you are looking for a second element which when multiplied by the first element will yield 1. The multiplicative identity is another name for "reciprocal." For example the multiplicative identity of 2/3 is 3/2 because when those two elements are multiplied, your answer will be 1 (which is the identity element for multiplication). In this problem you are asked for the multiplicative inverse of 3i. You know that the answer has to be $\frac{1}{3i}$. Looking at the choices you don't seem to find one that matches. This means that you can change your answer to match one of the choices. It is simply a matter of rationalizing your answer--getting rid of the i in the denominator.

$\frac{1}{3i}$ Multiply numerator and denominator by i.

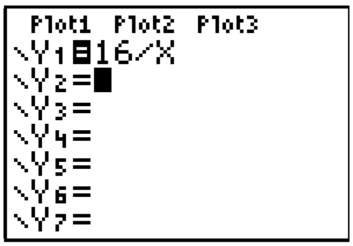
$\frac{1}{3i} \bullet \frac{i}{i}$ Remember that **$i \bullet i = i^2 = -1$**

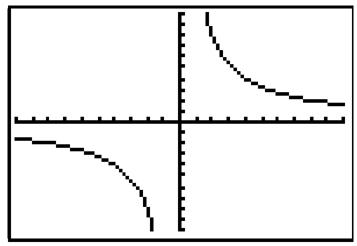
$\frac{i}{3(-1)} = \frac{i}{-3}$ or $-\frac{i}{3}$

ANSWER: (4)

15) $i^0 = 1$ $i^1 = i = \sqrt{-1}$ $i^2 = (\sqrt{-1})(\sqrt{-1}) = -1$ $i^3 = i^2(i) = -i$ $i^4 = (i^2)(i^2) = 1$
Now, to simplify a power of i, simply divide the power by 4 and remember the remainder.
In our case you are looking for the choice which will equal -i. That means that after you divide by 4 you should have a remainder of 3 since **$i^3 = -i$**. Your answer is **choice 2** which reads **i^{47}**. When 47 is divided by 4, your remainder will be 3. **ANSWER: (2)**

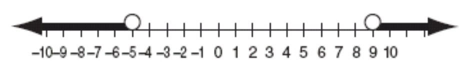
16) Choice 1 can be transposed to $x^2 + y^2 = 16$. You should recognize this as representing a circle whose center is the origin, and whose radius is 4. Choice 2 can be rewritten as $y = -x^2 + 16$. which represents a parabola that opens to the bottom, and will have as its maximum point (0,16). Choice 3 represents a parabola that will open to the top and have a minimum turning point (0,0). **Choice 4 represents the hyperbola**. It can be transposed to xy = 16, which is an equation for inverse variation, which is represented by one type of hyperbola. Below you can see the equation entered into your calculator and a screen capture of the graph it generates.

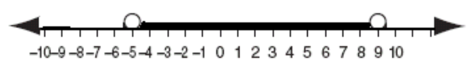




ANSWER: (4)

17) You are presented with the graph at the right and asked to select the inequality it represents. You should immediately notice that it is the graph of a disjunction (or). The graph of a conjunction (and) would look like the graph below it, to the right.





Here is a simpler way to remember what the graph of an absolute inequality will look like. First set it up so that it is either greater than 0 or less than 0 (the same rule will follow if it is greater than or equal, or less than or equal). Now for each absolute inequality you will have two solution sets. If the inequality reads > 0 (pointing to the right), you know that the graph will look like the first one pictured for this problem. You will have two answers where x will be **greater** than the larger root, or x will be **less** than the smaller root. If however, the original inequality reads < 0 (pointing to the left), then x will be **less** than your larger root and **greater** than your smaller root, as pictured by the second graph for this problem.

Based on the above discussion you know that your answer will be either choice 1 or 3. If you look carefully at choice 3 you see that it makes no sense. How can the absolute value of an expression equal a negative number? **That leaves you with choice 1 as the answer.**

BTW here is how you would solve this problem algebraically had you started with the inequality. (Remember that each absolute inequality can be expressed as two inequalities)

$|x-2| > 7$ Following are the two derived equations:

$x-2 > 7$	Add two to both sides.	$-(x-2) > 7$	Distribute the minus sign
$\mathbf{x > 9}$		$-x+2 > 7$	Subtract 2 from both sides.
		$-x > 5$	Divide by -1 (symbol switches)
		$\mathbf{x < -5}$	

That's exactly what the graph accompanying the problem pictures. x is a number greater than 9, or a number less than -5.

ANSWER: (1)

18) When two quantities are inversely proportional, their product will equal a constant. In this problem, pressure and volume are inversely proportional. You are told that when the pressure is 20, the volume is 500. Their product is 500(20) or 10,000. This makes 10,000 your constant. You are now asked to determine the pressure when the volume is 400. This "boils" down to 400 times what will give you that constant of 10,000. Divide 10,000 by 400 and you have your answer of **25**. **ANSWER: (2)**

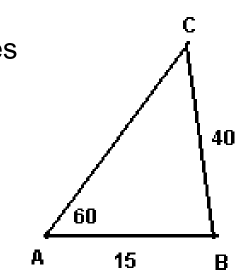
19) You are asked for the 4th term of $(y-1)^7$.
When raising a binomial to a power n, realize that your resulting answer will contain n+1 terms. In this problem, $(y-1)^7$ will therefore result in a polynomial 7+1 or 8 terms. This problem is asking you for the fourth term--term number 4. Each term can be represented as the product of a numerical coefficient, an x term, and a y term.
Here is how to determine the numerical coefficient. The numerical coefficient of term 1 will be represented by ${}_nC_0$, followed by ${}_nC_1$ for term 2, and so on. Term number 4's numerical coefficient will be represented by $\mathbf{{}_nC_3}$. In our case where n is 7, the numerical coefficient of the 4th term will be: $\mathbf{{}_7C_3}$, which is equal to **35**.
Following the numerical coefficient will be the variables x and y raised to some power. Realize that if they are raised to the 0 power they will be equal to one and not appear. All that remains now is how to determine these two variables—the x and y.
The y term's exponent will always be one less then whichever term you are working on. The 1st term will have y^0, the 2nd will have y^1, the 3rd will have y^2., and the 4th which is our case will have y^3. Once you know the y exponent, it is easy to determine the x exponent. Both exponents always have to add up to that exponent to which you are raising the binomial—in our case 7. Hmm…perhaps I should explain that the first term in the parenthesis, regardless of what it looks like, is your x term, and the second is your y term. What this means is that in our case the variable y is really going to be considered our x term as it appears first in the parenthesis, while our y term is -1 which appears second in the parenthesis.
So if our y term (the -1) is raised to the 3rd power, our x term (the y) will be raised to the 4th power since 3 + 4 = 7 which is the power to which we are expanding our binomial. So this is what our 4th term will look like: $\mathbf{{}_7C_3\,(y)^4(-1)^3}$ or $\mathbf{35y^4(-1)}$ **which equals** $\mathbf{-35y^4}$**. (Choice 3).**
If you are curious, this is what each term of $(y-1)^7$ would look like:

$$\mathbf{{}_7C_0(y)^7(-1)^0 + {}_7C_1(y)^6(-1)^1 + {}_7C_2(y)^5(-1)^2 + {}_7C_3(y)^4(-1)^3 + {}_7C_4(y)^3(-1)^4 + {}_7C_5(y)^2(-1)^5 + {}_7C_6(y)1(-1)^6 + {}_7C_7(y)^0(-1)^7}$$

ANSWER: (3)

20) This type of problem generally requires you to use the Law of Sines. To the right, not drawn to scale, is a representation of a triangle that satisfies the given conditions. Below you see the Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



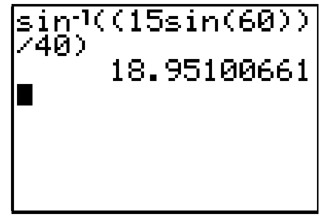
Little "a" is the side opposite angle A, little "b" is opposite angle B, and little "c" is opposite angle C. Using this law on the triangle at the right, we can determine the measure of angle C by setting up the proper proportion.

(The solution continues on the next page)

$\frac{40}{\sin 60} = \frac{15}{\sin C}$ Cross multiply.

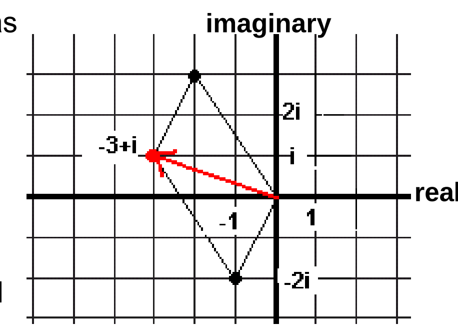
40 sin C = 15 sin 60 Divide both sides by 40.

$\sin C = \frac{15 \sin 60}{40}$ Use the **SIN^{-1}** key to solve for angle C.

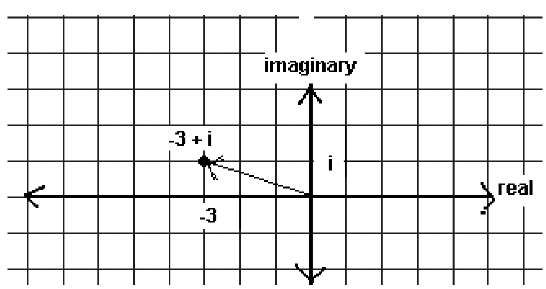


Angle C is approximately 19 degrees. We now know that angle A equals 60 (it was a given), angle C equals 19, and angle B will equal 180 minus the sum of angles C and A, or 180 - 79 which equals 101. Those would be the three angles in the given triangle.
Here is how to determine if another triangle is possible. Using the Law of Sines, we determined that angle C was 19 degrees, but actually it can be greater because the sine of a 19 degree angle will be equal to the sine of a 19 degree angle in the second quadrant. Such an angle would equal 180-19 or 161 degrees. You can check it now on your calculator sin19 = sin161 = .3255681545.
So before when we said angle C equals 19, angle C actually could have been 161 degrees. Let us suppose that angle C is 161 and angle A is our given 60. We immediately notice that their sum is already greater than 180. They cannot therefore be the angles of a triangle! So although the sines of a 19 degree and 161 degree angle are equal, we cannot use the 161 in this triangle. This means that **only 1 triangle is possible.** **ANSWER: (1)**

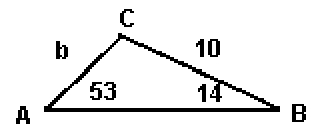
21) One way of doing the problem using graphing techniques is as follows. First graph the two given points, -2+3i and -1-2i. Then complete a parallelogram as seen at the right. Finally, connect with a line the origin to the opposite point (diagonal). That point which is labeled -3+I is the resultant. It is represented by a line from the origin of the axis drawn with an arrow to the point. It also happens to be the sum of the two given complex numbers.
You can also treat the i term as a number with a variable i, and simply add as you would algebraic expressions.



-2 + 3I
-1 - 2I
-3 + i **To complete this question you would still have to graph that point -3+I the way you see in the graph below:**



22) Looks like another problem dealing with the Law of Sines. To the right, again not drawn to scale, is your triangle with the givens noted To review, the Law of Sines sets up a proportion involving the measure of the side of a triangle with the sine of the angle opposite that side. Look back at problem 20 to refresh your memory. In this triangle, a side of 10 is opposite an angle of 53, while the unknown side b is opposite an angle of 14. Let us set up the proportion:



$\frac{10}{\sin 53} = \frac{b}{\sin 14}$ Cross multiply.

$b \sin 53 = 10 \sin 14$ Divide both sides by sin 53.

$b = \frac{10 \sin 14}{\sin 53}$ Use your calculator to solve for b.

(10sin(14))/sin(53)
3.02919032

b= 3.02919032

ANSWER: To the nearest integer, b = 3.

23) You are given the following equation and asked to solve for x: $\log_2 (x+1) = 3$
All you have to do here is convert the logarithmic equation to exponential form, and solve.
Remember this: $\log_{10} 100 = 2$ because $10^2 = 100$.
Following the same procedure: $\log_2 (x+1) = 3$ because $2^3 = x + 1$
Now solve this equation:

$2^3 = x + 1$ You know that $2^3 = 8$, so substitute that value.

$8 = x + 1$ Subtract 1 from both sides.

7 = x

ANSWER: x = 7

24) You are asked to evaluate the following: $$\sum_{k=1}^{2} \frac{(-1)^{k-1}}{(2k-1)!}$$

This is a summation problem that requires you to evaluate the sum of all the values of $\frac{(-1)^{k-1}}{(2k-1)!}$ as the value of k goes from 1 thru 2.

Let us evaluate the expression at

k=1: $\frac{(-1)^{k-1}}{(2k-1)!} = \frac{(-1)^{1-1}}{(2(1)-1)!} = \frac{(-1)^{0}}{(2-1)!} = \frac{1}{1!} = \frac{1}{1} = 1$

k=2: $\frac{(-1)^{k-1}}{(2k-1)!} = \frac{(-1)^{2-1}}{(2(2)-1)!} = \frac{(-1)^{1}}{(4-1)!} = \frac{-1}{3!} = \frac{-1}{6} = -\frac{1}{6}$ Note: 3! = 3•2•1= 6

The sum of 1 and $-\frac{1}{6}$ is $\frac{5}{6}$ **ANSWER:** $$\sum_{k=1}^{2} \frac{(-1)^{k-1}}{(2k-1)!} = \frac{5}{6}$$

25) The first thing necessary to realize is that if the probability for rain is 0.3, then the probability for no rain is 0.7. The probability of something happening plus the probability of that event not happening has to always equal 1. The question you are being asked to determine here is what is the probability that there will be **no rain on exactly 3 of the 5 days** they will spend on the island.

The first part of your answer will involve $_5C_3$, because you are selecting any 3 days out of 5. The answer to $_5C_3$ can easily be found using your calculator or by knowing that $_5C_3$ means:

$$\frac{5P3}{3!} = \frac{(5)(4)(3)}{(3)(2)(1)} = \frac{60}{6} = \mathbf{10}$$

Next you have to determine the probability of no rain on exactly 3 days. You already know that the probability of no rain on one day is .7. This means that the probability of no rain on 3 days will be:

$\mathbf{(.7)^3} = (.7)(.7)(.7) = \mathbf{.343}$

Now if you just figured out the probability for exactly 3 days of no rain, you still have to figure out the probability of "yes" rain for the remaining 2 days. Probability of rain is .3. For two days this would be:

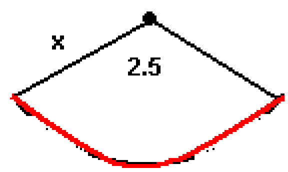
$\mathbf{(.3)^2} = (.3)(.3) = \mathbf{.09}$

You now have the information necessary to solve this problem it is:

$\mathbf{_5C_3\ (probability\ of\ no\ rain)^3\ (probability\ of\ rain)^2 = {}_5C_3\ (.7)^3(.3)^2 = 10(.343)(.09) = .3087}$

ANSWER: .3087

26) You are told that a pendulum swinging through an angle of 2.5 radians travels through an arc of 50 centimeters. You are asked to determine the length, which I am calling x, of the pendulum.



Here is what you should understand about radian measure and its relationship with the intersected arc. A measure of one radian means that the length of the intersected arc is equal in measure to the radius of the circle.

In our example, had the angle have been one radian rather than 2.5, then the arc of 50 would have been the length of the radius and the length of the pendulum, which is actually the radius. Had we been told that the angle is 2 radians, that would mean it intersects an arc equal to twice the radius. In our case that would mean that the arc of 50 is twice the radius. That would make the radius, or the length of the pendulum, equal to 25. So we see that we can actually divide the length of the arc by the radian measure to determine the length of the radius or pendulum in our case. **50 divided by 2.5 = 20** **ANSWER: 20**

27) You are presented with two equations and required to solve them algebraically.
The equations are: $9x^2 + y^2 = 9$ and $3x - y = 3$.
Let us solve the second equation for y in terms of x:

$3x - y = 3$	Subtract 3x from both sides.
$-y = -3x + 3$	Divide both sides by -1.
$\mathbf{y = 3x - 3}$	

You can now substitute this value for y in the first equation.

$9x^2 + y^2 = 9$	Substitute for 3x -3 for y.
$9x^2 + (3x - 3)^2 = 9$	Square 3x - 3 using FOIL or any other method.
$9x^2 + 9x^2 - 18x + 9 = 9$	Combine like terms.
$18x^2 - 18x + 9 = 9$	Subtract 9 from both sides.
$18x^2 - 18x = 0$	Divide both sides by 18.
$x^2 - x = 0$	Factor
$x(x - 1) = 0$	Set both factors equal to 0 and solve for x.

$\mathbf{x=0}$ $\quad x - 1 = 0$
$\mathbf{x = 1}$

You now have two answers for x. Use each one in either of the equations to solve for the corresponding y value. I will use the linear equation as it is easier.

Solve for y when **x = 0**

$3x - y = 3$	Substitute 0 for x.
$3(0) - y = 3$	Simplify.
$0 - y = 3$	Divide both sides by -1.
$\mathbf{y = -3}$	

Solve for y when **x = 1**

$3x - y = 3$	Substitute 1 for x.
$3(1) - y = 3$	Simplify
$3 - y = 3$	Subtract 3 from both sides
$-y = 0$	Divide by -1
$\mathbf{y = 0}$	

You now have your two answers:
When x=0, y = - 3 and when x=1, y = 0

28) This problem asks you to simplify the following complex fraction: $\dfrac{1-\frac{2}{a}}{\frac{4}{a^2}-1}$
Let's first simplify the numerator, then the denominator, and finally complete the division.
First of all you can treat the numerator as a mixed numeral. For example, do you recall how to change a mixed numeral to an improper fraction? Imagine if you have $1\frac{2}{5}$. You can now multiply the whole number (in this case 1) by the denominator (in this case 5), then add the numerator (in this case 2), and put it all over the denominator (in this case 5). Here is how to change $1\frac{2}{5}$:
1 times 5, plus 2, over 5 = $\frac{7}{5}$. What's nice about this method is that it works for $1+\frac{2}{5}$ as well as for $1 - \frac{2}{5}$. $1+\frac{2}{5}$ is simply treated the same way as $1\frac{2}{5}$, while for $1 - \frac{2}{5}$ you do the following:
1 times 5, minus 2, over the denominator of 5 or $1 - \frac{2}{5} = \frac{3}{5}$. Now let's get on to our problem. The numerator reads $1 - \frac{2}{a}$. Here goes: 1 times a, minus 2, over a. This means that $1 - \frac{2}{a} = \mathbf{\frac{a-2}{a}}$

The numerator has now become $\frac{a-2}{a}$. Now let's work on the denominator.

At this point it is $\frac{4}{a^2}-1$. Let's just switch it around to look as follows: $-1+\frac{4}{a^2}$.

Now we can treat it as a mixed numeral: -1 times a^2, plus 4, over a^2, or $\frac{-a^2+4}{a^2}$. We can now rewrite the original complex fraction as our new numerator divided by our new denominator, as follows:

$$\frac{a-2}{a} \div \frac{-a^2+4}{a^2} = \frac{a-2}{a} \cdot \frac{a^2}{-a^2+4} = \frac{a-2}{\cancel{a}} \cdot \frac{\cancel{a}(a)}{4-a^2} = \frac{a-2}{1} \cdot \frac{a}{(2+a)(2-a)}$$

At this point, allow for a brief explanation.
(a-2) over (2-a) will reduce to -1. You can put that -1 either in the numerator or denominator. I will keep it as the numerator of the first fraction. Here goes:

$$\frac{a-2}{1} \cdot \frac{a}{(2+a)(2-a)} = \frac{-1}{1} \cdot \frac{a}{2+a} = \mathbf{\frac{-a}{2+a} \text{ or } -\frac{a}{2+a} \text{ or } \frac{a}{-2-a}}$$

29) You are asked to solve the following, algebraically for x:

$\sqrt{3x+1}+1=x$	Subtract 1 from both sides.
$\sqrt{3x+1}=x-1$	Square both sides.
$3x+1=x^2-2x+1$	Subtract 3x + 1 from both sides
$x^2-2x+1-3x-1=0$	Combine like terms.
$x^2-5x=0$	Factor
$x(x-5)=0$	Set each factor equal to 0, and solve for x.
x=0 x-5=0	Add 5 to each side.
x=5	

When solving a radical equation you have to check your answers to make sure the roots work and are not extraneous

For x = 0

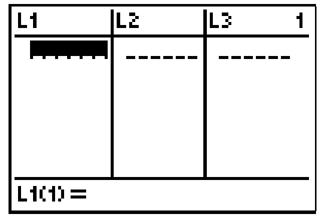
$\sqrt{3x+1}+1=x$
$\sqrt{3(0)+1}+1=0$
$\sqrt{0+1}+1=0$
$\sqrt{1}+1=0$
$1+1=0$
$2=0$ **reject**

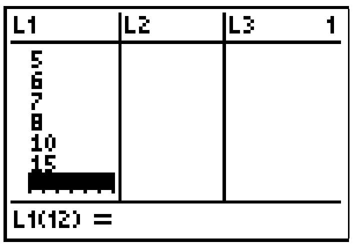
For x=5

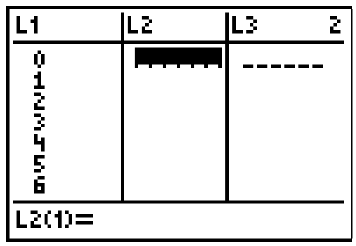
$\sqrt{3x+1}+1=x$
$\sqrt{3(5)+1}+1=5$
$\sqrt{15+1}+1=5$
$\sqrt{16}+1=5$
$4+1=5$
$5=5$ **T**

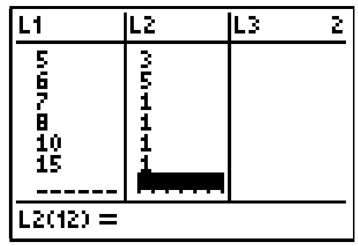
ANSWER: x = 5

30) Step number one will involve entering the information into your calculator. Press **STAT** **ENTER** and you will see the following screen . It is called the STAT LIST EDITOR screen. This is the screen where you will enter your data. You can enter all the data for **Number of Children** into L1 (list 1), and all the data for Number of **Presidents** into L2 (list 2). The simplest way to enter the data is to first enter the data for the number of years into L1. This is done by simply typing each number and then hitting the **ENTER** key. Below, to the left is a screen capture of what your screen will look like after you have entered all the data into L1. You can scroll up and down to make sure you entered all the data properly. Now hit the ► right scroll key once and you will immediately see the see the middle screen pictured below. You are now ready to enter the data for L2. Enter the data the same way you entered it into L1. Enter each number followed by the ENTER key. The final screen capture shows all the data entered into L1 and L2.

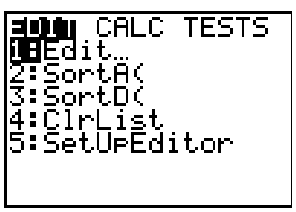


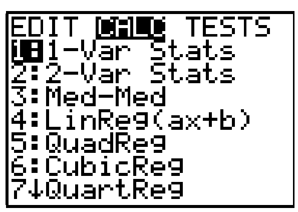




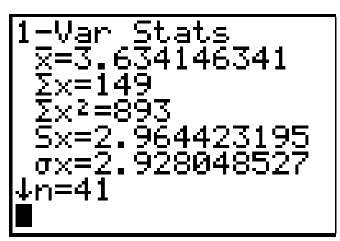


Your first task is to determine mean and standard deviation for this data to the nearest tenth. Again, press **STAT** followed by the ► right scroll key and you will access the **CALC** menu. Now the item you want is the first one 1-Var Stats. However, don't forget that you have entered two lists. The second list was the frequency of the data entered in L1. You have to take this into account when determining the mean and standard deviation. Below you see the first two screen captures of what your calculator screen will look like. as soon as you hit **ENTER** or **1** to select 1-Var Stats your screen will look like the third one below. It is now important to enter L1, L2 the way you see below on the final screen capture. This makes sure you are incorporating the frequency of the data.



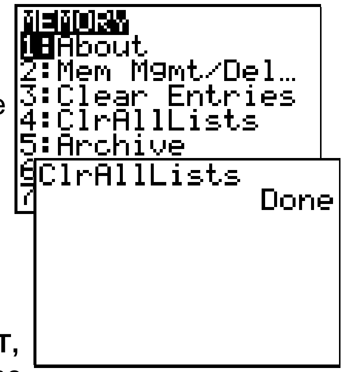


At this point hit **ENTER** and you will have the screen you see at the right, and your answer. Notice, by the way, that your data consists of 41 items. That is indicated by that n on the last line. If you add up the numbers you entered in L1, they should equal 41, your total frequency. At the right you see that your **mean, ü, equals 3.6** and the **standard deviation, Fx, equals 2.9**, all to the nearest tenth. Now, since the mean is 3.6, one standard deviation above the mean will be 3.6 + 2.9 or **6.5**. One standard deviation below the mean will be 3.6 - 2.9 or **.7**. So how many presidents have between .7 and 6.5 children? In essence you will now simply count the number of presidents who have 1, 2, 3, 4, 5, or 6 children. Look at the table presented with the problem, or at the two lists you entered. The total is: 2+8+6+7+3+5 or **31**.

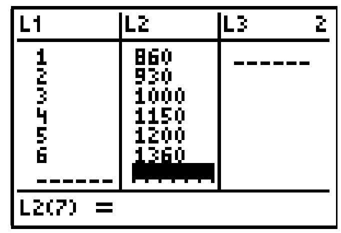


ANSWER: Mean = 3.6 Standard Deviation = 2.9 31 presidents fall within one SD of the mean.

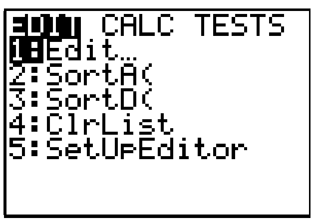
31) For this problem you again have to enter data into lists. You can use L3 and L4 since L1 and L2 still have the information from the previous problem, but here is a quick way to clear all lists.
Hit **2nd** followed by **+** the plus key. Remember that after you hit the 2nd key, the next key you will be accessing will be the "yellowish" one above the key you are actually hitting. In this case you will actually be accessing the **MEM** key. At this point select choice 4 and hit **ENTER**. You will see the second screen at the right. The data you entered for L1 and L2 will no longer be there.

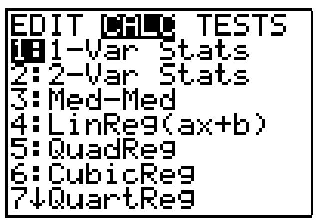


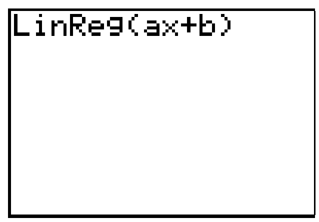
You can now follow the instructions as in the last problem in order to enter the data for this problem into the two lists. In summary, hit **STAT,** make sure the cursor is at the top of column L1, and begin to enter the data for that column. As before, it is probably easiest to enter all the Day data into L!, and then enter all the data for L2. To the right is how your screen will look after you've entered the last item in L2.

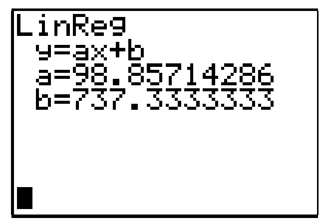


You are asked to write the linear regression equation for this set of data rounding your numerical coefficients to four decimal places.
Once your data is entered, hit the following keys:
STAT ► **4** **ENTER** Below is a screen capture for each one of the screens that you will be seeing as you hit the above keys:

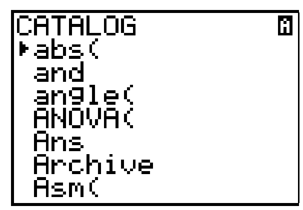




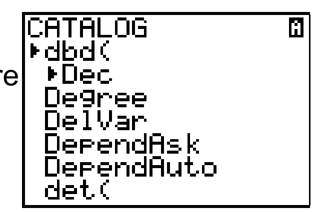




There is your answer on the last screen capture above. But first it's time for a little break.
You may sometimes encounter a problem in which you are required to know the correlation coefficient in addition to the linear regression formula. Let me show you how to get it if it does not automatically appear on screen.
Hit the **2nd** key followed by the **0** key. This allows you to access the **CATALOG** menu. The little shaded A at the top corner of the screen lets you know that you are in ALPHA mode and can now access the alphabet keys on your calculator. They are the ones you see in teal on the upper right of some keys. You are looking now to turn Diagnostics On.

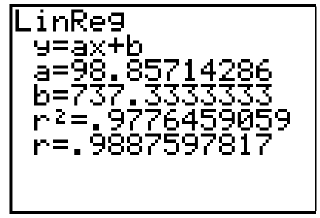


You can scroll down till you get to what you are looking for. However here is how you can jump down directly to the D's. Simply hit **X^{-1}** key.
If you look carefully above it, you will see a teal D, which is the key you are really now accessing. Now scroll down until you see **DiagnosticOn** and hit **ENTER ENTER**.



OK, that's the end of the break. The problem continues on the next page.

Hit STAT ► 4 ENTER again and look at the screen capture now. The little r that you see is your correlation coefficient. It tells you how accurate your regression formula is and whether it is a positive or negative correlation. Now the a and b that you see in the screen at the right are the coefficients in the linear equation y = a x + b. (You are used to seeing it as y= m x + b) Substituting the a and b values into the equations (rounded to 4 decimal places) you get a linear regression equation of:
y = 98.8571x + 737.3333



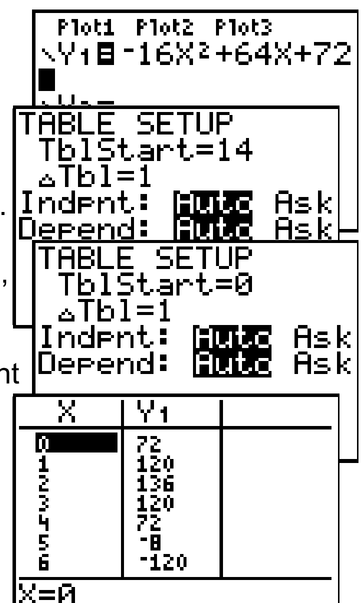
Now you are to use the above equation to determine the day metal sheets will be shipped. You are told that the factory ships when there is a minimum of 2,050 sheets in stock. You are told that x represents the day. So, let y = 2050 and set up your equation.

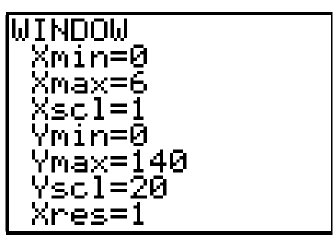
2050 = 98.8571x + 737.3333	Subtract 737.3333 from both sides.
1312.667 = 98.8571x	Divide both sides by 98.8571.
13.2784 = x	

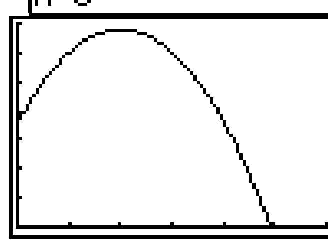
The sheets will be shipped on day 14.

ANSWER: Linear regression equation y = 98.8571x + 737.3333
Sheets will be shipped on day 14.

32) This problem requires you to draw a parabola that opens to the bottom. In other words it will have a maximum turning point. First use your calculator and enter the given equation using the **Y=** editor. Enter it the way you see it, except to make inputting easier let's use the variable x instead of t. When entering the negative sign in front of the 16 make sure to use the **(-)** which is on the bottom row of your calculator, to the left of the **ENTER** key. Now let's display the table this graph generates. We can then use the table to help us graph the equation. Before we display the table, we know that when x (which represents seconds) is 0, y will equal 72. This is so because the rocket is launched from a height of 72 feet. To save some scrolling you can use **TBLSET** (the table set up screen). Hit the following keys: **2nd** followed by **WINDOW.** You want the value TblStart= to read 0. So simply type in a 0 and hit **ENTER**. When you now hit 2nd followed by GRAPH, you will see the TABLE generated by this graph. You can use these points to draw your graph. (0,72), (1,120), (2,136), (3,120), and (4,72) You are not being asked for the turning point, but you clearly see it at (2,136). You can now hit the **WINDOW** key and input the information on your screen as you in the screen capture to the right. All the way to the right is the screen capture of the graph as generated by your calculator using the window you have just set up. On the next page you will see what the graph would look like on the graph paper provided by the regents.



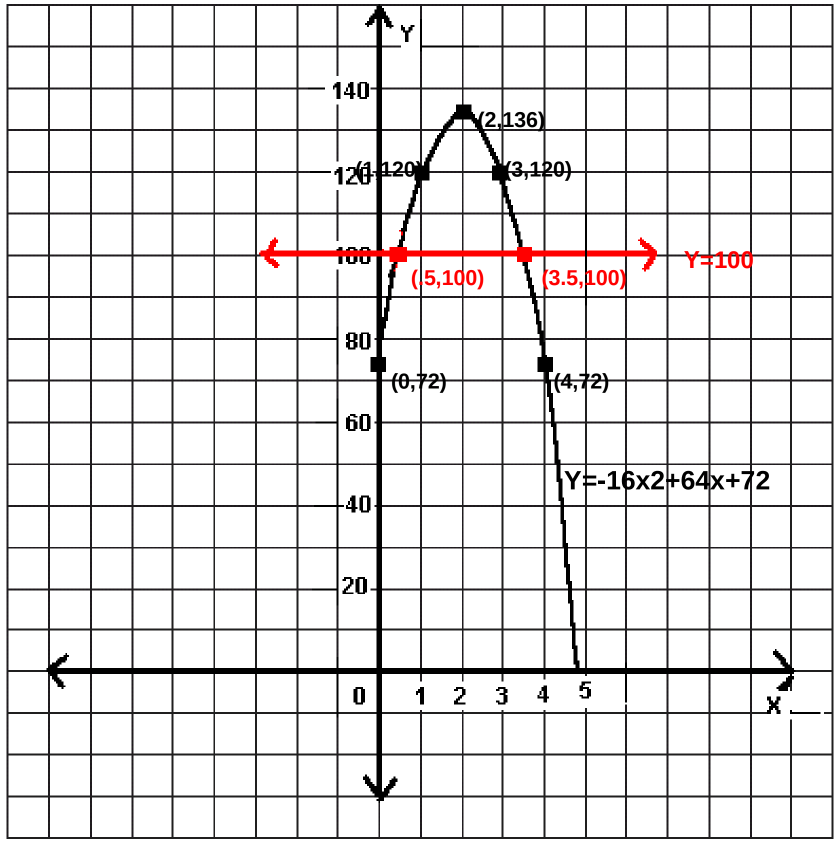




To the right you see a graph that represents the given equation:

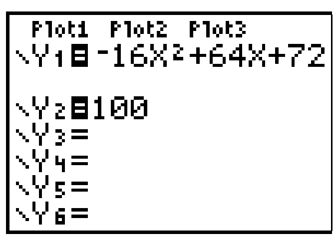
$h(t) = -16t^2 + 64t + 72$

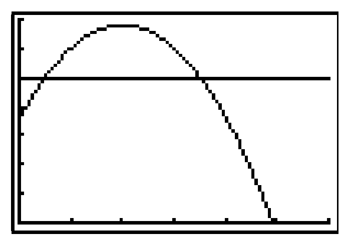
I should have mentioned earlier that on the graph I have drawn, the x-axis is your t, which represents time in seconds. The Y-axis is your h(t), which represents the height of the rocket at each point in time. The points I used in the graph are the the ones shown on your calculator when you generated your table as shown in a screen capture on the page before this one.



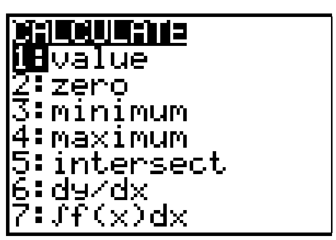
One more question remains to be answered. That is, to determine the number of seconds that the rocket will remain at or above 100 feet. Notice that red line which is the equation of y=100. In essence what you want to now determine is the value of the points of intersection of the parabola and that line where y=100. The x values of those two points signal where the height first hits 100, continues to rise, and then descends to 100 again. The distance between these two points will therefore be your answer to this question.

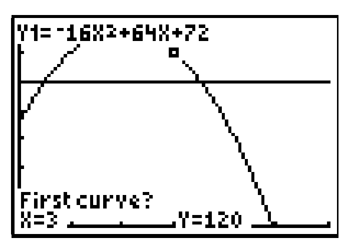
Let's use the calculator to determine the points of intersection which I have already indicated on the graph in red. This will take a while to explain but the method is quite simple. First enter the second equation, y=100 into the y= editor. You will now have two equations that will be graphed simultaneously when you hit the graph key.

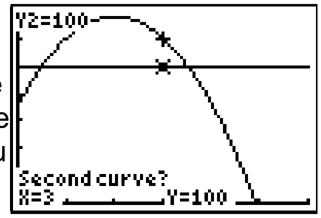


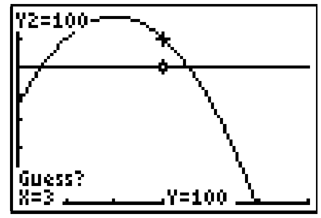


Now here are the instructions for finding the points of intersection. You will be using the the **intersect** command which is found on the **CALCULATE** menu. Here are the steps for finding an intersection. First hit 2nd followed by **TRACE**. This will get you to the **CALCULATE** menu as you see at the right. Now select item **5: intersect**. As soon as you do this, you will see the graph displayed again, but prompting you at the bottom left of your screen with **First curve?** Notice that little blip that just appeared on the parabola. You also know that it is a point on the parabola because the screen gives you its equation.

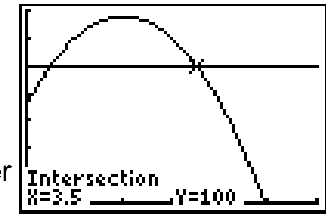




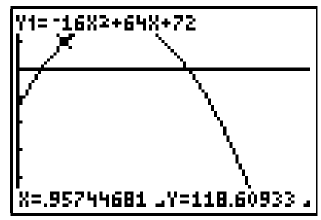




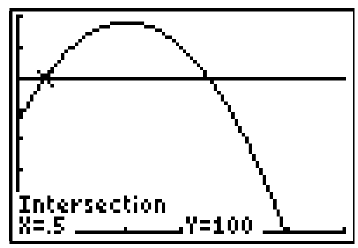
OK…you have just selected 5 and are at the last screen shown on the previous page. Hit **ENTER** and you will see a similar screen but this time the blip will be on the next graph, and you will see the prompt **Second curve?** Hit ENTER again and you will see almost the same screen again, but this you will see the prompt **Guess** at the lower right.



Hit **ENTER** one final time and WOW! The calculator gives you the point of intersection and its coordinates! You see that the coordinates of the intersection at the right are **(3.5 ,100)**
Now finding the intersection of the other side is just as simple.



First hit the **TRACE** key. Then use the ◀ or ▶ key to move in the proper direction closer to the second point of intersection. (It happens to be that if your calculator was set up the way mine was, you will not be moving to to the left, but you could just as easily have found the left intersection point and have to move to now move towards the right.) When your blinking blip (no, I am not cursing. It is blinking.) is close to the other point of intersection, follow the instructions you used to find the first point of intersection: Hit **2nd TRACE 5 ENTER ENTER ENTER** and you will see the other point of intersection. It is the point **(.5 , 100)**.

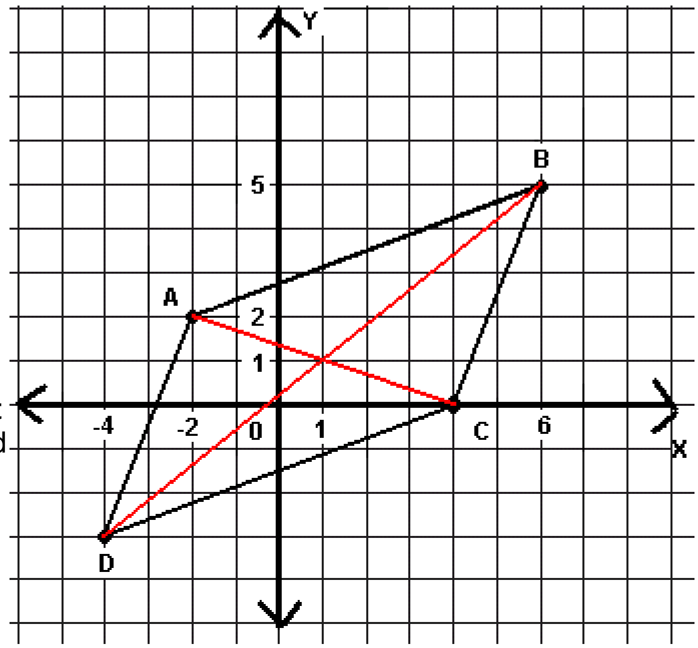


To review quickly, your two points of intersection are:
(.5 , 100) and **(3.5 ,100)**
What this means is that at **.5** seconds the rocket will reach a height of 100 feet, continue rising until the turning point, begin its descent, and again reach 100 feet at **3.5** seconds into flight.
3.5 - . 5 = 3

The rocket will remain at or above 100 feet for a total of 3 seconds.

Problem 33 starts on the next page

33) You are given the coordinates of points A, B, C, and D, and are asked to prove ABCD is a parallelogram but not a rectangle. It really is not necessary to graph the quadrilateral but it does help give you a clearer picture of which sides you will be working with.



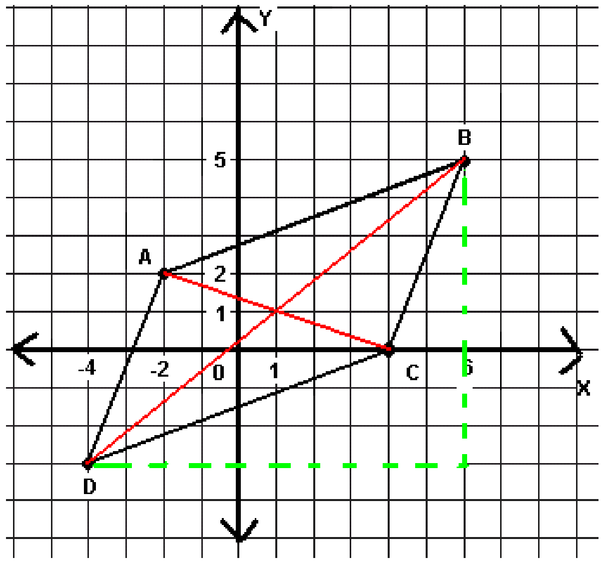
A(-2,2) B(6,5) C(4,0) D(-4,-3)
At the right you see the quadrilateral formed when these points are connected. Now here is the plan. In order to prove that ABCE is a parallelogram, all that is necessary to show is that its diagonals bisect each other. I have drawn the diagonals in red so that you can see their endpoints. The diagonals are line segments AC and BD. **In order to prove that they bisect each other, we have to show that they share a common midpoint.** We find the midpoint of a line segment by adding its coordinates and dividing by 2. To find the x-coordinate at the midpoint, we add the two x-coordinates and divide by 2. To find the y-coordinate at the midpoint, we add up the two y-coordinates and divide by 2.

Let's first find the midpoint of $\overline{AC}$. A(-2,2) C(4,0) (-2 + 4) = 2, divided by 2 = 1
(2 + 0) = 2, divided by 2 = 1
The coordinate of the midpoint is (1,1)

Now let's find the midpoint of $\overline{BD}$. B(6,5) D(-4,-3) (6 + -4)= 2, divided by 2 = 1
(5 + -3)=2, divided by 2 = 1
The coordinate of the midpoint is (1,1)

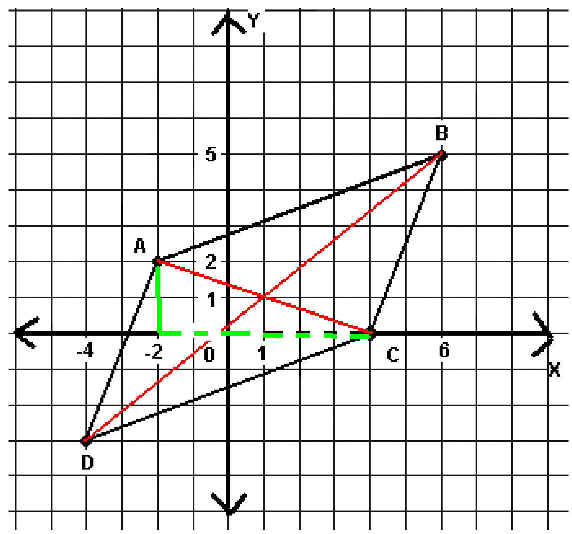
Since diagonals $\overline{AC}$ and $\overline{BD}$ share the same midpoint of (1,1), they bisect each other, and ABCD is a parallelogram.

One way to now prove that ABCD is not a rectangle is to show that one of its angles is not a right angle. Let's use angle D. For angle D to be a right angle, side AD and CD would have to be perpendicular to each other. To show graphically whether or not they are perpendicular requires you to compare their slopes. The slopes of perpendicular lines are negative reciprocals. That means their product equals -1, For example 2/3 and -3/2 would be negative reciprocals. Generally when you are given the coordinates of the endpoints of a line, you can determine the slope of the line by subtracting one y-coordinate from each other and then one x-coordinates from the other. The difference of the y-coordinates becomes the numerator of the slope, while the difference of the x- coordinates becomes the denominator of the slope. However, in a case like ours where you actually have the polygon graphed it is much easier to simply determine the slope by counting units. I will explain the method on the next page.



The slope of a line is the change in y divided by the change in x. Follow the dotted green lines at the right. Begin from point B, and move down 8 units. You are now at the point where you can make a left turn and move to point D. You would be moving 10 units to the left from the point you turn to get to point D. Here is how you now determine the slope. You first moved down from point B for a total of 8 units. That is represented by -8 (negative 8).
You then turned and moved left for a total of 10 units to get to point D along the x-axis That is represented by -10 (negative 10).

The **slope of BD** is therefore $\frac{-8}{-10} = \frac{8}{10} = \mathbf{\frac{4}{5}}$



In the same manner we can determine the slope of AC. Follow the green line again, this time from A moving down, and then to the right, to point C. We move down the y-axis for a total of 2 units. That is -2 (negative 2) in the y-direction, followed by 6 units to the right till point C. That is +6 along the x-axis.

The **slope of AC** is therefore $\frac{-2}{6} = \mathbf{-\frac{1}{3}}$

$\frac{4}{5}$ and $-\frac{1}{3}$ are not negative reciprocals.
Therefore BD is not perpendicular to AC.
Angle D is therefore not a right angle.
ABCD is therefore not a rectangle since all the angles of a rectangle have to be right angles.

34) First for the cool way of doing this problem and then for the way most students probably would have completed it. There is an old formula known as Heron's formula that is tailor made for this type of problem. You are presented with the three sides of a triangle and asked for its area.
Here is Heron's formula:

$$\mathbf{A = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{a+b+c}{2}}$$

s is known as the semiperimeter, because that is what it is--half of the perimeter.
a, b, and c are the lengths of the sides of the triangle.
The triangle in our problem has sides with measures of 5, 7, and 10.

Go on to the next page please.

Let your a, b, and c equal 5,7,and 10 respectively. Your semiperimeter, s, will equal $\frac{5+7+10}{2} = 11$

Let us now use Heron's formula:

A= $\sqrt{s(s-a)(s-b)(s-c)}$	Substitute the known values.
A = $\sqrt{11(11-5)(11-7)(11-10)}$	Simplify.
A = $\sqrt{11(6)(4)(1)}$	Continue simplifying.
A = $\sqrt{264}$	Use your calculator.
A = 16.24807681	Round to nearest tenth.

ANSWER: AREA = 16.2 square meters

Alternate method using the formula given in your Regents booklet for the area of a triangle.

$K = \frac{1}{2} ab \sin C$

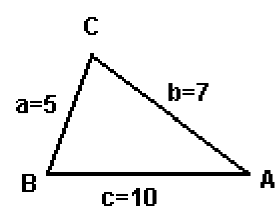
In addition, in order to use the above formula you have to use the Law of Cosines to find one of the angles of your triangle. The general Law of Cosines as presented in your booklet is:

$a^2 = b^2 + c^2 - 2bc \cos A$ where a, b, and c are the sides of a triangle, while the capital letters (in the case at the left, A) represent the angle opposite that particular side. Capital A would be the angle opposite side a.

Basically the formula begins with the square of one of the sides being equal to the sum of the squares of the remaining two sides, minus 2 times the product of these remaining two sides multiplied by the cosine of the angle opposite the side the formula begins with.
So if instead of beginning with a^2 it would begin with b^2 , it would look as follows:

$b^2 = a^2 + c^2 - 2ac \cos B$

Picture the triangle at the right, and imagine it having sides of 5, 7, and 10. Here would be the three versions of the Law of Cosines that you would be able to use, depending on whether you are looking to find angles A, B, or C

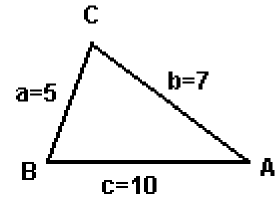


$5^2 = 7^2 + 10^2 - 2(7)(10) \cos A$
$7^2 = 5^2 + 10^2 - 2(5)(10) \cos B$
$10^2 = 5^2 + 7^2 - 2(5)(7) \cos C$

In our case, it makes no difference at all which angle we find. Whichever angle we find we will then be able to use to determine the area of the triangle by using the formula mentioned earlier.

$K = \frac{1}{2} ab \sin C$ It too, can be rewritten as $K = \frac{1}{2} bc \sin A$, or $K = \frac{1}{2} ac \sin B$. (This formula, like the Law of Cosines depends on the relationship of 2 sides and the included angle.)

Ok…let's use the Law of Cosines to determine angle A.



$5^2 = 7^2 + 10^2 - 2(7)(10)\cos A$	Square and multiply.
$25 = 49 + 100 - 140\cos A$	Simplify.
$25 = 149 - 140\cos A$	Subtract 149 from both sides.
$-124 = -140\cos A$	Divide both sides by -140
$\frac{-124}{-140} = \cos A$ or simply $\cos A = \frac{124}{140}$	In division, two negatives become positive.

Now you can use your **COS^{-1}** key to determine angle A. Angle A will be that angle whose cosine equals $\frac{124}{140}$. Let's not round till after we find the area.

```
cos⁻¹(124/140)
     27.6604499
```

So as far as we are concerned, **angle A = 27.6604499 degrees.**
Now let's use angle A to determine the area of the triangle. Angle A is between sides b, and c, so we can use the formula for the area as follows.

$K = \frac{1}{2}(7)(10)\sin A$	Substitute your value for angle A.
$K = \frac{1}{2}(7)(10)\sin 27.6604499$	Complete your multiplication.
$K = 35\sin 27.6604499$	Find the sine.
$K = 35(.464230766)$	Complete the multiplication.
$K = 16.24807681$	Round to the nearest tenth.
K= 16.2	

```
sin(27.6604499)
      .464230766
```

ANSWER: The area of the triangle is 16.2 square meters.