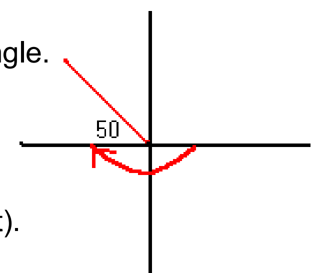
ANSWERS MATH B – June 23rd 2005

1) The range is all the y values, from lowest to highest, as x runs through a specific interval. Here you are asked for the range of the heart rate during a 4-minute interval. You can see on the graph that before the jogger begins, his heart rate is at 60. At 4 minutes it is at 110. Therefore, the range is 60 -110. **ANSWER: (4)**

2) Each trigonometric function will be positive in two quadrants and negative in two quadrants. sine is positive in quadrants 1 and 2. It is negative in 3 and 4. You are asked in which quadrant will sine and cosine be negative. You already know that sine is negative in quadrants 3 and 4. To answer this question you now need to know in which of these two quadrants is cosine negative as well. Cosine is negative in quadrants 2 and 3. So your answer is **quadrant 3** as that is the quadrant where both sine and cosine are negative. One mnemonic that can help you recall the signs of the trigonometric functions in the various quadrants is: **A**ll **S**tudents **T**ake **C**alculus. This sentence consists of 4 words, one for each quadrant. **A**ll: **ALL** trigonometric functions are positive in quadrant 1. **S**tudents: **Sine** which begins with the letter S is positive in quadrant 2. **T**ake: **Tangent** which begins with a T is positive in quadrant 3. **C**alculus: **Cosine** which begins with the letter C is positive in quadrant 4. Now once you know in which quadrants a particular function is positive, you know that it is negative in the other two quadrants. For example since cosine is positive in quadrants 1 and 4, it will be negative in quadrants 2 and 3. **ANSWER: (3)**

3) You are asked to express sin (-230°) as a function of a positive acute angle. Imagine the angle formed by the red arc which moved in a clockwise direction as being -180°. If it were to continue for another -50°, you would have your angle of -230°. You see that angle by the red terminal side in quadrant 2. So for all intents and purposes, an angle of –230 is the same as an angle of 50° (but in the second quadrant). Now, is sine positive or negative in the second quadrant? It is positive. Therefore, sin (-230°) would equal sin 50°



ANSWER: (1)

4) You are asked to simplify: $\frac{x^2-9x}{45x-5x^2}$ Factor an x from the numerator and 5x from the denominator.

$\frac{x(x-9)}{5x(9-x)}$ Now, the (x-9) and (9-x) are opposites, so their quotient is -1. You can put the –1 in the numerator or denominator. It makes no difference. Let's put it in the denominator. You now have:

$\frac{x}{5x(-1)}$ The x in the numerator cancels with the x in the denominator.

$\frac{1}{5(-1)} = \frac{1}{-5}$ or $-\frac{1}{5}$. **ANSWER: (2)**

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5) You are asked to solve the following absolute inequality:
$|2x+3| > 7$ What this really means is that $2x+3>7$ and $-(2x+3)$ will also be greater than 7.
Let us solve both inequalities:

$2x+3>7$ Subtract 3 from both sides
$2x > 4$ Divide both sides by 2
$x > 2$

$-(2x+3) > 7$ Distribute the negative sign.
$-2x - 3 > 7$ Add 3 to both sides
$-2x > 10$ Divide both sides by –2 (symbol will switch)
$x < -5$

Your answer is that $x < -5$ or $x > 2$. This solution set is depicted by choice 2. **ANSWER: (2)**

6) The general equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$
The center of the above circle would be (h,k) and the radius would be r.
As an example, if the equation were $(x - 3)^2 + (y + 5)^2 = 36$ then
the center would be (3,-5) (always switch those signs),
and the radius would be 6 (the square root of 36).
You are given the equation $\mathbf{(x + 3)^2 + (y - 4)^2 = 25}$ Its center would be (-3,4) **ANSWER: (3)**

7) Perhaps the easiest way to do this problem is to add all the scores and to divide their sum by the number of scores there are. Keep in mind that you can not simply add the 4 scores as there are more than 4 scores. There are really 3 25's, 2 20's, and so on. So the way to do this is to use your calculator to input 25x3 + 20x2 + 11x5 + 10x4. Your sum should be: 210 Next, divide this sum by the number of scores there were. This would be the total of the frequency column which adds up to 14. 210 divided by 14 equals 15. **15 is your mean.** **ANSWER: (3)**

8) In an inverse variation the product of 2 variables is a constant. For example xy=60.
If x is doubled then y will have to be divided by 2 so that the product will still remain 60
ANSWER: (1)

9) You are given the formula $Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2}$ You are also given that $Z_1 = 1+2i$ and $Z_2 = 1-2i$

All you have to next is substitute and simplify by multiplying in the numerator and adding in the denominator.

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(1+2i)(1-2i)}{(1+2i) + (1-2i)} = \frac{1-4i^2}{2} = \frac{1-4(-1)}{2} = \frac{1+4}{2} = \mathbf{\frac{5}{2}}$$

ANSWER: (3)

10) Use the laws of logarithms for this problem. (Remember that $\sqrt{b} = b^{½}$)
Product Rule: The log of a product will be equal to the sum of the log of its factors.
For example, log (4)(6) will equal log 4 + log 6.
Power Rule: The log of a number raised to a power will equal to the power times the log of that number. For example, $\log 6^3 = 3 \log 6$. For our problem, remember that $\sqrt{b} = b^{½}$

$\log a\sqrt{b} = \log a + \frac{1}{2} \log b$ Now substitute the given values for logs a and b.

$x + \frac{1}{2} y$ or $x + \frac{y}{2}$ **ANSWER: (4)**

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11) You are given 4 relations and asked to identify the one that is a function. For a relation to be a function, there must be exactly one specific y-value for any x-value. One simple way of determining if a relation is a function is called the “vertical line” test. If when drawing a vertical line, you will intersect exactly one point on the graph of the relation, then the relation is a function. If however you can draw a vertical line that will cross more than one point, or no points, then the relation is not a function. A circle or ellipse are examples of relations that are not functions. When drawing a vertical line you will generally cross 2 points. In this problem, **Choice 1 is the answer**. It will be a hyperbola in quadrants 1 and 3. You can ascertain this by entering into your calculator’s y-editor the equation y=7/x. For any given x value, there will be only one specific y value, such that the product of xy will be 7. If you graph it you will see that any vertical line will intersect the graph at only one distinct point. Choice 2, x = 7, represents a vertical line which will have an infinite number of y values for x, such as (7,0), (7,1), (7,2) to name a few. Choice 3 and choice 4 both contain a y^2. They are ruled out for that reason. If you were to solve these relations for y, at some point you would end up with y^2 on one side of the equation and something with x on the other side. Your next step would entail getting the square root of both sides so that you can solve for y. At this point you would end up with y = plus or minus something with x. For example, imagine ending up with something like $y^2 = x^2 - 7$. Taking the square root of both sides would result in $y = \pm\sqrt{x^2-7}$. Now lets try the value of 4 for x. This would result in $y = \pm\sqrt{4^2-7} = \pm\sqrt{16-7} = \pm\sqrt{9} = \pm 3$. This means that you would have the 2 points for your solution set: (4,3) and (4,-3). This therefore can not be a function as there are two different y values for the same x. **ANSWER: (1)**

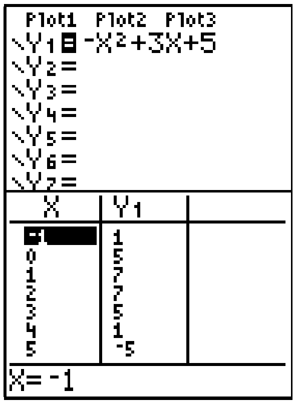
12) You are asked to determine which of the 4 given equations would be represented as a ellipse. Choice #4 is your inverse variation which is represented by a hyperbola in quadrants 1 and 3. Choice #3 is a linear equation.
Choices #2 and #1 differ in the sign before the second term. Here is a brief rundown on the equation of a circle, ellipse and hyperbola. $ax^2 + bx^2 = c$ represents a circle only if a and b are equal. If a and b are not equal, and the signs of a, b, and c are the same then the equation represents an ellipse whose center is the origin. If you have $ax^2 + bx^2 = c$, and the signs of a and b are different then the equation represents a hyperbola. This is depicted by the equation in Choice #2. Choice #1 represents an ellipse since the signs of a, b, and c are the same. **ANSWER: (1)**

13) To simplify the given expression, multiply the numerator and denominator by the conjugate of the denominator. The denominator is 3+i. Its conjugate is 3-i. **(Remember that $i^2 = -1$)**

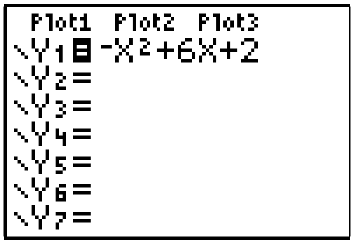
$$\frac{2+i}{3+i}\bullet\frac{3-i}{3-i} = \frac{6-2i+3i-i^2}{9-i^2} = \frac{6+i-(-1)}{9-(-1)} = \frac{6+i+1}{9+1} = \mathbf{\frac{7+i}{10}}$$

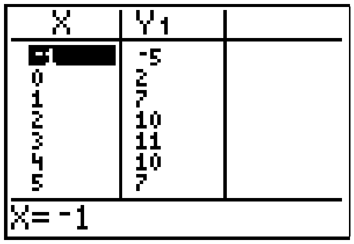
ANSWER: (4)

14) There is more than one way to this problem. I prefer the following. If you know that the axis of symmetry of an equation is x=3 then you know that its turning point will also have an x-coordinate of 3. By looking at a table of values you can tell which point is the turning point since the y values on either side of this x will match (be symmetric). So let’s begin this problem by inputting the equation using the y= editor of your calculator. To begin, hit the y= key. Then enter the first equation as you see it. (Don’t forget that to indicate a negative sign you use the key to the left of the ENTER . It is the negative sign in parentheses). On the next page is a screen capture of what your screen should look like after having entered the first equation.



Now let's take a look at the table generated for this equation. Hit 2nd GRAPH **(Remember that after hitting the yellow 2nd key, you will be accessing the symbol to the upper left of the key you are actually hitting.)** This will bring up the table for the equation you just entered. The second screen capture to the left shows what you screen will look like. Look carefully at this table and you will see that the turning point for this equation will be right between x=1 and x=2. It is on either side of these values that the y-values match. So choice #1 is not your answer. Now do the same for choice #2. Below you will see the screen capture after entering the equation, and a screen capture of the table.





Now look at the table at the left. You clearly see that the turning point for the entered equation is x=3. The y-values match on either side. This means that the axis of symmetry for this equation is x=3. **ANSWER: (2)**

15) You are presented with the following equation: Mw cos θ = w sin θ
If both sides are now divided by w, your equation becomes M cos θ = sin θ
Now let's look at the choices that are given. You should know the following relationships:

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\csc\theta = \dfrac{1}{\sin\theta}$

Now back to our problem.
M cos θ = sin θ Divide both sides by cos θ
$M = \dfrac{\sin\theta}{\cos\theta}$ or substituting **M= tanθ** **ANSWER: (1)**

16) You are asked to determine the value of x.

$(a^x)^{\frac{2}{3}} = \dfrac{1}{a^2}$ The way to do this is to express both sides as powers of the same base.

Here is how to do it for this problem.
When raising a power to a power we actually multiply the powers. Therefore:

$(a^x)^{\frac{2}{3}} = (a)^{\frac{2x}{3}}$ and $\dfrac{1}{a^2} = (a)^{-2}$ The bases are now both equal to "a".

Here is what you now have: $(a)^{\frac{2x}{3}} = (a)^{-2}$ Now once the bases are equal the powers have to be equal to each other so you now have a new equation to solve:

$\dfrac{2x}{3} = -2$ Multiply both sides by 3.
2x = -6 Divide both sides by 2.
x = -3 **ANSWER: (3)**

17) You are asked to find the third term in the expansion of $(\cos x + 3)^5$

This is a problem using the Binomial Theorem. Here is a brief explanation of the theorem. When raising a binomial to a power you will have numerical coefficients. If you are raising a binomial to the 3rd power, your coefficients will be ${}_3C_0$, ${}_3C_1$, ${}_3C_2$, ${}_3C_3$. This shows you that you will always have one more term than the number of the power. In the case above when raising to the 3^{rd} power you will end up with 4 terms. If you are raising a binomial to the 5^{th} power, as in our problem, you will have 6 terms, and their coefficients will be ${}_5C_0$, ${}_5C_1$, ${}_5C_2$, ${}_5C_3$, ${}_5C_4$, ${}_5C_5$.

Each term in the expansion will contain a numerical coefficient, an "x" factor, and a "y" factor. The sum of the powers of the "x" and "y" factors will always be the exponent to which you are raising the binomial. In our problem, the X-term is cos x and the Y-term is 3, and the sum of the powers will always equal 5.

The exponent of the y-term will always match the subscript of the numerical coefficient. And once you know the exponent of the y-term you will automatically know the exponent of the x-term because both exponents have to add up to the exponent to which you are raising the binomial.

OK…let's begin… You are being asked for the 3^{rd} term of $(\cos x + 3)^5$

You know that the numerical coefficient of the third term will be: $\mathbf{{}_5C_2}$. ${}_5C_0$, ${}_5C_1$, $\mathbf{{}_5C_2}$

The exponent of the y-term will match this subscript and therefore the y-term will be: $\mathbf{3^2}$

The exponent of the x-term will therefore be 3 because the exponents of the x and y-terms have to add up to 5 in our case. The x-term will therefore be: $\mathbf{(\cos x)^3}$

Put it all together for the final answer:

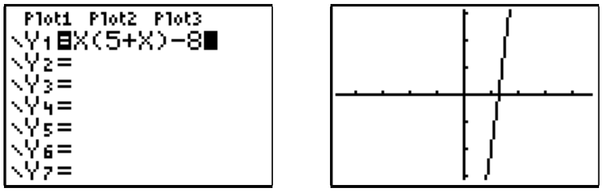
$\mathbf{{}_5C_2(\cos x)^3 (3^2) = 10 \cos^3 x\ (9) = 90 \cos^3 x}$ ${}_5C_2=10$ and $(\cos x)^3 = \cos^3 x$ **ANSWER: (4)**

18) You are presented with 4 equations and asked to determine which one has imaginary roots. One way to do this problem would be to set each equation equal to zero and find the value of its discriminant. If the discriminant is negative then the equation has imaginary roots. The discriminant is represented by b^2-4ac. Here is how you would check choice #1 using this method.

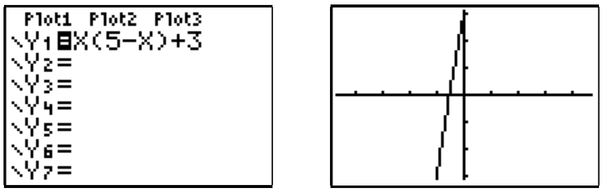
$x(5+x) = 8$	Use the distributive property.	
$5x + x^2 = 8$	Subtract 8 from both sides.	
$5x + x^2 - 8 = 0$	Put into the form of $ax^2 + bx + c$	
$x^2 + 5x - 8 = 0$	a = 1 b = 5 c = -8	Now determine the value of the discriminant
b^2-4ac	Substitute the values for a, b, and c.	
$(5)^2 - 4(1)(-8)$	Multiply	
25 + 32		
57	The discriminant is not negative. (It happens to be positive and not a perfect square. This means that the roots of this particular equation will be irrational and unequal).	

You can now use this method for the remaining 3 choices. The one with the negative discriminant will be your answer as that will be the equation with the imaginary roots.

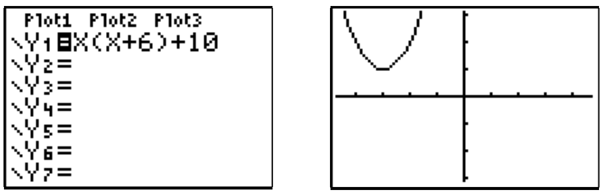
Here is another way to do this problem using your calculator. Enter each equation into your y= editor, and then hit the GRAPH key. If any part of the graph crosses the x-axis then its roots are not imaginary. All you have to do first is set the equation equal to 0, you don't have to set it into into the form of $ax^2 + bx + c$. This means that you can enter the first equation into your y= editor as x(5+x)-8. On the next page you will see these equations entered and the graphs they produce.



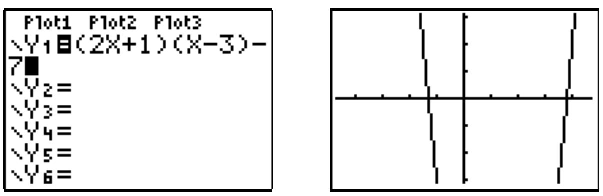
To the left you see how to enter equation 1 into your calculator. You also see enough of the graph that lets you know that it does cross the x-axis and therefore has real and not imaginary roots.



Equation 2 is pictured at the left. By now you understand how to set it equal to zero. The –3 became a +3 when you moved it from one side of the equal sign to the other. And again, this graph also crosses the x-axis and therefore does not have imaginary roots.



Here to the left you see equation 3. The parabola opens upwards and does not intersect the x-axis at any point. This indicates that its roots are imaginary.

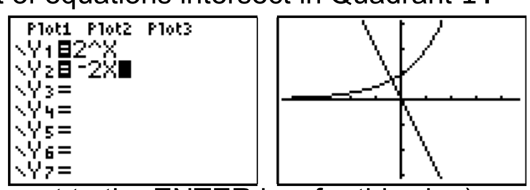


And finally here to the left you see the last equation. You clearly see that it intersects the x-axis and therefore has real roots. By the way, all four equations happen to be parabolas but you would have to adjust the window to see them.

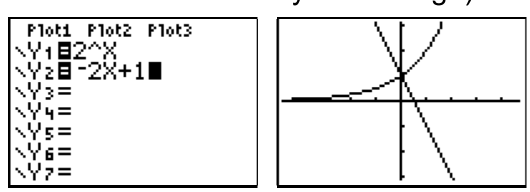
ANSWER: (3)

19) You are asked for what value of "a" will the following set of equations intersect in Quadrant I:

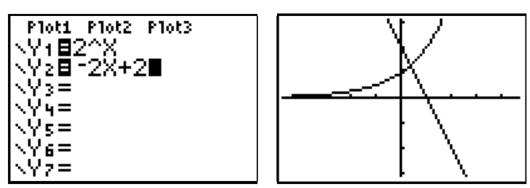
Equation 1: $y = 2^x$ Equation 2: y= -2x + a



If a=0 then the graphs of the two equations would look like the ones at the right. They are now intersecting in quadrant II. Let's see what happens if we substitute a 1 for "a". This time input into the y= editor the equation –2x+1 (Remember to use the negative symbol next to the ENTER key for this sign)

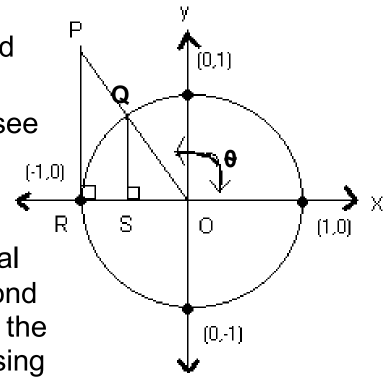


Now look to the right and see where the graphs of the equations will now intersect. They now intersect at (0,1) You see that the graph of y=-2x+a moved to the right as we increased the value of "a" from 0 to 1. It makes sense now that if we increase the value of "a" even more then the line the equation represents will move even more to the right. When it moves more to to the right it will finally intersect the exponential curve in quadrant I. You can see at the right what happens when "a" is 2. As soon as "a" is greater than 1, the graphs intersect in quadrant I. At a=1 it intersects on the y-axis.



ANSWER : (4)

20) The diagram at the right is the one you are more or less presented with for this problem. You are asked to determine which measure represents sin θ . You should immediately realize that what you see at the right is a unit circle. This means that its radius is 1. In addition, the angle represented by θ terminates in the second quadrant. Let us use line segment OQ as its terminal side. Sin pQOS, which is an angle in the second quadrant, will be equal to sin θ . (It is 180 - θ). Since sine is positive in the first and second quadrant, you don't have to worry about the signs. You recall that the sine of an angle is the ratio of opposite divided by hypotenuse. Using triangle QOS, and angle QOS: QO is the hypotenuse, QS is the opposite.



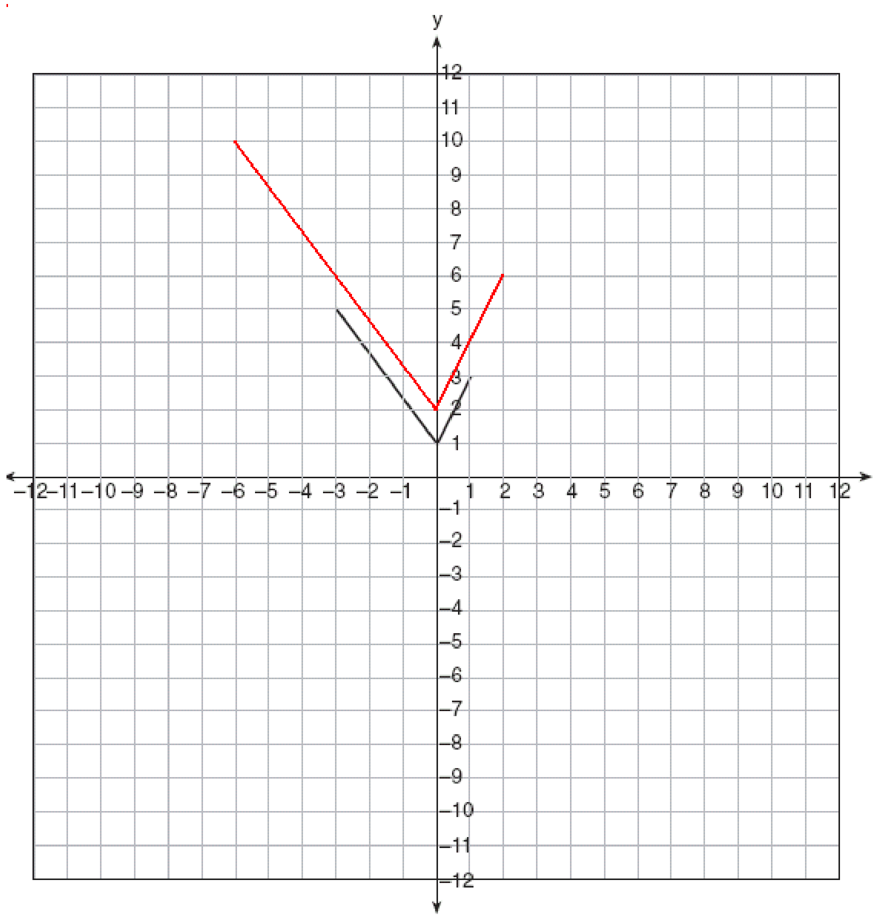
Therefore sin θ = sin pQOS = $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{QS}{QO} = \frac{QS}{1} = \mathbf{QS}$ **ANSWER: (4)**

21) To the right you see in black the function g(x). You are asked to sketch its image under the transformation D_2. This means under a dilation of 2.
Three of its coordinates are:
(-3,5), (0,1), (1,3)
Under a dilation of 2, each point (x,y) becomes (2x,2y).

Under a dilation of 2:
(-3,5) → (-6,10)
(0,1) → (0,2)
(1,3) → (2,6)

On the lines above you see to the left the original point, and to the right you see the image of the point after its x and y were dilated (multiplied) by a factor of 2.

On the graph to the right you see the sketch of this new image in red.



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22) You are given the following equation and asked to solve for m:
(The plan will be the same as what we did in problem 16. You want to try to get the bases equal to each other.)

$3^{m+1} - 5 = 22$	Add 5 to both sides
$3^{m+1} = 27$	27 can be expressed as 3^3
$3^{m+1} = 3^3$	Once the bases are equal, set the exponents equal to each other.
$m+1=3$	Subtract 1 from both sides.
m = 2	

23) Evaluate $\sum_{k=0}^{3}(3\cos k\pi + 1)$ What you have to do here is evaluate the sum of all the values of

3cos k π + 1 as the value of k goes from 0 to 3. In order to get the correct answer when solving the cosine of a constant times π, make sure that you put your calculator into radian mode. Press **MODE** ▼ ▼ At this point the choice for Radian will be blinking on your calculator screen. Now hit the ENTER key and you are set. Just to repeat again, you will now have to calculate the value of 3cos k π + 1 when k goes from 0 thru 3.

$3\cos 0\pi + 1 = 3(1) + 1 = 4$	$\cos 0\pi = 1$
$3\cos 1\pi + 1 = 3(-1) + 1 = -2$	$\cos 1\pi = -1$
$3\cos 2\pi + 1 = 3(1) + 1 = 4$	$\cos 2\pi = 1$
$3\cos 3\pi + 1 = 3(-1) + 1 = -2$	$\cos 3\pi = -1$

The sum of all these values would be 4-2+4-2 which equals 4.

Therefore: $\sum_{k=0}^{3}(3\cos k\pi + 1)$ **= 4**

24) Express in simplest form:

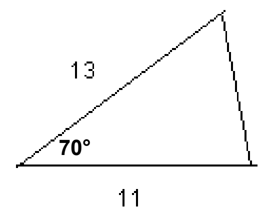
$\frac{1}{x} + \frac{1}{x+3}$ Get both denominators equal to x(x+3), the common denominator.

$\frac{1}{x}\cdot\frac{x+3}{x+3} + \frac{1}{x+3}\cdot\frac{x}{x}$ Both fractions now have equal denominators as you see on the next line.

$\frac{x+3}{x(x+3)} + \frac{x}{x(x+3)}$ Now combine the numerators.

$\mathbf{\frac{2x+3}{x(x+3)}}$ **or** $\mathbf{\frac{2x+3}{x^2+3x}}$

25) The diagram at the right represents the information given provided in this problem. You are given 2 sides and the included angle and asked to find the area of the garden represented by this triangle to the nearest integer. The formula for the area of a triangle when you know two sides and the included angle is:



$K=\frac{1}{2}ab\sin C$ K represents the area, a and b represent the two given sides, and C represents the included angle.

$K=\frac{1}{2}(13)(11)\sin 70^\circ$

$K = 71.5\sin 70^\circ$ Use your calculator to multiply $\sin 70^\circ$ by 71.5

K = 67.18802239 Now round to the nearest integer.

K = 67 **The area of the garden would be 67 square feet to the nearest integer.**

26) You are given the function h(x)= 70 + 0.2x and also the function $g(t)= 300(0.8)^t$. You are then asked to find the value of $h(g(4))$. What this requires you to do, is to first solve g(4) and then to find h of that answer.

$g(t)= 300(0.8)^t$ To find g(4) substitute the value of 4 for t.

$g(4)= 300(0.8)^4$ $(0.8)^4 = .4096$

g(4)= 300(.4096) Multiply

g(4) = 122.88 Now substitute 122.88 for x to solve h(x)

h(x)= 70 + 0.2x

h(122.88)= 70 + 0.2 (122.88)

h(122.88) = 70 + 24.576

h(122.88) = 94.576 **To the nearest whole number this is 95.**

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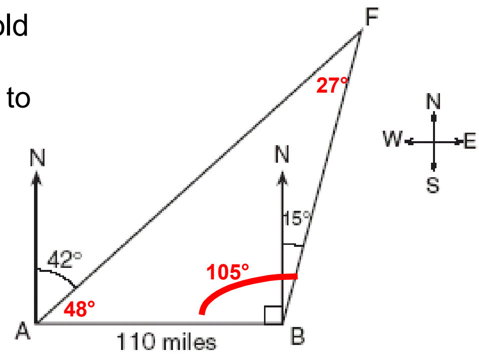
27) You are presented with the diagram at the left. You are told that AB is an east-west line. AN is indicating a northerly direction. This means that pNAB is a right angle and equal to 90°. You are given that pNAF is 42°. This leaves (90-42) or 48° for pFAB. I have indicated the 48 in red. You also know that pFBA is equal to (15+90) or 105°. We now know that pF is (180 – (48+105)) or 27°. There is now enough information available so that we can use the Law of Sines to determine the length of side AF, which represents the distance the fire is from station A.



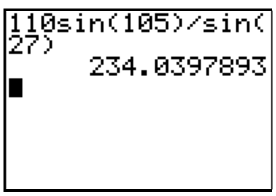
The Law of Sines states that in a triangle, all sides and the sines of the angles opposite these sides are in proportion: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

In our triangle above, we see the angle of 27° and the side of 110 opposite this angle. We want to determine the length of AF which is opposite the 105° angle. So all we have to do now is set up a proportion using the Law of Sines.

$$\frac{\text{side}}{\text{sine of angle opposite this side}} = \frac{AF}{\sin 105} = \frac{110}{\sin 27}$$ Cross multiply.

$AF (\sin 27^\circ) = 110 (\sin 105^\circ)$ Divide both sides by $\sin 27^\circ$.

$AF = \frac{110 (\sin 105)}{\sin 27}$ Use your calculator in **degree mode!**



AF = 234.0397893 Round to nearest mile.

AF = 234 miles

28) You are asked to solve for all values of q that satisfy the following equation:

$\sqrt{3q+7} = q + 3$	Square both sides.
$3q + 7 = (q+3)(q+3)$	Multiply
$3q+7 = q^2 + 6q + 9$	Set equal to 0.
$3q + 7 - q^2 - 6q - 9 = 0$	Simplify and put into standard form.
$-q^2 - 3q - 2 = 0$	To make life easier, multiply each term by –1.
$q^2 + 3q + 2 = 0$	Factor
$(q+1)(q+2) = 0$	Set both factors equal to 0, and solve.

$q+1 = 0$ $q + 2 = 0$

q = -1 **q = - 2**

It is a good idea to check for extraneous roots by substituting your answers in the original equation. In this case, both solutions do check.

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29) You are told that there is a ¾ or .75 probability that a seed will sprout. That means that the probability that it won't sprout is ¼. This is the information necessary to solve this problem. You are asked to determine the probability that if 7 seeds are planted **at least 5** will sprout. To answer this question you will have to figure out the probability that 5 will sprout. Then you will have to figure out the probability that 6 will sprout, and then the probability that all 7 will sprout. Finally you will add all these probabilities to get the answer to "at least 5" will sprout.

Let's begin by computing the probability of 5 seeds sprouting. You are planting 7 seeds. To determine the probability for 5 out of 7 you begin with a $\mathbf{{}_7C_5}$. Then you have to raise ¾ or .75 to the 5^{th} power because that is the probability that 5 seeds will sprout. This will result in $\mathbf{(.75)^5}$. You will now be left with 2 seeds that will not sprout. This will be expressed as $\mathbf{(.25)^2}$ which is the probability that 2 seeds will not sprout. So far you have the probability for 5 seeds sprouting as being $\mathbf{{}_7C_5\,(.75)^5\,(.25)^2}$

Using the same method, here is the probability for exactly 6 seeds sprouting: $\mathbf{{}_7C_6\,(.75)^6\,(.25)^1}$
And here is the probability for all 7 sprouting: $\mathbf{{}_7C_7\,(.75)^7\,(.25)^0}$

Now compute these 3 probabilities, add them together and round to the nearest ten thousandth.
$\mathbf{{}_7C_5\,(.75)^5\,(.25)^2 + {}_7C_6\,(.75)^6\,(.25)^1 + {}_7C_7\,(.75)^7\,(.25)^0}$ = .7564086914 **ANSWER .7564**

```
7 nCr 5*(.75)^5*
(.25)^2+7 nCr 6*
(.75)^6*(.25)^1+
7 nCr 7*(.75)^7*
(.25)^0
      .7564086914
```

You can use your TI-83 Plus calculator to solve combinations. To evaluate $_7C_5$ use the following sequence of keys:
[7] **MATH** ► ► ► You have just accessed the Probability menu. Choice 3 is the one you want **.** Now continue:
[3] [5] ENTER
Your screen should now read **7 nCr 5**.

30) You are asked to find to the nearest degree all values of θ in the interval $0^\circ \le \theta < 360^\circ$ that satisfy the equation $3\cos 2\theta + \sin\theta - 1 = 0$.
Your first objective in solving this type of problem is to convert the equation into a quadratic with only one trigonometric function. Now it contains cos and sin. You should immediately notice the cos 2θ . It is known as the cosine of a double angle. There are 3 formulas that are given for the cosine of a double angle. They are:

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2\cos^2 A - 1$$
$$\cos 2A = 1 - 2\sin^2 A$$

We will use the last one because it contains only a $\sin^2$ and our equation also contains a sin term. So now, where you see the cos 2θ you will substitute $1 - 2\sin^2\theta$.

$3\cos 2\theta + \sin\theta - 1 = 0$	Substitute
$3(1 - 2\sin^2\theta) + \sin\theta - 1 = 0$	Use the distributive property.
$3 - 6\sin^2\theta + \sin\theta - 1 = 0$	Simplify and put in standard form.
$-6\sin^2\theta + \sin\theta + 2 = 0$	Multiply all terms by –1
$6\sin^2\theta - \sin\theta - 2 = 0$	Factor into two binomials (imagine this as if it were $6x^2 - x - 2 = 0$)

(If you have difficulties factoring, you can always use the quadratic formula. See problem #32.)
Continue on to the next page…

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$6\sin^2\theta - \sin\theta - 2 = 0$ Remember that $(\sin\theta)(\sin\theta) = \sin^2\theta$

$(3\sin\theta - 2)(2\sin\theta + 1) = 0$ Set each factor equal to 0 and solve for $\sin\theta$

$3\sin\theta - 2 = 0$ Add 2 to both sides

$3\sin\theta = 2$ Divide both sides by 3

$\sin\theta = \frac{2}{3}$

$2\sin\theta + 1 = 0$ Subtract 1 from both sides

$2\sin\theta = -1$ Divide both sides by 2

$\sin\theta = -\frac{1}{2}$

What remains to be done now is to figure out for what values of θ will sin θ be equal to $\frac{2}{3}$, and for what values will it be equal to $-\frac{1}{2}$. Let's take care of the $-\frac{1}{2}$ first.

By now you should know that $\sin 30^\circ = \frac{1}{2}$. Sine is positive in quadrants I and II. It will be negative in quadrants III and IV. A 30° angle in quadrant III is equivalent to (180+30) or **210°**. You can check that on your calculator. **Sin 210° = -.5 or – ½**. In quadrant IV a 30° angle is equivalent to (360 – 30) or 330°. **Sin 330° = -.5 or – ½**. By the way, had you not known the sine of which angle equals to .5, you could have figured it out by using the $\sin^{-1}$ key of your calculator.

```
sin⁻¹(.5)
                    30
```

Now determine for which values of θ will sine equal $\frac{2}{3}$

Use the $\sin^{-1}$ key which is accessed by first hitting the 2nd key of your calculator, followed by the sin key. You see the answer at the right. You are being asked for the answer to the nearest degree, so θ would be **42°**.

```
sin⁻¹(2/3)
            41.8103149
```

Now the same thing we did with the 30 angle has to be done with the 42° degree angle. This time, however, we are dealing with a positive value. Sine is positive in quadrants I and II. So one value for θ can be 42°. The second value will be a 42° angle in the second quadrant which is equal to (180 - 42) or **138°**.

To sum things up, you determined two values for sin θ : $\frac{2}{3}$ and $-\frac{1}{2}$

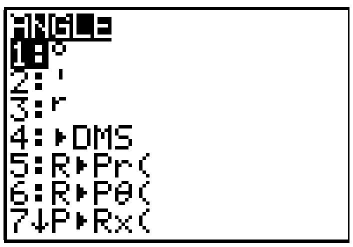
There are two values for θ so that sin θ will equal $\frac{2}{3}$. They are: **42°** and **138°**

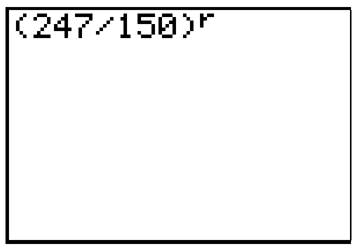
There are also two values for θ so that sin θ will equal $-\frac{1}{2}$. They are **210°** and **330°**

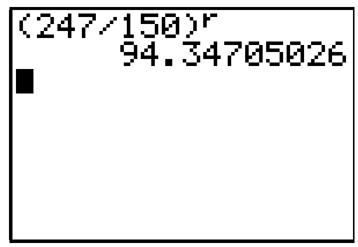
Your final answers are θ = 42°, 138°, 210°, 330°

31) This problem deals with radian measure, so here is a small explanation of what is meant by radian measure. If a central angle in a circle intersects an arc equal in length to the radius of that circle, then the angle is said to be 1 radian measure. In our particular problem, you are told that the radius is 150 ft. This means that if the arc in question, arc AS would also measure 150ft., then its radian measure would be 1. If it would measure 300 ft., its radian measure would be 300/150 or 2 radians. In our problem arc AS happens to measure 247 ft. To find its radian measure we would therefore divide 247/150 and get an answer of 1.646666667 radians. Now all that remains is to convert the radian measure to degree measure. Here is how to use your calculator for this task.
Begin by inputting (247 / 150) Now follow these steps:
2^{nd} **APPS** (this will access the ANGLE menu) **Select choice 3, and hit ENTER.**

Here is a screen capture of what the angle menu looks like and what your screen will show after you enter choice 3. Choice 3 which shows the small r stands for radians and will convert radian measure to degree measure. As you can see, to the nearest degree, a radian measure of 247/150 will equal **94°**.







(Without a calculator: $\frac{pi}{180} = \frac{\text{radian measure}}{\text{degree measure}}$ In our case $\frac{pi}{180} = \frac{1.64666}{x}$ Now cross multiply

$\pi x = 1.64666\,(180)$ Divide by π

$x = 1.64666\,(180)/\pi = 94.34666829$

94° to the nearest degree

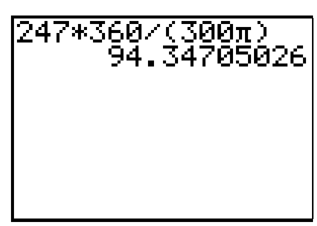
Just for the fun of it here is another way to do this problem. You know that the radius of the circle is 150. You can now compute the circumference of this circle by multiplying the diameter by pi. The diameter is twice the radius or 300. 300 π = 942.4777961
Now recall that AS which is a part of the circumference measures 247.
A complete circle, in this case one with a circumference of 942.4777961 would measure 360°. We can now set up a proportion:

$\frac{\text{Length of arc}}{\text{Degree measure}} = \frac{300(pi)}{360} = \frac{247}{x}$ Cross multiply

$300\pi(x) = 247\,(360)$ Divide both sides by 300 π.

$x = \frac{247(360)}{300pi}$ Use your calculator.

$x = 94.34705026$ Round to nearest degree



ANS: 94°

32) You are given the following equation $y = -2x^2 + 38x + 10$, where x represents time in seconds and y represents the height in feet. You are asked to determine during what interval of time will y be at least 125 feet. Set up your equation. Recall that at least means equal to or greater

$-2x^2 + 38x + 10 \geq 125$ Subtract 125 from both sides

$-2x^2 + 38x + 10 - 125 \geq 0$ Simplify

$-2x^2 + 38x - 115 \geq 0$ Multiply by –1 so that "a" of the x^2 term becomes positive. ($\mathbf{a}x^2+bx+c$)

Remember to switch the direction of the inequality sign when multiplying or dividing by –1.

$2x^2 - 38x + 115 \leq 0$

Now is to use the quadratic formula on the above inequality.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad a = 2 \;\; b= -38 \;\; c = 115$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad a = 2 \;\; b= -38 \;\; c = 115$$ Let us substitute

$$x = \frac{-(-38) \pm \sqrt{(-38)^2-4(2)(115)}}{2(2)} = \frac{38 \pm \sqrt{1444 - 920}}{4} = \frac{38 \pm \sqrt{524}}{4}$$

You now have 2 values for x. Let's solve each one separately.

$x = \frac{38 + \sqrt{524}}{4}$ =15.22276157 To the nearest tenth this is **15.2**

$x = \frac{38 - \sqrt{524}}{4}$ = 3.777238429 To the nearest tenth this is **3.8**

The answer is: $3.8 \leq x$ and $x \leq 15.2$ or $3.8 \leq x \leq 15.2$

Here is a way you can remember how to depict solution sets.

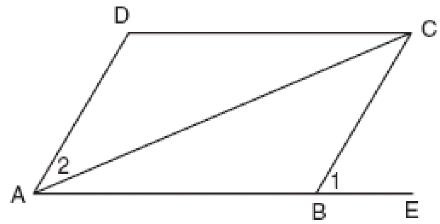
First make sure that the numerical coefficient of the first term is positive. That is why we multiplied our inequality by –1.

For an inequality greater than or equal to 0, think of the direction of the symbol. It is pointing to the right. This means that you will **move to the right from your greater root**, and in the **opposite direction (to the left) from the lower root.** Using the above example, had the inequality been $2x^2 - 38x + 115 \geq 0$, you would first solve it set equal to 0. You get 3.8 and 15.2. You would therefore have $x \leq 3.8$ and $x \geq 15.2$

However, our inequality ended up being $2x^2 - 38x + 115 \leq 0$ (less than or equal to 0). In such a case you will be **moving to the left from your greater term**: $x \leq 15.2$

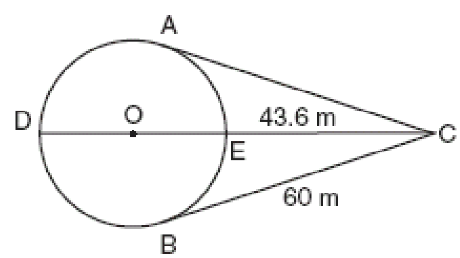
And in the **opposite direction (to the right) from your lower term**: $x \geq 3.8$

33) You are presented with the following parallelogram and asked to prove that mp1 > mp2. There are many ways to do this problem. Here is the way that first caught my eye:



STATEMENTS	REASONS
1. Parallelogram ABCD, diagonal AC, line ABE	1. Given
2. DC is parallel to BA	2. Opposite sides of a parallelogram are parallel.
3. mp1 ≅ mpDCB	3. If parallel lines are cut by a transversal, then alternate interior angles are congruent.
4. mpDCB ≅ mpBAD	4. Opposite angles of a parallelogram are congruent.
5. mp1 ≅ mpBAD	5. Substitution
6. mpBAD > mp2	6. The whole is greater than any of its parts.
7. **mp1 >m p2**	7. Substitution

34) In addition to the information you see in the diagram at the left, you are also told that the ratio of the measure of arc ADB to the measure of arc AEB is 3:2. This information enables you to find the measures of these two arcs. The 2 given arcs comprise the complete circle and therefore total 360°. So you can set up the following equation:

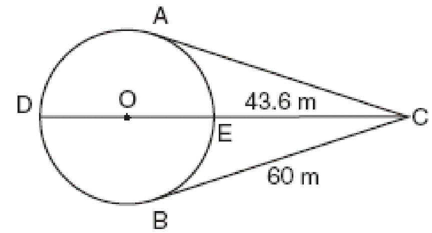


$3x + 2x = 360$ Simplify
$5x = 360$ Divide both sides by 5
$x = 72$

Arc ADB which is represented by 3x is therefore 3(72) or 216° , while arc AEB which is represented by 2x is equal to 2(72) or 144°. We can now calculate the degree measure of pACB.
It is an angle formed by the intersection of two tangents to a circle. The degree measure of such an angle is equal to ½ the difference of the two arcs it intercepts. In our case the two arcs are ADB and AEB. Their difference is 216 -144 or or 72. And ½ of 72 equals 36.

That is the answer to what is the measure of the angle between walkways CA and CB. The measure is 36°.

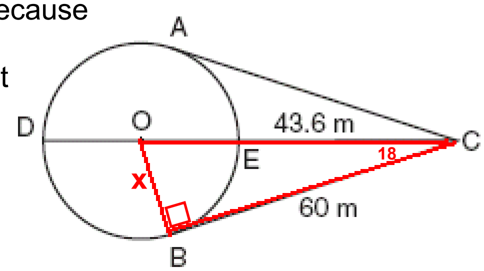
There is one more part to this problem. You are also asked to find the diameter of the circular garden pictured above. That solution continues on the next page.



Your final task is to find the diameter of the circular garden.
In the diagram pictured at the right DOE represents the diameter. DOE happens to be part of secant COD.
Remember that CB is a tangent to the circle.
The segment relationship involve here is:

$(CED)(CE) = (CB)^2$ Let ED = x
$(x+43.6)(43.6) = (60)^2$ Use distributive property.
$43.6x + 1900.96 = 3600$ Subtract 1900.96 from both sides.
$43.6x = 1699.04$ Divide both sides by 43.6.
$x = 38.96880734$ Round to nearest meter.
ED= 39

Here's another way to do this last part of the problem without even using the 43.6.
I've outlined right triangle COB. We know it is a right triangle because OB is perpendicular to CB. (A radius drawn to a tangent is perpendicular to it at the point of tangency.) We also know that pBOC is ½ of pACB. (A line from the center of a circle to the external point from which two tangents are drawn to the circle, will bisect the angle formed by those two tangents.)
We found the measure of pACB in the first part of the problem. It was 36°. Therefore, pBOC will be 18°. Now we can use the tangent ratio to solve for BO which also happens to be the radius of this circle. Once we determine the length of BO, we will double it as that will be the diameter of the circle.



$\tan 18^\circ = \frac{x}{60}$ Cross multiply

$X = 60(\tan 18)$ Use your calculator
$X = 19.49518177$ This is the radius. Now double it for the diameter.
Diameter = 38.99036355 or **39 to nearest meter**.