ANSWERS MATH B – August 16th, 2005

1) You are given the equation $y = 3x^2 + 6x - 1$, and asked for the turning point of the parabola its equation represents. The x coordinate of a parabola is given by the formula $x = -b/2a$.
In the above equation, a=3 and b= 6. Let us substitute to solve for x.

$x = \frac{-b}{2a} = \frac{-(6)}{2(3)} = \frac{-6}{6} = -1$ Once you know that the x-coordinate at the turning point is –1, you can substitute the –1 in the original equation to solve for the corresponding y-coordinate.

$y = 3x^2 + 6x - 1$ Substitute –1 for x.
$y = 3(-1)^2 + 6(-1) - 1$ Use the order of operations.
$y = 3(1) - 6 - 1$ Continue to simplify.
$y = 3 - 6 - 1$ Combine.
$y = -4$

The coordinates of the turning point are (-1,-4) **ANSWER: (2)**

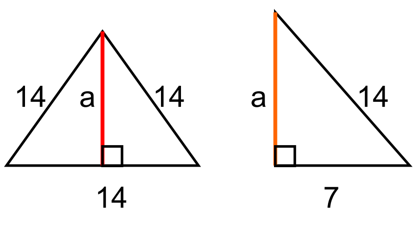
2) You are presented with the function $f(t) = 2^{\frac{t}{3}}$ and asked to determine for which value of t will f(t)=32.
What this problem now boils down to is 2 raised to which power will = 32? You know that $2^5 = 32$
This means that the exponent, t/3, in the original function has to equal 5.

Set $\frac{t}{3} = 5$ and by cross multiplying you get **t=15**.

(Had you not known that 2 has to be raised to the power of 5 here is a way to have figured that out.
$2^x = 32$ Use the power rule.
$x \log 2 = \log 32$ Divide both sides by log 2
$x = \frac{\log 32}{\log 2} = 5$ **ANSWER: (3)**

3) Choice #4 is the shape of your typical cosine curve. **ANSWER: (4)**

4) You are presented with the diagram at the right. Based on the given, it is obviously an equilateral triangle. You are asked to find the exact height of the pole, which is the altitude pictured in red at the right. One way to do this problem is to use the Pythagorean Theorem. Realize that the base of the triangle is bisected, so each half is 7, as pictured on the triangle all the way to the right. Now use the Pythagorean Theorem



$a^2 + b^2 = c^2$ Substitute the know values.
$a^2 + 7^2 = 14^2$ Square the 7 and 14.
$a^2 + 49 = 196$ Subtract 49 from both sides.
$a^2 = 147$ Take the square root of both sides, and simplify the resulting radical.
$a = \sqrt{147} = \sqrt{49} \cdot \sqrt{3} = \mathbf{7\sqrt{3}}$ **ANSWER: (3)**

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5) You are asked for the sum of: $(y-5) + \frac{3}{y+2}$. Here is one way of doing this type of problem. First a brief explanation. Think of the above as adding a whole number and a fraction, for example $5 + \frac{3}{4}$.

Obviously the answer would be $5\frac{3}{4}$ which can be changed to an improper fraction as follows.
The whole numeral times the denominator plus the numerator…over the denominator.
5 times 4 plus 3 over 4 or $\frac{23}{4}$. Now let's treat our original problem the same way. It is

$(y-5) + \frac{3}{y+2}$ think of the y-5 as being the whole numeral. Multiply that by the denominator y+2, and add to it the numerator of 3, and then put the whole answer over the denominator of y+2. Here it is step by step. (Then simplify by multiplying the binomials and combining like terms).

$(y-5) + \frac{3}{y+2} = \frac{(y-5)(y+2)+3}{y+2} = \frac{y^2-3y-10+3}{y+2} = \mathbf{\frac{y^2-3y-7}{y+2}}$ **ANSWER: (4)**

6) To simplify the given expression $\frac{1}{5-\sqrt{13}}$, multiply its numerator and denominator by the conjugate of the denominator: $5 + \sqrt{13}$. **(Remember $\sqrt{13} \bullet \sqrt{13} = 13$)**

$\frac{1}{5-\sqrt{13}} \bullet \frac{5+\sqrt{13}}{5+\sqrt{13}} = \frac{5+\sqrt{13}}{25-13} = \mathbf{\frac{5+\sqrt{13}}{12}}$ **ANSWER: (1)**

7) You are asked to express $2\sqrt{-32} - 5\sqrt{-8}$ in terms of i. First simplify the radicals.

$2\sqrt{-32} - 5\sqrt{-8}$ **Remember $\sqrt{-1} = i$**
$2\sqrt{-1}\sqrt{16}\sqrt{2} - 5\sqrt{-1}\sqrt{4}\sqrt{2}$
$2i \bullet 4\sqrt{2} - 5i \bullet 2\sqrt{2}$
$8i\sqrt{2} - 10i\sqrt{2}$
$\mathbf{-2i\sqrt{2}}$ **ANSWER: (3)**

8) The origin is the point (0,0). Under a certain translation its image becomes the point (2,-6). We can easily figure out the rule of this translation. 2 is added to the x-coordinate, and –6 is added to the y-coordinate. Now, given the point (-3,-2), you are asked for its image under the same translation. If we add 2 to its x-coordinate we get –3+2 or **–1**. If we add –6 to its y-coordinate we get –2-6 or **–8**. The image of the point (-3,-2) will now become **(-1,-8)**. **ANSWER: (4)**

9) You are asked for the solution of the following inequality |2x-3| < 5. This inequality is really stating the following: 2x-3 < 5 and -(2x-3) will also be less than 5. Let us solve both of these inequalities.

2x-3 < 5	Add 3 to both sides.	-(2x-3) < 5	Distribute the negative sign.
2x < 8	Divide both sides by 2.	-2x + 3 < 5	Subtract 3 from both sides.
x < 4		-2x < 2	Divide by –2 (symbol will change direction)
		x > -1	

Your answer is that **x > -1 and x < 4** **ANSWER: (2)**

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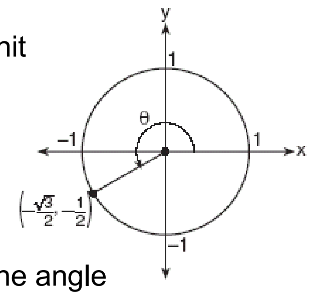

10) You are presented with a unit circle, and told the coordinates of the ordered pair which represents the point where the terminal side of θ intersects the unit circle. The x-coordinate is $-\frac{\sqrt{3}}{2}$, and the y-coordinate is $-\frac{1}{2}$. Now, once you know the x and y coordinates you also in essence know the sine, cosine, and tangent values of θ. The sine of the angle is represented by the y-coordinate, and the cosine of the angle is represented by the x-coordinate. The y-coordinate in this example is $-\frac{1}{2}$. So all you need to do now is to find the angle whose sine is $-\frac{1}{2}$.

Using your calculator (if you have to), hit 2nd SIN. You will now be accessing the SIN $^{-1}$ function of your calculator. The angle whose sine is -.5 is –30 degrees. This means that your answer will be a 30 degree angle in a quadrant where sin is negative. Sine is negative in quadrants III and IV. The angle pictured on the exam terminates in quadrant III. A 30 degree angle in the third quadrant is the same as (has a reference angle of) 180+30 or 210 degrees. **ANSWER: (1)**

11) You are told that an angle measures 125° and are asked for the expression that is equivalent to the cosine of this angle. An angle of 125° is a second quadrant angle. It can be converted to an equivalent acute angle by subtracting it from 180. 180 – 125 = 55. We can therefore treat an angle of 125° as an angle of 55° when dealing with trigonometric functions. But you do have to keep in mind the quadrant of the original angle. An angle of 125° is in the second quadrant. Cosine is negative in the second quadrant. Therefore the cosine of 125° is equivalent to **-cos 55° .**

ANSWER: (4)

12) You are asked for the sum of w and u, expressed in standard complex number form. w represents 4 + 2i, while u represents –1 + 5i. Added together the sum is 3 + 7i. **ANSWER: (2)**

13) You are presented with the complex fraction $\dfrac{1+\frac{1}{x}}{\frac{1}{x}-x}$.

Let's begin by simplifying the numerator and then the denominator. Remember #5 on this regents. We will simplify $1+\frac{1}{x}$ in that same manner: 1 times x, plus 1…over x = $\frac{x+1}{x}$

And now think of the denominator as follows $-x+\frac{1}{x}$ and follow above pattern. It simplifies to: $\frac{-x^2+1}{x}$.

This is now your problem: $\frac{x+1}{x} \div \frac{-x^2+1}{x}$ Think of the second fraction as follows: $\frac{1-x^2}{x}$

$\frac{x+1}{x} \bullet \frac{x}{1-x^2} = \frac{x+1}{x} \bullet \frac{x}{(1+x)(1-x)}$ At this point, the single x in the numerator will cancel with the single x in the denominator, and the x+1 in the numerator will cancel with the (1+x) in the denominator.

Therefore, $\frac{x+1}{x} \bullet \frac{x}{(1+x)(1-x)} = \mathbf{\frac{1}{1-x}}$ **ANSWER: (3)**

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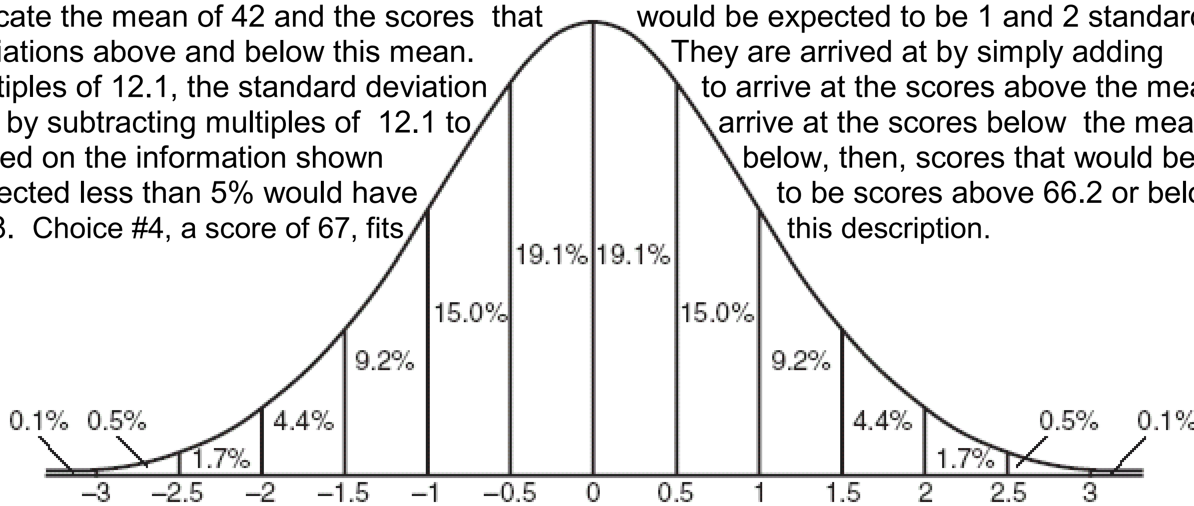
14) You are told that a radio wave travels in a path represented by the equation y= 5 sin 2x. You are asked to determine the period of this wave. The basic sine curve, y = sin x, has a period of 2**B**. What this means is that the sine curve will repeat itself every 2**B** radians or 360°. The curve you are given y= 5 sin 2x has a 2 in front of the x. That 2 tells you that the curve will repeat itself 2 times in 2**B** radians or 360°**. This means that it will complete its curve once in B radians or 180°.** This is what is meant by the period of a curve—in how many degrees will it complete one of its cycles.

ANSWER: (3)

15) You are told that the mean score on a normally distributed exam is 42, and that the standard deviation is 12.1. You are presented with 4 choices and asked which one would be expected to occur less than 5% of the time? Based on the bell curve included on the formula page in your regent's booklet, and reproduced below, you can see the percentages of scores that fall within three standard deviations above and below the mean. The mean is given as 42. Adding the percentages shown past 2 standard deviations you get 1.7+.5+.1. That equals 2.3% which is less than the 5% the problem is looking for. The numbers you see in bold below the curve indicate the mean of 42 and the scores that would be expected to be 1 and 2 standard deviations above and below this mean. They are arrived at by simply adding multiples of 12.1, the standard deviation to arrive at the scores above the mean, and by subtracting multiples of 12.1 to arrive at the scores below the mean. Based on the information shown below, then, scores that would be expected less than 5% would have to be scores above 66.2 or below 17.8. Choice #4, a score of 67, fits this description.



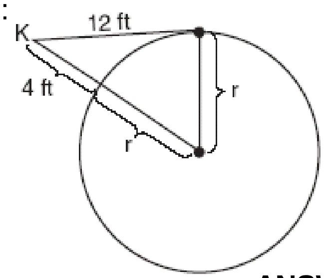
17.8 29.9 42 54.1 66.2 **ANSWER: (4)**

16) To determine the nature of the roots of a quadratic equation you have to determine the value of the discriminant. The discriminant is represented by the expression $b^2 – 4ac$. You are asked to determine for which value of m will the roots be real, equal, and rational**. This will occur when the discriminant equals 0.** You are given the equation $4x^2 + mx + 9 = 0$.
In the above equation, **a=4 b=m c=9**. Let us substitute these values for $b^2 – 4ac$.
$b^2 – 4ac = 0$
$m^2 – 4(4)(9)$
$m^2 – 144 = 0$ Subtract 144 from both sides.
$m^2= 144$ Find the square root of both sides.
$m = \pm 12$ **One of the given choices is 12** **ANSWER: (1)**

17) You are given the following equation as representing the shape of the orbit of a planet: $3(y + 1)^2 + 5(x+4)^2 = 15$. You are given the choices of hyperbola, ellipse, circle and parabola. You can see that the equation has an x^2 and y^2. In addition it will has 2 unequal numerical coefficients: 3 and 5. There is also a plus sign joining the x and y terms. **This makes it the equation representing an ellipse.** Had there been a minus sign then the equation would

have been that of a hyperbola. Had the coefficients been equal then you would have had the equation representing a circle. A parabola would have had only the x or y as to the second power—not both. **ANSWER: (2)**

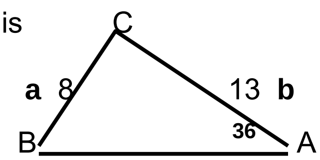
18) To the right is the diagram you are presented with on the Regents:
You are being asked to solve for the radius, r.
You can use the Pythagorean Theorem to solve for r.
Set up your equation:



$r^2 + (12)^2 = (r+4)^2$	Your hypotenuse is r + 4.
$r^2 + 144 = r^2 + 8r + 16$	Subtract r^2 from both sides.
$144 = 8r + 16$	Subtract 16 from both sides.
$128 = 8r$	Divide both sides by 8.
16 = r	**Radius equals 16 ft.**

ANSWER: (1)

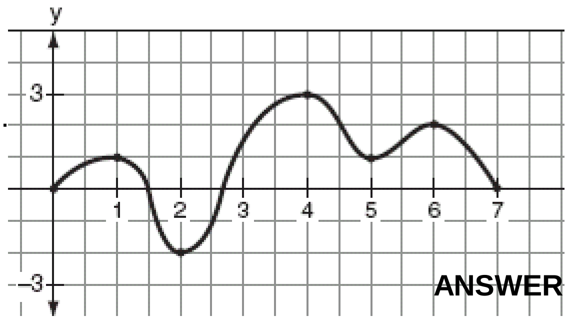
19) Whenever you are presented with this type of problem—how many distinct triangles can be constructed—use the Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. So that you set up your proportion correctly, imagine the triangle at the right . AC= 13 BC= 8 ∡A = 36°
The side opposite angle A is side a, and the side opposite angle B is side b. Now set up the proportion.



$\frac{a}{\sin A} = \frac{b}{\sin B}$	Substitute the given values.
$\frac{8}{\sin 36} = \frac{13}{\sin B}$	Cross multiply.
$8 \sin B = 13 \sin 36$	Divide both sides by 8.
$\sin B = \frac{13 \sin 36}{8}$	Use your calculator to solve right side of the equation.
$\sin B = .955151035$	Now use the 2nd function sine ($\sin^{-1}$) to find angle B.
∡B = 72.7753°	Let's round to 73 degrees and use that value for the remainder of this problem. If ∡A = 36° , and ∡B = 73° , we can figure out angle C by sub-
180-(36+73)=71	subtracting the sum of A and B from 180. ∡C = 71° This would be the angle measurements of 1 possible distinct triangle. It would have angles of 36, 73, and 71 degrees. Now to determine if another triangle is possible. We originally figured out that ∡B measured 73° based on its sine of .955151035. However, there is another angle with that same sine. Sine is also positive in the second quadrant. This means that a 73° angle has a second quadrant
180 –73 = 107	reference angle equal to the same sine. It can be found by subtracting 73 from 180. It is an angle of 107°. Now let us see if we can construct a triangle where ∡B = 107°. Angle A is given as 36. Let us figure out what
180 –(36+107)= 37	angle C would now be. Subtract the sum of A and B from 180. ∡C = 37° **.** This would be a second distinct possible triangle. Had the sum of the 3 angles been more than 180° , then no second triangle would have been possible.

ANSWER: (2)

20) You are shown the graph at the right and are asked to find the value of $(f \circ f)(6)$. This is read as "f of f of x." Another way of writing this would be: f (f(x)). This is problem dealing with composition of functions. What it requires you to do is to find the value of f(6), and then to find f(of that answer). Looking at the graph, you see that f(6) is 2. So now you have to find f(2). Based on the graph, **f(2)= -2**



ANSWER (4)

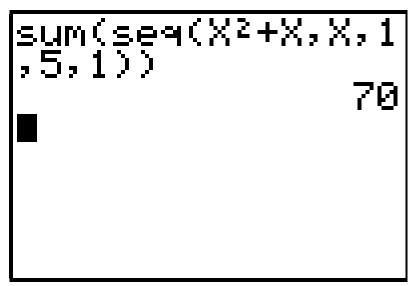
21) This problem asks you to evaluate: $\sum_{n=1}^{5}(n^2+n)$. This summation problem that is asking you to evaluate the sum of all the values of (n^2+n) as the value of n goes from 1 thru 5.

$n^2+n = 1^2 + 1 = 1 + 1 = 2$
$n^2+n = 2^2 + 2 = 4 + 2 = 6$
$n^2+n = 3^2 + 3 = 9 + 3 = 12$
$n^2+n = 4^2 + 4 = 16 + 4 = 20$
$n^2+n = 5^2 + 5 = 25 + 5 = 30$

The sum of all these values would be 2+6+12+20+30, which equals 70.

Therefore, $\mathbf{\sum_{n=1}^{5}(n^2+n) = 70}$

You can actually do summation problems using your TI-83 graphing calculator. Here is a screen capture of this problem and the answer. What it does is get the sum of the sequence that you enter. Here are the instructions to enter some of those key strokes.



To enter the **sum** function: Hit 2nd STAT This accesses the LIST menu. Your screen will show the following drop-down menus: NAMES OPS MATH

Move your cursor over MATH and hit ENTER, then hit the 5 key. The 5 key will put the **sum(** function on your screen.

To enter the **seq** function: Follow the same instruction as above but this time don't select the MATH drop-down menu. Move your cursor over the OPS selection and then hit the 5 key. This will enter the **seq(** function on your screen. It is easier to enter the letter x on your calculator screen than using the ALPHA key to enter the letter N, so I used X above. To enter an X simply hit the X,T, θ,n key , and to square that X simply hit the x^2 key. Following are the keystrokes necessary to do this problem.

2nd STAT ► ► 5 2nd STAT ► 5 X,T, θ,n x^2 + X,T, θ,n ,

X,T, θ,n , 1 , 5 , 1)) ENTER

As you see on the screen capture, the answer is 70.

(Remember that after you've hit the 2nd function key you are really accessing the function of what is in yellow above the key you will hit next. What this means, for example, is that when you hit 2nd followed by STAT, you are really hitting the LIST key.)

22) Given the channels 3, 6, 7, 10, 11, and 13, you are asked to determine the probability that a family will select exactly three even-numbered channels in five nights.
First you have to know the probability of choosing an even-numbered channel. There are a total of 6 channels, and 2 of them are even. The P(even-numbered channel) = $\frac{2}{6}$.

And the probability of not choosing an even-numbered channel would be $\frac{4}{6}$.

Out of 5 attempts to chose exactly 3 means that you will have to figure out $\frac{2}{6} \bullet \frac{2}{6} \bullet \frac{2}{6}$ or $\left(\frac{2}{6}\right)^3$ and

also 2 attempts that will not be even-numbered which would be $\frac{4}{6} \bullet \frac{4}{6}$ or $\left(\frac{4}{6}\right)^2$.

Since the 3 even-numbered channels can be selected on any one of 5 nights you also have to multiply by ${}_5C_3$. So now we have all the information we need to solve this problem:
${}_5C_3 \bullet$ P(Even-numbered channel)$^3 \bullet$ P(non even-numbered channel)^2

$${}_5C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right)^2 \qquad {}_5C_3 = \frac{(5)(4)(3)}{(3)(2)(1)} = 10$$

Using your calculator, these are the key strokes for ${}_5C_3$.

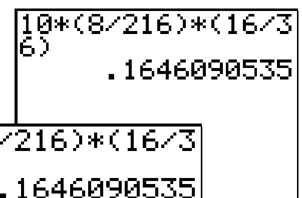
$$10 \bullet \frac{8}{216} \bullet \frac{16}{36}$$

5 MATH ► ► ► 3 3 ENTER

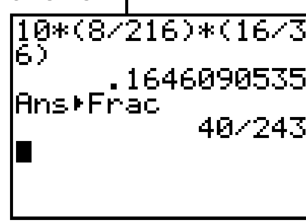
.1646090535

To the nearest four places this is .1646 The first screen capture is how to get your decimal answer.

As a fraction it is $\frac{40}{243}$



To convert a decimal to a fraction use the following key strokes:
MATH ENTER ENTER Here is what it will look like on your screen:

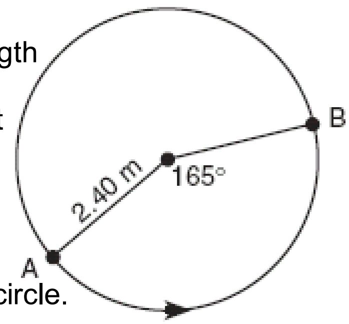


ANSWER: .1646 or $\frac{40}{243}$

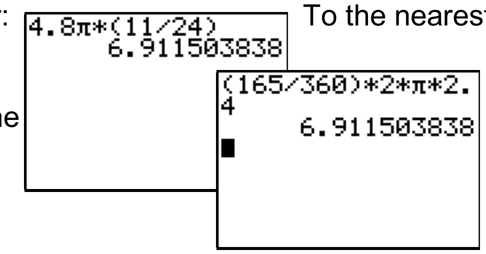
(**After finding ${}_5C_3$ above : ∗ (2 ÷ 6) ^ 3 ∗ (4 ÷6) ^ 2 ENTER** answer will be: .1646090535)

23) In a direct proportion, when one quantity gets larger, the other will get proportionally larger as well. In an inverse proportion, on the other hand, when one quantity gets larger, the other will get proportionally smaller. For example, if one were to double, the other would be halved**. In an inverse proportion, therefore, the product of the two quantities equal a constant.** For example xy=24 would represent an inverse proportion. If x=6 then y=4, and if x were doubled to 12 then y would be halved to 2. So in every inverse proportion the product of the two quantities will be equal to a constant. In this problem the two quantities being dealt with are the pressure of a compressed gas and its volume. You are told that they are inversely proportional to each other. As an example, you are told that when the pressure of a certain gas is 16 kilopascals its volume is 1,800 liters. You can now easily figure out the constant as being (16)(1800) or 28800. The problem asks you what the pressure of the gas would be when its volume is 900 liters? All you now have to figure out is 900 times what will get you that constant of 28800. The answer is gotten by dividing 28800 by 900.
28800 ÷ 900 = 32 **ANSWER: The pressure would be 32 kilopascals.**

24) The diagram at the right shows a circular track with a radius of 2.40 meters. You are asked to find to the nearest tenth of a meter the length of minor arc AB. The central angle is given as 165 degrees. Here is the plan for solving this problem. First find what part of the circle that minor arc is. It will be the part of the circle intercepted by the 165° angle, which is $\frac{165}{360}$ of the complete circle. This reduces to $\frac{11}{24}$.
All you now have to do is find the circumference of the circle and multiply it by 11/24, since the arc you are looking for is 11/24 of the circle.



Circumference = 2**B** r = 2**B** (2.40) = 4.8**B** Now multiply this by 11/24 and you will have your answer. Here is a screen capture of the work and answer: To the nearest tenth, this is **6.9 meters**. Obviously you could have done the same work without first reducing 165/360 as you can see from the screen capture all the way to the right.



ANSWER: 6.9 meters

25) You are told that the graph of the function y = f(x) lies completely above the x-axis. You are asked to explain why the equation f(x)=0 has no real solutions.
f(x)=0 means that y equals 0. But that can only be on the x-axis. One of the stipulations was that the graph lies completely above the x-axis, which rules out any case of y being equal to 0.

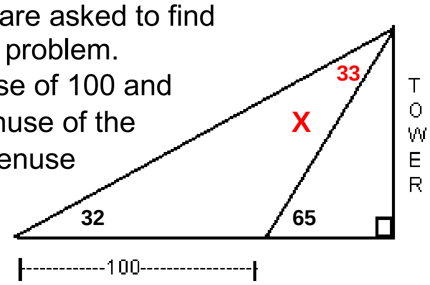
26) Express in simplest terms: $\frac{2 - 2\sin^2 x}{\cos x}$ First factor the numerator: $2 - 2\sin^2 x = 2(1 - \sin^2 x)$

So you now have: $\frac{2(1 - \sin^2 x)}{\cos x}$ Since $\sin^2 x + \cos^2 x = 1$, $\cos^2 x = 1 - \sin^2 x$. Substitute.

You now have: $\frac{2\cos^2 x}{\cos x}$ $\cos^2 x = \cos x(\cos x)$ One cos x cancels with the denominator.

FINAL ANSWER: 2 cos x

27) To the right is a diagram that shows a cell phone tower. You are asked to find its height to the nearest foot. Here is the plan for solving this problem. First you can use the law of sines on the triangle with the base of 100 and the angle of 32°. The Law of Sines will give you the hypotenuse of the small right triangle that contains the 65°. I marked the hypotenuse with a red X. In order to use the Law of Sines, we also need the angle opposite the side of 100. We know that the 65° angle is an exterior angle to the triangle containing the 32° angle. As such, the angle opposite the side of 100 can be found by knowing that together with the 32 it has to equal 65. It is 33°, and I entered it in red as well on the diagram.



Now for the Law of Sines. (The procedure is the same as that used in #19 in this regents.)

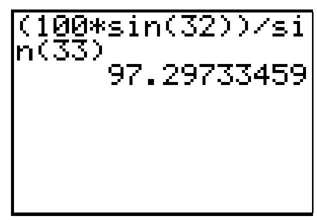
$\frac{a}{\sin A} = \frac{b}{\sin B}$ Substitute the known values.

$\frac{100}{\sin 33} = \frac{X}{\sin 32}$ Cross multiply

$X \sin 33 = 100 \sin 32$ Divide both sides by sin 33.

$X = \frac{100 \sin 32}{\sin 33}$ Use your calculator to solve for X.

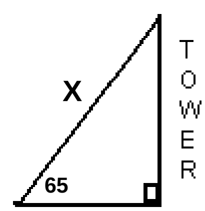
X = 97.29733459 Here is a screen capture of the work. Be careful entering those parentheses.



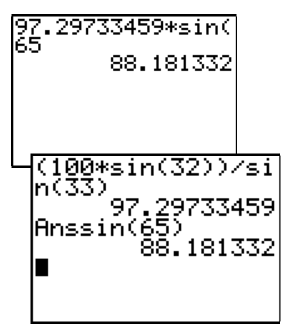
Look back at the original diagram now. Once you know what X is, you can use your basic trig to solve for the height of the tower.

A the right is a diagram of the triangle you will now be using. We know the angle of 65° and the hypotenuse X. We want to find the height of the tower to the nearest foot. We can use the sine relationship. It is the ratio of the opposite over the hypotenuse.

In this problem $\sin 65° = \frac{TOWER}{X}$ Let's use the full value we obtained for X. Cross multiply above and you end up with



TOWER = X sin 65 Substitute for X, and use your calculator for sin 65.
TOWER = 97.29733459 (sin 65) = 88.181332 **88 to the nearest foot**
ANSWER: Height of tower to nearest foot is 88 feet.



By the way, at the point where you have the answer to X on your calculator, you don't have to retype that value when you want to multiply it by sin 65. You can simply hit the 2nd key followed by the (-) key and you will automatically use the last answer you see on screen for your next calculation. Look at the screen capture the right. (Actually in this case all you had to do was hit the multiplication key and the calculator would have automatically used the answer that was on the screen from the last calculation).

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28) You are presented with the equation: $s = \mathbf{B}r\sqrt{r^2+h^2}$ In addition, you are given values for s and r, and are asked to solve for h to the nearest hundredth of a centimeter.
First let's transpose the equation to isolate h.

$s = \mathbf{B}r\sqrt{r^2+h^2}$ Divide both sides by **B**r.

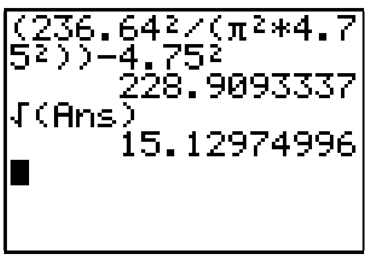
$\frac{s}{\pi r} = \sqrt{r^2+h^2}$ Square both sides.

$\frac{s^2}{\pi^2 r^2} = r^2 + h^2$ or $r^2 + h^2 = \frac{s^2}{\pi^2 r^2}$ Subtract r^2 from both sides.

$h^2 = \frac{s^2}{\pi^2 r^2} - r^2$ Now substitute the known values. s = 236.64 r = 4.75

$h^2 = \frac{(236.64)^2}{pi^2(4.75)^2} - (4.75)^2$ Now use your calculator and you will get the answer for h^2.

Get the square root of that answer and you will have solved for h.



ANSWER to the nearest hundredth: 15.13

(Be careful entering those parentheses. You want the 4.75^2 subtracted from the answer you get when you divide the numerator by the denominator.)

29) Solve for all values of x: $\frac{9}{x} + \frac{9}{x-2} = 12$

One way of doing this problem is to cancel the denominators. This can be done by multiplying each term by x(x-2).

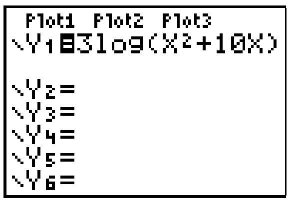
$x(x-2)\frac{9}{x} + x(x-2)\frac{9}{x-2} = x(x-2)\ 12$ In the first term the x's cancel, and in the second the x-2 cancels.

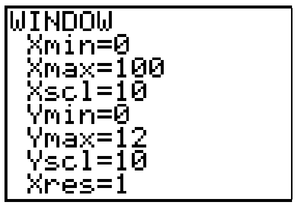
$(x-2)\ 9 + x \cdot 9 = x(x-2) \cdot 12$ Complete the multiplications.

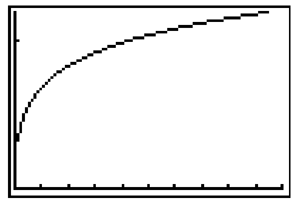
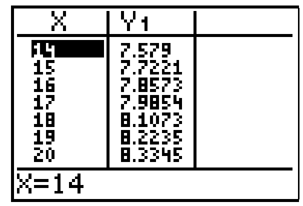
$9x – 18 + 9x = (x^2 – 2x) \cdot 12$ Combine like terms and distribute the 12.
$18x – 18 = 12x^2 – 24x$ Transpose to set equation equal to 0
$12x^2 – 24x – 18x + 18 = 0$ Combine like terms.
$12x^2 – 42x + 18 = 0$ Divide all terms by 6.
$2x^2 – 7x + 3 = 0$ Factor
$(2x - 1)(x – 3) = 0$ Set each factor equal to 0 and solve.

$2x – 1 = 0$ $x – 3 = 0$
$2x = 1$ **$x = 3$** (You should check each answer in the original equation)
$x = \frac{1}{2}$

30) You are presented with the function R = 3 log (n^2 + 10n), where R represents the total annual revenue, and n is the number of rooms occupied daily by guests. Your first task is to graph the function over the interval $0 < n \leq 100$. Enter the function into your calculator using the Y= editor. The first screen capture to the right below shows your calculator screen after the function has been entered. So that you can have an idea of what this graph will look like, hit your **WINDOW** key and set up an appropriate graphing window based on the information the problem provides. Set the x -axis from 0 thru 100, and set your y-axis as going from 0 thru 13. (When x=100, Y will equal 12.1). The second screen capture below show what the information in the window should look like once you are done. Finally hit your **GRAPH** key and you will see what your graph should look like. It is the third screen capture below. There is one more important key to hit. That is the **TABLE** key. The **TABLE** key is accessed by hitting the **2nd** key followed by the **GRAPH** key. This will show you a table of values for your graph. You can scroll up and down to see any value you wish. The final screen capture below shows you an example of the **TABLE** window with values starting at 14 thru 20.

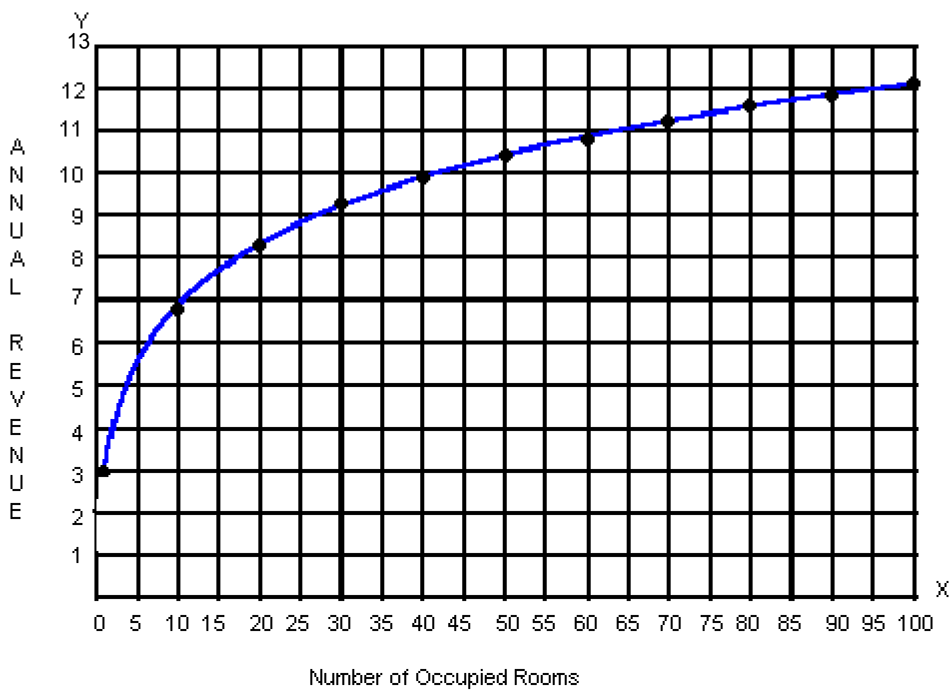




Sketch your graph as follows. Label your x and y-axis. The number of rooms, represented by n in the function is your x-axis, while R, the annual revenue in millions, is your y-axis. You can number your x-axis in increments of 5 or 10. I used 5 below but only plotted the points for 1 and then multiples of 10. Here is a table of the points I used rounded to the nearest tenth.

X	Y
1	3.1
10	6.9
20	8.3
30	9.2
40	9.9
50	10.4
60	10.9
70	11.2
80	11.6
90	11.9
100	12.1



The problem continues on the next page

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X	Y_1
94	11.97
95	11.997
96	12.023
97	12.048
98	12.074
99	12.099
100	12.124

X=94

The second part of this problem asks you to calculate the minimum number of rooms that must be occupied daily for the hotel to be profitable. You are told that the hotel needs an annual revenue of $12 million to be profitable. So in essence all you have to do is look at the table on your calculator and see at which point the y value (in our problem R) will equal 12. To the right is a screen capture of the **TABLE** screen with the Y values visible for X going from 94 through 100. **You can see that when X= 95, the Y-value is still under 12. When X hits 96, the Y-value is 12.023. The answer is therefore 96. When 96 rooms are occupied daily, the hotel will realize its annual revenue of at least $12 million.**
(Please note that the X=94 you see in the above screen capture is there because the calculator's cursor is over the 94 in the X column. It is not your answer.)

(BTW you can obtain the y-value for any x without scrolling. All you have to do is hit the **2nd** key followed by **WINDOW** to access the **TABLESET** function and type in whatever value you are looking for where you see the words **TblStart=** . You don't even have to hit ENTER at this point. Simply hit the **2nd** key followed by **GRAPH** to access the **TABLE** function where you will see the corresponding X and Y values beginning at the value you entered earlier.)

Alternate method for solving part 2 of this problem:
You can set up and solve the following equation: $12 = 3 \log (n^2 + 10n)$. Divide both sides by 3 and your result is $\log (n^2 + 10n) = 4$
The above equation can be rewritten in exponential form.
Remember that $\log 100 = 2$ is the same as $10^2 = 100$. Using this pattern,
$\log (n^2 + 10n) = 4$, can be rewritten as:

$10^4 = n^2 + 10n$ — $10^4 = 10{,}000$

$10{,}000 = n^2 + 10n$ — Set equal to 0.

$n^2 + 10n - 10000 = 0$ — Use quadratic formula to solve for n (x will be our n).

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In our equation, a=1 b= 10 c= -10,000

Substitute these values for into the quadratic formula:

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(-10000)}}{2(1)} = \frac{-10 \pm \sqrt{100 + 40000}}{2} = \frac{-10 \pm \sqrt{40100}}{2}$$

There are 2 answers: $(-10 - \sqrt{40100}) / 2$ and $(-10 + \sqrt{40100}) / 2$
We can reject the first as it will yield a negative number of rooms.
Use your calculator to calculate the second answer of $(-10 + \sqrt{40100}) / 2$
To the right is a screen capture of your answer:

```
(-10+√(40100))/2
            95.12492197
```

Since n represents the number of rooms we have to round up to 96. (You can not rent fractions of a room). **The answer is 96 rooms.**

(Please note the outer set of parenthesis. They are there to ensure that the complete numerator is divided by 2, not just the square root of 40,100).

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31) You are presented with the following equation: $P= -10x^2 +750x – 9,000$.
P represents the profit, while x represents the selling price. You are asked to determine what range of selling prices will allow the manufacturer to make a profit.
First you should realize that the above equation is a quadratic equation in the form of $y=ax^2+bx+c$. Since the "a" term, -10, is negative, the graph of this equation will open to the bottom and have a maximum turning point. Its solution set will be the two points it intersects on the x-axis where the y-coordinate equals 0. Which really means that in the above case, the part of the graph that is above the x-axis will signal the range where a profit can be made. We can easily solve the above equation to determine its solution set. We set P equal to 0 and factor.

$-10x^2 +750x – 9,000 = 0$ Divide both sides by 10.
$-x^2 + 75x – 900 = 0$ Factor.
$(-x +15) (x - 60)= 0$ Set both factors equal to 0, and solve for x.

$-x +15= 0$ $x - 60 = 0$
$-x = -15$ **$x = 60$**
$x = 15$

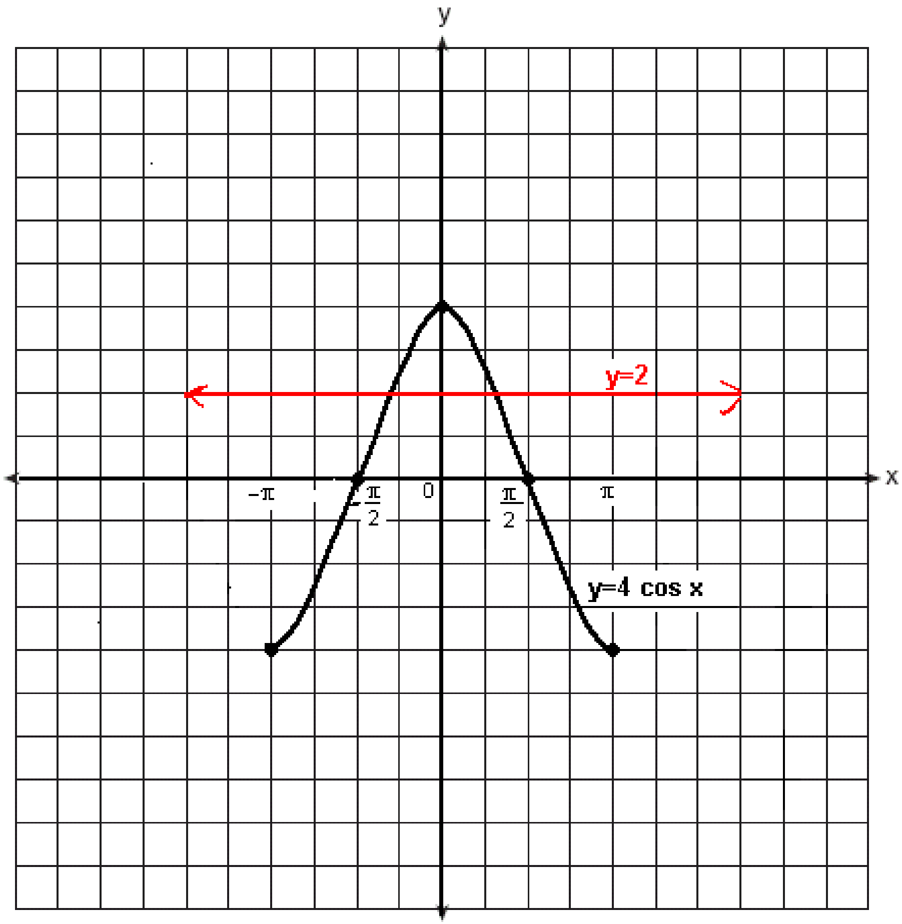
What this means is that when x, the selling price, is $15 or $60 there will be 0 profit. When x is less than 15 or greater than 60 there will be a negative profit or loss. But anywhere between that range, where x is greater than $15 and less than $60, there will be a profit.
ANSWER: $15 < x < 60$
(At the point above where you have the equation $-x^2 + 75x – 900 = 0$, you can actually multiply both sides by –1 and end up with the following equation which is easier to factor and solve.
$x^2 - 75x + 900 = 0$
$(x - 15) (x - 60) =0$
$x – 15 = 0$ $x – 60 = 0$
$x = 15$ $x = 60$
But you have to realize that although its intersects are 15 and 60, it is not the same equation were you to graph it. The graph of this equation would open to the top and it would have a minimum turning point.)

32) You are asked to graph the following two equations on the same axes:

$y = 4 \cos x$ and $y = 2$ in the domain $-\pi \le x \le \pi$

and then to express in terms of π the interval for which $4 \cos x \ge 2$.

To begin with, you should at this point know how to sketch the graph of the sine, cosine, and tangent functions. The simplest way is to plot the points for the quadrantal angles which are 0, 90,180, and 360 degrees. In this problem you are required to graph the cosine function from $–180^\circ$ thru 180°. You can actually use your calculator, if you have to, to get the following values:
$\cos (-180^\circ)= -1$ $\cos (-90^\circ) = 0$ $\cos (0^\circ) = 1$ $\cos (90) = 0$ $\cos (180)= -1$
Since we are graphing $y= 4 \cos x$, the value of the function at each angle is multiplied by 4. That is why at $–180^\circ$ the y-value will be –4, at -90° it will remain 0, at 0° it will be 4, at 90° it will again be 0, and at 180° it will again be –4. These are the points that you should plot. You will see them on the grid on the next page. In addition you are also required to plot on the same axis, the graph of y=2. It is represented by the red line that you see 2 units above the x-axis. One more thing you should know is how to label the x-axis. You can actually use degrees but radian measure seems to be preferred. Since π radians equals 180°, $\pi /2$ equals 90°. Now go on to the next page where you will see the answer to the first part of this question.



At the left you see in black the graph of the function $y= 4 \cos x$. It is graphed in the domain of $-\mathbf{B} \leq x \leq \mathbf{B}$

In addition you see in red the graph of the function y=2

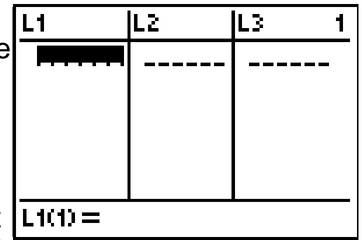
The next part of the question asks you express in terms of **B** the interval for which $4 \cos x \geq 2$.

Divide both sides by 4 and you have: $\cos x \geq ½$

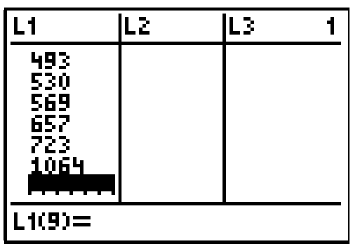
Using your calculator you can quickly see that $\cos 60^\circ = ½$ Therefore, the section of the graph that you see going above the line represented by y=2, starts at -60° and ends at $+60^\circ$. One thing now remains, that is to convert 60° to radian measure. The easiest way to convert degree measure to radian measure is to divide by 180 and then to append **B**. For example 60° , which is our answer, equals in radian measure, 60/180 or $\frac{1}{3}$ **B** or $\frac{\pi}{3}$.

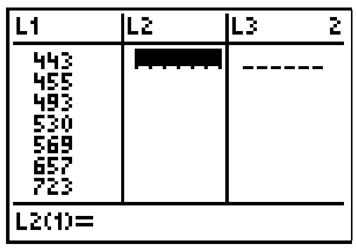
The answer to the second part of the question is: $\frac{-\pi}{3} \leq x \leq \frac{\pi}{3}$

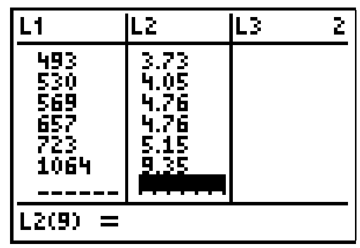
33) You are given a set of data and asked to find the linear regression for the set, rounding to five decimal places. Step number one involves the entering of this data into your calculator. Press **STAT** **ENTER** and you will see the following screen .



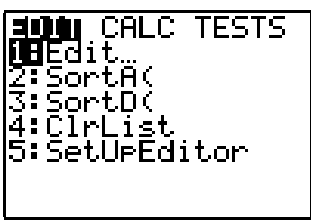
It is called the STAT LIST EDITOR screen. This is the screen where you will enter your data. You can enter all the data for **Number of Theaters** into L1 (list 1), and all the data for **Gross Earnings** into L2 (list 2). (As indicated in the problem, the data representing the number of theaters will be your x, while the data representing gross earnings will be your y. The simplest way to enter the data is to first enter the data for the number of theaters into L1. This is done by simply typing each number and then hitting the **ENTER** key. Below, to the left is a screen capture of what your screen will look like after you have entered all the data into L1. You can scroll up and down to make sure you entered all the data properly. Now hit the ► right scroll key once and you will immediately see the see the middle screen pictured below. You are now ready to enter the data for L2. Enter the data the same way you entered it into L1. Enter each number followed by the ENTER key. The final screen capture shows all the data entered into L1 and l2.

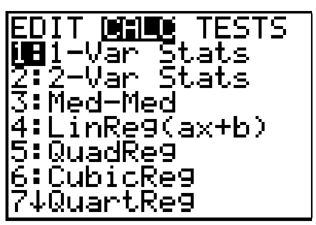


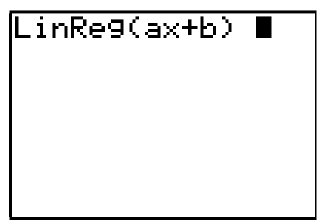


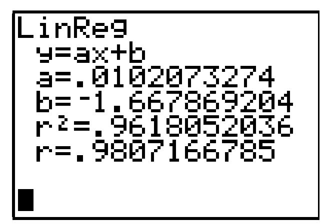


Once your data is entered, hit the following keys: **STAT** ► **4** **ENTER** Below is a screen capture for each one of the keys that you will be hitting:









The final screen above shows your answer, after you have hit the ENTER key. A linear equation is in the form of y= ax + b. You are shown the values for a and be and are expected to round them off to the five decimal places. **ANSWER: Your equation is: y = .01021x – 1.66787**

The second part of this question asks you to approximate the y-value, given an x-value of 610 theaters. This is simply a matter of substitution. All you have to do is substitute 610 for x in the above linear equation. Use your calculator to solve the following:
.01021 (610) – 1.66787. Your **ANSWER: 4.56** (rounded to 2 decimal places).

The final part to this question can be solved in a similar manner. This time you are given the y-value and asked to calculate the x-value. You are asked to find the minimum number of theaters necessary to generate **at least** 7.65 million dollars in gross earnings in one week. Your equation is: **y = .01021x – 1.66787.** What you now want is that **.01021x – 1.66787** should yield a y of at least 7.65 million. (This translates to greater than or equal). The solution continues on the next page.

.01021x – 1.66787 = y	Substitute 7.65 for y, and set up your inequality.
$.01021x - 1.66787 \geq 7.65$	Add 1.66787 to both sides.
$.01021x \geq 9.31787$	Divide both sides by .01021.
$x \geq 912.6219393$	You can not have fractions of theaters, so round up

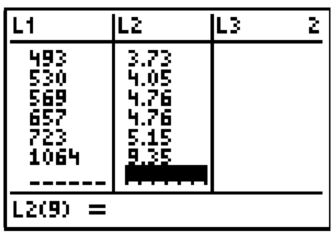
.

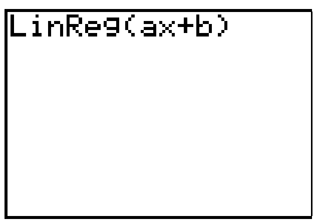
ANSWER: The minimum number of theaters necessary to generate at least.7.65 million dollars will be 913.

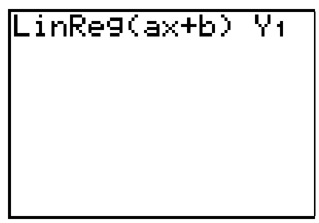
ALTERNATE METHOD USING YOUR CALCULATOR:

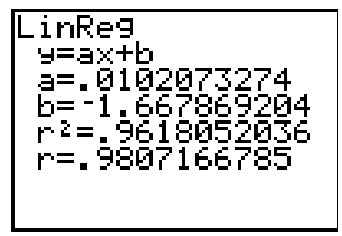
Here is an explanation of how you can do this question completely using your calculator. You have already typed in the data into L1 and L2. Now before you actually find the linear regression equation, it can automatically be entered into your y-editor as an equation. Here is how it is done. As mentioned before, we begin at the point after you have entered the data into L1 and L2. The first screen capture below to the right is what your screen looks like at that point. Now follow these keystrokes.

Press STAT ► 4 and **don't** hit ENTER. At this point your screen will look like the second screen below. Now continue with the following key strokes which will store the regression equation, in this case the linear equation, into the y-editor's y_1 VARS ► 1 1 . At this point your screen will look like the third screen below. The regression equation has been entered into y_1. Now hit **ENTER** and you will see the final screen.

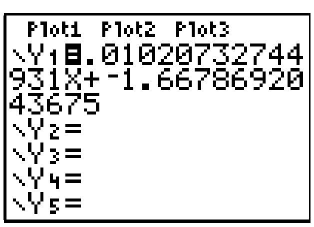


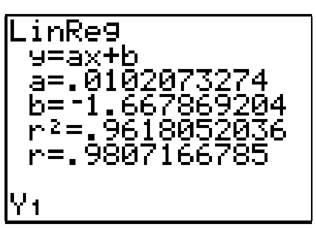


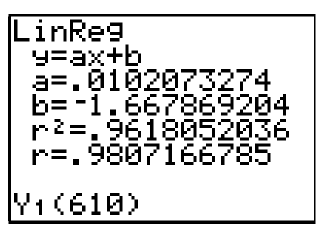


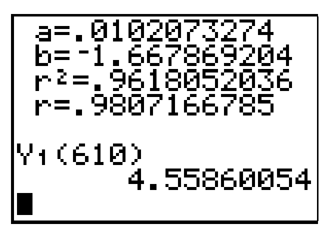


If you were to now hit the **Y=** key, you would see the first screen below to your left. That is your answer to the first part of the problem. You would have to round the decimals to five place, though. Now here is the nice part about finding the second part of the question. (First exit the y-editor by hitting **2nd** followed by the **MODE** key. You will be back at the last screen above). Now, what would be the value of y when x is 610? Hit the following keys. VARS ► ENTER ENTER . At this point you will see the second screen below. Now type in parenthesis your x value of 610. That is the third screen you see. Now hit **ENTER** and you will have your answer! (Don't forget to round it).



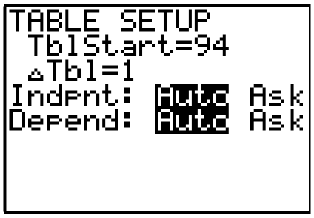




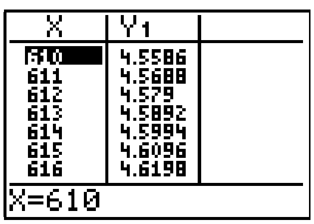


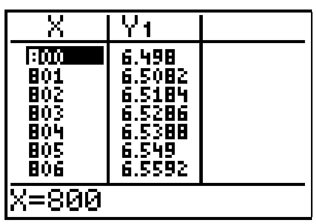
Actually, once the equation is input for y_1, you could simply check the table generated automatically for this equation in order to answer parts two and three of this question.
This is explained on the next page.

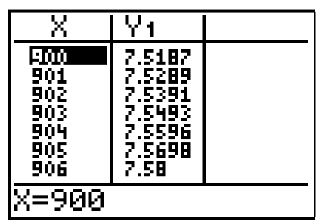
Hit the following keys: 2nd WINDOW This will access the TABLE SETUP screen. Below is a screen capture of that screen. Your screen may have a different number where you see the 94 below. Instead of the 94, type in 610—the value we want, and hit **ENTER**.

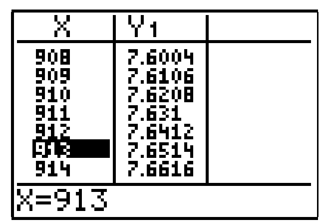


Now hit 2nd GRAPH This accesses the TABLE window. What you will now see is shown in the screen capture below. You clearly see the Y value that corresponds with an x of 610. (Again make sure to round your answer.) Had you not entered 610 into the TABLE SETUP window, then the first x you would have seen would not necessarily have been 610. As you see above, we started out with a 94 there. At that point, you could have scrolled to 610 and arrived at the same screen you see below. Now in a similar manner you can get the answer to the final part of this question. Find the minimum number of theaters necessary to generate **at least** 7.65 million dollars in gross earnings in one week. This means that you are looking for the X value that will result in a Y value of at least 7.65. What you can now do is continue scrolling to higher X values until you reach a Y-value of 7.65, or go back to the TABLE SETUP window and enter for example 800 for X and check the corresponding Y. The second screen blow shows you what the TABLE will look like at X=800. Let's see the table at 900. Almost there! At this point, scroll a bit until you see a Y value of at least 7.65. That is the final screen below.







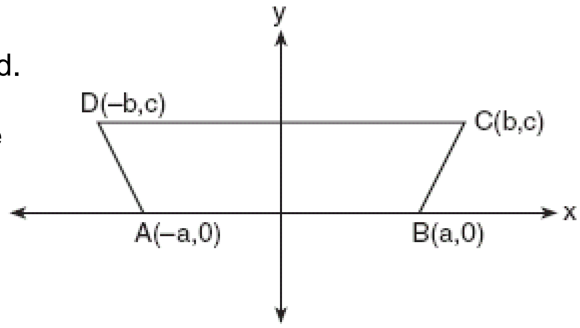


You clearly see that when X=912, you have not yet reached 7.65. When X= 913 you hit 7.65. That is why 913 is your answer.

NUMBER 34 BEGINS ON THE NEXT PAGE

ANSWERS MATH B – August 16th, 2005

34) Your objective is to prove the diagram at the right an isosceles trapezoid. What you should obviously know first is the definition of a trapezoid. A trapezoid is a quadrilateral that has ONLY one pair of parallel sides. Therefore, in order to prove something an isosceles trapezoid, you first have to prove it a trapezoid and then prove that it contains a pair of legs that are congruent. To prove the diagram a trapezoid, you will have to prove that one pair of sides IS parallel, while the other pair of sides IS NOT parallel.



Let us begin:

You will first prove that the slopes of line segment DC and AB are equal. Once you know their slopes are equal, the lines are parallel. This is true because parallel lines have equal slopes. The slope of a line is the change in y over the change in x. Subtract your y-coordinates, and divide this difference by the difference you get when you subtract your x-coordinates.

Slope DC $= \frac{y1-y2}{x1-x2} = \frac{c-c}{b-(-b)} = \frac{0}{2b} = 0$ **Slope AB** $= \frac{y1-y2}{x1-x2} = \frac{0-0}{a-(-a)} = \frac{0}{2a} = 0$

You have just shown that DC is parallel to AB because their slopes are equal.
(Actually since both line segments DA and CB are horizontal, they are parallel.)

Now you will show that the slopes of DA and CB are not equal, which make the other pair of sides non-parallel.

Slope DA $= \frac{y1-y2}{x1-x2} = \frac{c-0}{-b-(-a)} = \frac{c}{-b+a}$ **Slope CB** $= \frac{y1-y2}{x1-x2} = \frac{c-0}{b-a} = \frac{c}{b-a}$

You have just shown that DA and CB are not parallel because their slopes are not equal.

Now all that remains is to prove that they are equal in length. In order to do that you can use the distance formula: $d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$ Let's first find the length of DA and then CB.

Distance DA $= \sqrt{(-b-(-a))^2 + (c-0)^2} = \sqrt{(-b+a)^2 + c^2} = \sqrt{b^2-2ab+a^2+c^2}$

Distance CB $= \sqrt{(b-a)^2 + (c-0)^2} = \sqrt{(b-a)^2 + c^2} = \sqrt{b^2-2ab+a^2+c^2}$

Distance DA = Distance CB

Diagram ABCD is therefore an isosceles trapezoid. It has ONLY one pair of opposite parallel sides, and the other pair of sides are congruent.

(Please note above that squaring (-b+a) will yield the same answer as when (b-a) is squared.)