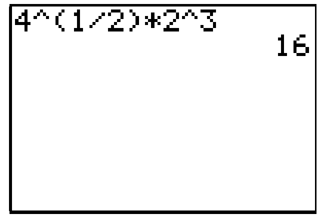
1) The problem given is: $4^{\frac{1}{2}} \cdot 2^3$



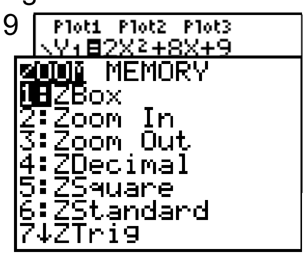
Perhaps the easiest way to do this is using your calculator. There are still more ways to do this problem. Here is one more way. 4 raised to the one-half is the same as finding the square root of 4. The square root of 4 is **2**. You know that 2 to the 3 is **8**. The problem therefore is basically 2 times 8 which is **16**. **ANSWER: (3)**

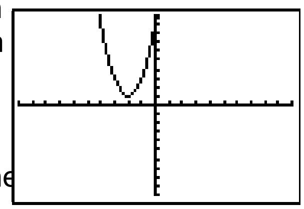
2) To solve an equation containing a radical, first isolate the radical to one side.

$\sqrt{2x-3} - 3 = 6$	Add 3 to both sides.
$\sqrt{2x-3} = 9$	Square both sides.
$2x - 3 = 81$	Add 3 to both sides.
$2x = 84$	Divide both sides by 2.
$x = 42$	

ANSWER: (1)

3) Perhaps the easiest way to do this problem is with the use of your graphing calculator. Enter the equation using the **Y=** editor. Enter it as you see it $2x^2 + 8x + 9$ Next hit the **ZOOM** key, and select 6 for a standard window. You will immediately see the graph generated by the given equation. You see a screen capture of the graph here to the right. You are asked for the minimum point. What this means is the lowest point. The lowest point of this graph will be its turning point. Now look at the choices provided in your test booklet. The only one that makes any sense is the point **(-2,1)** which is choice **4**.





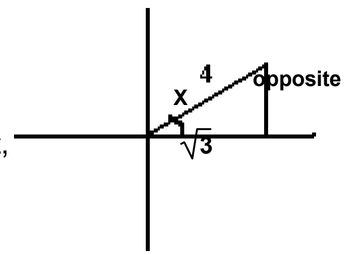
You can also have the calculator find the maximum or minimum point of any parabola but I will save that for another time.

Here is how you would do this problem algebraically. The general form of a quadratic equation is: $ax^2 + bx + c = 0$. In the equation presented for this problem, $y = 2x^2 + 8x + 9$, when y = 0: $a = 2$ $b = 8$ $c = 9$ You can easily find the minimum point of this equation as it will always lie along the parabola's axis of symmetry. The axis of symmetry is found by substituting the values into the following equation. $x = \frac{-b}{2a}$ Substituting, we get: $x = \frac{-8}{2(2)} = \frac{-8}{4} = -2$. This means that the **x-coordinate** at the turning point is **-2**. Once you know the x-coordinate, go back into the original equation and substitute -2 for x and solve for the y-value.

$y = 2x^2 + 8x + 9$	Substitute -2 for x.
$y = 2(-2)^2 + 8(-2) + 9$	Square and multiply.
$y = 2(4) - 16 + 9$	Multiply
$y = 8 - 16 + 9$	Combine

y = 1 This means that at the point where x = -2, y will equal 1. Your turning point, or minimum point in this case will be (-2,1) **ANSWER: (4)**

4) You are told that x is a positive acute angle. This means that it is an angle in the first quadrant. You are also told that $\cos x = \frac{\sqrt{3}}{4}$. The cosine of an angle is the ratio of the adjacent side to the hypotenuse. Given the triangle at the right, the side adjacent to the angle is the $\sqrt{3}$**,** while the hypotenuse is 4. Therefore the cosine of angle $x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{4}$



You are asked for the exact value of sin x.
Sine is the ratio of the opposite side to the hypotenuse. So all you have to do now is to figure out the measure of the side opposite the angle. You can use the Pythagorean Theorem to determine this side. Whenever you are dealing with a right triangle and know two of the sides, the third side can always be determined using the Pythagorean Theorem. The Pythagorean Theorem states: $(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$. In the diagram above to the right, the hypotenuse is 4, and one of the legs is $\sqrt{3}$. So let us substitute the values we know:

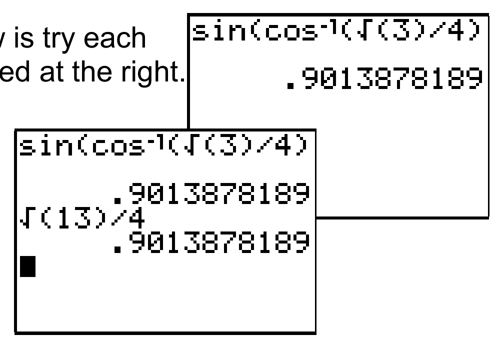
$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	Substitute
$4^2 = (\sqrt{3})^2 + (\text{opposite})^2$	Square
$16 = 3 + (\text{opposite})^2$	Subtract 3 from both sides.
$13 = (\text{opposite})^2$	Find square root of both sides.

$\sqrt{13}$ = opposite   Once you know that the opposite is the $\sqrt{13}$, you know the sine x.

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{13}}{4}$$

ANSWER: (2)

Alternate method using your calculator: You are asked to find the sine of an angle. Which angle? The angle whose cosine equals $\sqrt{3}$ divided by 4.
There to the right is your answer. All you have to do now is try each given choice to see which one equals the answer obtained at the right.
$\frac{\sqrt{13}}{4}$ is your answer.



5) An equation that represents a function will result in a specific y-value for each x-value that you input. In simple English, you cannot end up with two different y-values for the same x-value. For example, given the equation y = x +2, when x = 3 y = 5. When x =7 y = 9. There is a specific y for each x. At no time will the SAME x value yield two DIFFERENT y-values. Two **different** x-values, however,can result in the same y-value. For example, given the equation $y=x^2$, when x=2 y=4, and when x = -2 y=4. This does not contradict the definition of a function. However, an equation like x = 4, where x equals a constant would not represent a function. The graph of such an equation would be a vertical line that would contain points such as x=4, y=1, as well as the point x = 4 while y =7. It would therefore not represent a function. Choice number 1 therefore does not represent a function as it is an equation where x represents a constant value. **ANSWER: (1)**

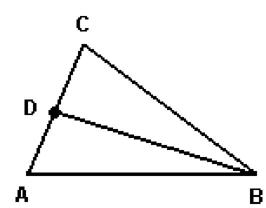
6) You are asked to find the equivalent of $\frac{12}{3+\sqrt{3}}$. To arrive at your answer, you have to simplify or rather rationalize the denominator. To do this you have to multiply both the numerator and denominator by the conjugate of the denominator. The denominator is $3+\sqrt{3}$ and its **conjugate** is **$3-\sqrt{3}$**

$\frac{12}{3+\sqrt{3}}\left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right)$ Multiply numerator and denominator by the conjugate $3-\sqrt{3}$.

$\frac{12}{3+\sqrt{3}}\left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right)=\frac{36-12\sqrt{3}}{9-3}=\frac{36-12\sqrt{3}}{6}=\mathbf{6-2\sqrt{3}}$ **ANSWER: (2)**

7) You are presented with an equation in exponential form and asked to rewrite it in logarithmic form. A log is another way to write an exponent. Try to remember the following example:
$\log_{10}100 = 2$ or $2 = \log_{10}100$ because $10^2=100$ or $100 =10^2$.
The above logarithmic expression is read as "the log of 100 to the base 10 equals 2, or 2 is the log of 100 to the base 10.
You can now rewrite our equation $y = 2^x$ as $\log_2 y = x$ or $\mathbf{x = \log_2 y}$ **ANSWER: (2)**

8) Imagine the triangle at the right being the one representing this problem. You are told that BD is a median. This makes D the midpoint of AC. Let's go through all the choices to determine which one must be true.



Choice 1 states that triangle's ABD and CBD are congruent. This does not have to be true because we do not know whether or not angles A and C are congruent.

Choice 2 states that angles ABD and CBD are congruent. This would only be true if choice 1 were proven to be true.

Choice 3 states that segments AD and CD are congruent. This statement has to be true as point D has to be a midpoint so that BD can be the median. Once you know that D is the midpoint of AC, then by definition, it divides the line AC into to congruent segments so that AD and CD are congruent.

Choice 4 states that BD is perpendicular to AC. This would be true only if AC would be the base of isosceles triangle ABC or equilateral triangle ABC. We are not told that ABC is isosceles or equilateral. **ANSWER: (3)**

9) You are presented with four equations and are asked to determine which one represents an ellipse.

Choice 2 is the equation whose graph is a circle with its center at the point (0,0). In our case it would have a radius of 12. It is represented by $x^2 + y^2 = r^2$.

Choice 1 also contains an x^2 and y^2 but here is what it would look like if both variables are transposed to one side of the equation: $x^2 - 36y^2 = 144$ The numerical coefficient of the y is negative. In a case where **one** of the coefficients is negative you will have the graph of a hyperbola.

Choice 4 is an equation that can represent a horizontal parabola. It would not be a function. The general form of a quadratic equation has an x^2 . The equation given for this choice has a y^2.

Choice 3 is the equation whose graph would represent an ellipse. It is similar to the equation of a circle but with one difference. The numerical coefficients of the x and y terms are not equal.

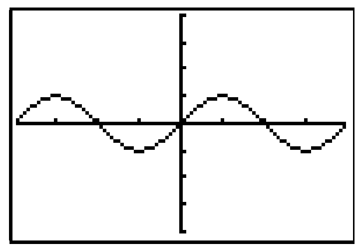
ANSWER: (3)

10) You are given the equation

$5 \sin \theta + 3 = 3$ Subtract 3 from both sides.

$5 \sin \theta = 0$ Divide both sides by 5.

$\sin \theta = 0$



You can see at the right that the sin of 0 degrees is 0. It is again 0 at 180 degrees, and at 360 degrees--multiples of 180.

ANSWER: (4)

11) Working algebraically you can set $x^2 = -x^2$ since both are equal to y.

$x^2 = -x^2$ Add x^2 to both sides.

$2x^2 = 0$ Divide both sides by 2.

$x^2 = 0$ Find the square root of both sides.

$x = 0$ Now substitute 0 for x in either original equation and your y will = 0.

The above proves that there is only **one point of intersection** (0,0). **ANSWER: (1)**

12) The general form of a quadratic equation is $ax^2 + bx + c = 0$
In such an equation, the sum of the roots will equal -b/a, while their product will equal c/a.
We are looking for the equation where the sum of the roots will equal their product.

Let's examine the four choices. We are looking for the equation where $\frac{-b}{a} = \frac{c}{a}$.

Choice 1: $x^2 + x + 1 = 0$ a = 1 b = 1 c = 1 Is $\frac{-b}{a} = \frac{c}{a}$? Substitute: $\frac{-1}{1} \neq \frac{1}{1}$

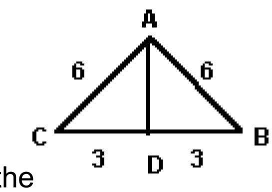
Choice 2: $x^2 + 3x - 6 = 0$ a = 1 b = 3 c = -6 Is $\frac{-b}{a} = \frac{c}{a}$? Substitute: $\frac{-3}{1} \neq \frac{-6}{1}$

Choice 3: $x^2 - 8x - 4 = 0$ a =1 b= -8 c = -4 Is $\frac{-b}{a} = \frac{c}{a}$? Substitute: $\frac{-(-8)}{1} \neq \frac{-4}{1}$

Choice 4: $x^2 - 4x + 4 = 0$ a = 1 b= -4 c = 4 Is $\frac{-b}{a} = \frac{c}{a}$? Substitute: $\frac{-(-4)}{1} = \frac{4}{1}$

ANSWER: (4)

13) To find the perimeter of a triangle, one simply finds the sum of the measures of its three sides. The sides of an equilateral triangle are congruent. In this problem you are told that the perimeter of an equilateral triangle is 18. You therefore now know that each side measures 6. You are asked to determine the altitude of this triangle. At the right you see our equilateral triangle. Let the altitude be represented by AD. The altitude to the base of an equilateral triangle will bisect the base. That is why you see that each part of the base measures 3.



Let's now consider right triangle ADB. One leg, DB, measures 3, while the hypotenuse, AB, measures 6. We are looking for the measure of AD, the other leg of this triangle, which also happens to be the altitude.
We can now use the Pythagorean Theorem which states that the square of the sums of the measures of the legs of a right triangle will equal the measure of the hypotenuse squared.

In our example this translates as $(AD)^2 + (BD)^2 = (AB)^2$. AB is the hypotenuse while AD and BD are the legs of our right triangle. Let us now substitute our known values.

$(AD)^2 + (BD)^2 = (AB)^2$	Substitute known given values.
$(AD)^2 + (3)^2 = (6)^2$	Simplify.
$(AD)^2 + 9 = 36$	Subtract 9 from both sides.
$(AD)^2 = 27$	Take square root of both sides.
$AD = \sqrt{27}$	That is your answer.

Now there seems to be a problem as the answer above does not seem to match any of the given choices. You can use your calculator to calculate the square root of 27, and then calculate the values for the choices and see which one matches. The other way is to simplify radical 27.
This process involves finding two factors of 27, one of which will be a perfect square. We can use 9, as 9 is a perfect square (3 times 3 is 9).

$\sqrt{27}$	Factor
$\sqrt{9}\sqrt{3}$	Simplify radical 9 to 3.
$3\sqrt{3}$	**CHOICE 4.**

ANSWER: (4)

Number 14 continues on the next page

14) Sigma notation is the summation symbol. It is a shorthand way of indicating that you wish to find the sum of a series. At this point, the easiest way to explain what this means would be to show you what happens in each of the given choice. Of the 4 given choices, only choice 2 will be different than the other three.

Choice 1 $\sum_{k=3}^{7} \frac{k-1}{k}$ What this means is that we are to find the sum of the series generated by $\frac{k-1}{k}$ as the value of k changes from 3 to 7. Here is what that would look like:

$\left(\frac{3-1}{3}\right) + \left(\frac{4-1}{4}\right) + \left(\frac{5-1}{5}\right) + \left(\frac{6-1}{6}\right) + \left(\frac{7-1}{7}\right)$ This would simplify to what we want: $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7}$

Choice 3: $\sum_{k=1}^{5} \frac{k+1}{k+2}$ This means we are to find the sum of the series generated by $\frac{k+1}{k+2}$ as the value of k changes from 1 to 5. Here is what it would look like:

$\left(\frac{1+1}{1+2}\right) + \left(\frac{2+1}{2+2}\right) + \left(\frac{3+1}{3+2}\right) + \left(\frac{4+1}{4+2}\right) + \left(\frac{5+1}{5+2}\right)$ Again, it is the series we are looking for: $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7}$

Choice 4: $\sum_{k=2}^{6} \frac{k}{k+1}$ This means we are to find the sum of the series $\frac{k}{k+1}$ as the value of k changes from 2 to 6. It will end up looking the same as the other two choices:

$\left(\frac{2}{2+1}\right) + \left(\frac{3}{3+1}\right) + \left(\frac{4}{4+1}\right) + \left(\frac{5}{5+1}\right) + \left(\frac{6}{6+1}\right)$ Simplified, here is the same series again $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7}$

Choice 2: $\sum_{k=1}^{5} \frac{k}{k+1}$ This means we are to find the sum of the series generated by $\frac{k}{k+1}$ as k changes from 1 to 5. Finally, **this series will be different than the other three**, and will be the answer to this question.

$\left(\frac{1}{1+1}\right) + \left(\frac{2}{2+1}\right) + \left(\frac{3}{3+1}\right) + \left(\frac{4}{4+1}\right) + \left(\frac{5}{5+1}\right)$ This series simplifies to: $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6}$

ANSWER: (2)

15) An equation of the form y = a sin bx has a period of 2π/b radians or 360/b degrees. The period is the number of degrees that it will complete its curve once. In general, when presented with a trigonometric equation, you can determine its period by dividing 2π by "b". In this problem you are given the equation $y = 2 \sin \frac{1}{3}x$. In this case, "a" which is the amplitude, is 2. The frequency "b" is 1/3. The period is 2π/b. It this case it is 2π divided by $\frac{1}{3}$.

$2\pi \div \frac{1}{3} = 2\pi \bullet \frac{3}{1} = 6\pi$

Its period is 6π. This means that this particular sine curve will complete itself in 6π or 1,080 degrees. (6 times 180 = 1080) **ANSWER: (3)**

16) You are presented with the following absolute value quadratic and asked for its solution set:

$|x^2 - 2x| = 3x - 6$

The above absolute equation yields two derived equations:

$x^2 - 2x = 3x - 6$ and $-(x^2 - 2x) = 3x - 6$ Let's solve them one at a time:

$\mathbf{x^2 - 2x = 3x - 6}$	Subtract 3x from both sides.
$x^2 - 5x = - 6$	Add 6 to both sides.
$x^2 - 5x + 6 =0$	Factor.
$(x - 2) (x - 3) = 0$	Set both factors equal to 0 and solve.

$x-2 = 0$ Add 2 to both sides. $x - 3 = 0$ Add 3 to both sides.

$\mathbf{x = 2}$ $\mathbf{x = 3}$

$\mathbf{-(x^2 - 2x) = 3x - 6}$	Distribute the negative.
$- x^2 + 2x = 3x - 6$	Subtract 3x from both sides.
$-x^2 - x = - 6$	Add 6 to both sides.
$-x^2 - x + 6 = 0$	Multiply all terms by -1.
$x^2 + x - 6 = 0$	Factor.
$(x + 3) (x - 2)$	Set both factors equal to 0 and solve.

$x + 3 = 0$ Subtract 3 from both sides $x - 2 = 0$ Add 2 to both sides.

$\mathbf{x = - 3}$ $\mathbf{x = 2}$

You've ended up with three answers. Always check your roots in absolute value equations!

x = 2 yes	**x=3** yes	x= -3 no
$\|x^2 - 2x\| = 3x - 6$	$\|x^2 - 2x\| = 3x - 6$	$\|x^2 - 2x\| = 3x - 6$
$\|2^2 - 2(2)\| = 3(2) - 6$	$\|3^2 - 2(3)\| = 3(3) - 6$	$\|(-3)^2 - 2(-3)\| = 3(-3) - 6$
$\|4 - 4\| = 3(2) - 6$	$\|9 - 6\| = 9 - 6$	$\|9 + 6\| = -9 - 6$
$\|0\| = 6 - 6$	$\|3\| = 3$ **T**	$\|15\|= -15$ **REJECT**
$0 = 0$ **T**		

ANSWER: (4)

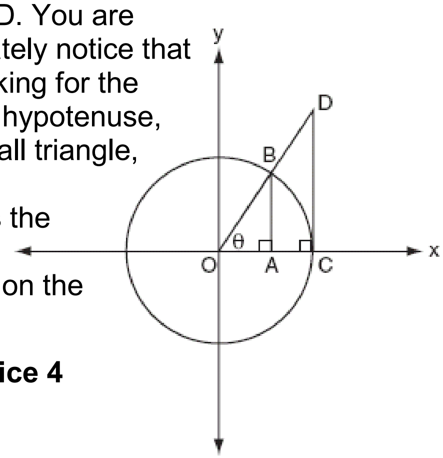
17) This problem is really asking you to simplify $\frac{\sin 2\theta}{\sin^2 \theta}$

The numerator is known as the sine of a double angle. You are given its equivalent on the formula page of the Regents. It is : 2sinθ cosθ The denominator can be simplified to sinθ (sinθ) .

$\frac{\sin 2\theta}{\sin^2\theta} = \frac{2\sin\theta\cos\theta}{\sin\theta\sin\theta} =$ at this point the sinθ 's will cancel $= \frac{2\cos\theta}{\sin\theta} =$ (cos over sin is cot) = $\mathbf{2 \cot \theta}$

ANSWER: (3)

18) Pictured at the right you can see two triangles, AOB and COD. You are told that OB, the radius, is 1. In addition, you should immediately notice that OB is also the hypotenuse of triangle AOB. Since we are looking for the cosine relationship, which is the adjacent side divided by the hypotenuse, we need the side adjacent to the central angle. Using the small triangle, OA would be the adjacent side. Using the larger triangle, OC would be the adjacent side. Using the smaller triangle, OB is the hypotenuse, and using the larger triangle, OD is the hypotenuse. There are therefore only two possibilities based on the given diagram for cosθ . Using the small triangle, AOB:



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{OA}{OB} = \frac{OA}{1} = OA$$ **This is choice 4**

Using the larger triangle COD:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{OC}{OD}$$ No choice matches this answer. **ANSWER: (4)**

19) You are asked for the equivalent of the following expression: $\frac{3y^2 - 12y}{4y^2 - y^3}$

The first step involves some factoring. Let us first factor the numerator using the distributive law: (or greatest common factor)

$3y^2 - 12y = \mathbf{3y(y - 4)}$

Now let's factor the denominator:

$4y^2 - y^3 = \mathbf{y^2 (4 - y)}$

Your new fraction now becomes $\frac{3y(y-4)}{y^2(4-y)}$

Now, the (y-4) in the numerator and the (4-y) in the denominator are opposites and reduce to -1, while the y in the numerator will cancel with one of the y's from the y^2 in the denominator. Here is the result so far:

$$\frac{3y(y-4)}{y^2(4-y)} = \frac{3(-1)}{y} = -\frac{3}{y}$$ **ANSWER: (2)**

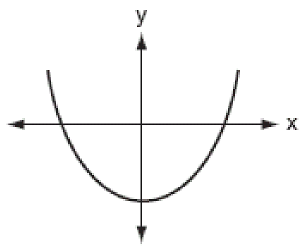
20) You are shown the graphs of 4 quadratic functions and asked to determine which one represents the function with a negative discriminant.

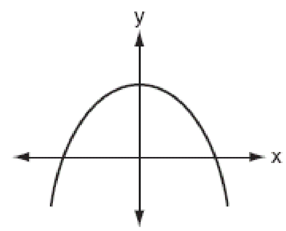
To begin with, you should know that the discriminant, the part of the quadratic formula that is under the radical sign, is called the discriminant because it allows you discriminate or determine the nature of the roots of a quadratic equation.
There are 4 possibilities regarding the roots of a quadratic equation. The roots can be:

1. Real, rational and unequal
2. Real, rational and equal
3. Real and irrational.
4. Imaginary.

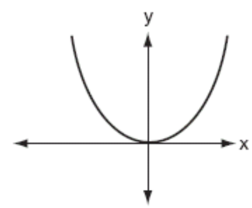
The quadratic formula is one method that can be used to solve quadratic equations. However, in this case, since you are given the graphs of the equations, you can answer the question without using the formula. Actually to use the formula you would need the equations.
The roots of a quadratic equation are the points where the parabola intersect the x-axis. The parabola can intersect the x-axis at one point, two points, or no points.

To the right you see choices 1 and 3. Both of them depict parabolas that intersect the x-axis at two points. We know that these two points will be real, although we cannot tell whether they will be rational, or irrational. Regardless, though, the discriminant in either case will not be negative. If the two points are rational, the discriminant would be a perfect square. If the two points are irrational, then the discriminant would not be a perfect square.



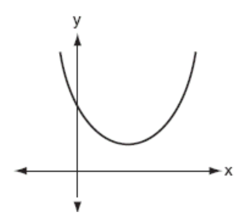


Here to the right you see a parabola that intersects the x-axis in only 1 point. It is choice number 2 and depicts an equation whose discriminant will equal 0. This will result in the solution set being two roots that are real and equal.



A parabola such as choice number 4 depicted here to the right represents a quadratic equation whose roots are imaginary. There are no points of intersection with the x-axis. The discriminant of such an equation would be negative. In other words, a negative discriminant tells you that the roots are imaginary.

ANSWER: (4)



21) Your first step here should be to simplify $(2+i)^2$.

$(2+i)^2 = (2+i)(2+i) = 4 + 4i + i^2 = 4 + 4i + (-1) = 3 + 4i$

Now, given the complex number c + di, **c corresponds to 3**.

22) You are given the equation $V = \frac{4}{3}\pi r^3$ and asked to rewrite it for r in terms of v and π.

$V = \frac{4}{3}\pi r^3$ Multiply both sides by 3.

$3V = 4\pi r^3$ Divide both sides by 4π

$\frac{3V}{4\pi} = r^3$ Take cube root of both sides

ANSWER: r = $\sqrt[3]{\frac{3V}{4\pi}}$

23) Given an angle with a radian measure of:

$$\frac{7\pi}{12}$$

You are asked to convert it to degree measure.
Remember that π radians equals 180 degrees. To convert our problem to degree measure, therefore, first multiply 7 by 180 and then divide by 12.

7(180)/12 = 105 degrees **ANSWER: 105**

24) This problem requires you to know one of the rules of logarithms.
$\log_b (10 \div 2) = \log_b 10 - \log_b 2$ This is known as the quotient rule.
Also as long as the logarithms are with the same base, you can then drop the log and base. For example if $\log_b X = \log_b Y$ then X=Y

In our problem you are presented with: $\log_b 36 - \log_b 2$
Using the quotient rule, this is the equivalent of $\log_b (36 \div 2)$

Instead of writing $\log_b 36 - \log_b 2 = \log_b x$, you can now rewrite the equation as:

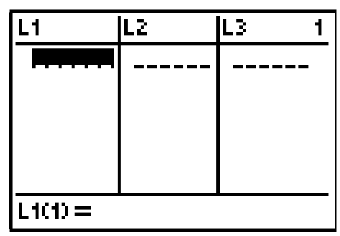
$\log_b (36 \div 2) = \log_b x$ Simplify

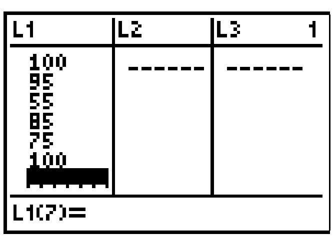
$\log_b 18 = \log_b x$ And now as long as the bases are equal

$18 = x$ **ANSWER: (18)**

25) Step number one involves finding the mean, and step number two involves finding the standard deviation. You can use your calculator to find both. Your first step is to enter the given data into a list. Here are the steps to do that:

Press **STAT** **ENTER** and you will end up at the screen to the right. It is called the STAT LIST EDITOR screen. This is the screen where you will enter the given test scores. You will enter all the scores into L1 (list 1). This is done by simply typing each number and then hitting the **ENTER** key. Below, to the left is a screen capture of what your screen will look like after you have entered all the data into L1.

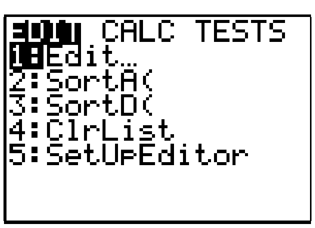


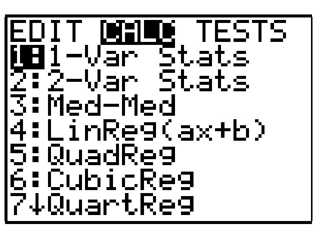


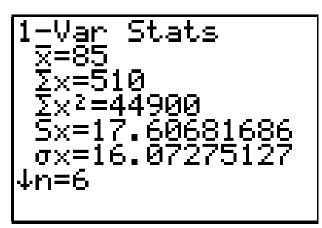
Now all you have to do is hit the following keys:

STAT ▶ **ENTER** **ENTER**

On the next page you will see a screen capture of what each key will generate.







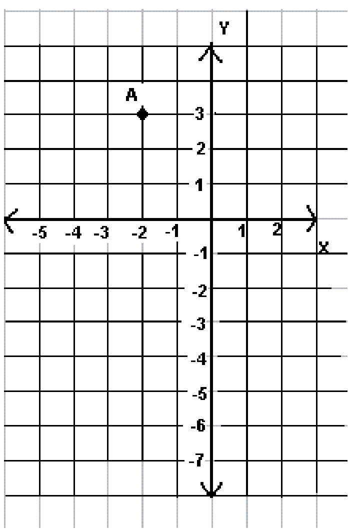
The last screen has your answer. **The mean is given by ü (x bar) and is 85.**

The **standard deviation** for the population is given by **σ** (lower case sigma) and is **16.07275127**

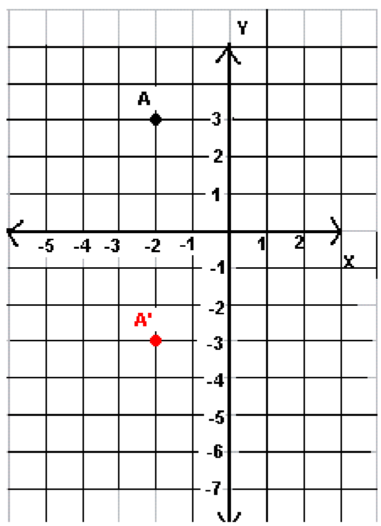
The question you have to answer here is how many scores fall within 1 standard deviation of the mean? That will be all the scores that fall between
85 minus16.07275127, and 85 plus16.07275127.
In other words, how many scores fall between **68.92724873** and **101.0727513.** Look at your original scores again. They are: 100, 95, 55, 85, 75, 100. That is a total of 6 scores. The only one that does not fall within the required parameters is the 55. In other words:
5 scores fall within one standard deviation of the mean.

By the way, looking at the last screen capture above, the line beginning with Sx is the sample standard deviation. Had you used that value in your calculations, you would still have arrived at the correct answer but you would have lost a point because this question asks for the population standard deviation.

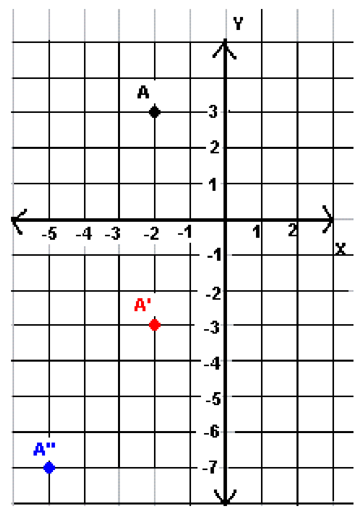
26) This problem is asking you to do 2 transformations on 1 point. You are given the point A(-2,3) and asked what its image will be under a composite transformation. You are asked to translate it **following** a reflection under the x-axis. In other words, what is the image of **A(-2,3) after a translation of $T_{-3,-4} \circ r_{x\text{-axis}}$**. You first reflect (-2,3) under the x-axis, and then perform a translation of $T_{-3,-4}$. As you can see in the diagrams below, the final image is **(-5,-7)**.



The point **A(-2,3)**



The point **A'(-2,-3),** which is the reflection of A under the x-axis



The point **A''(-5,-7)**, which is **A'(-2,-3)** after a translation of $T_{-3,-4}$ (3 to the left, 4 down)

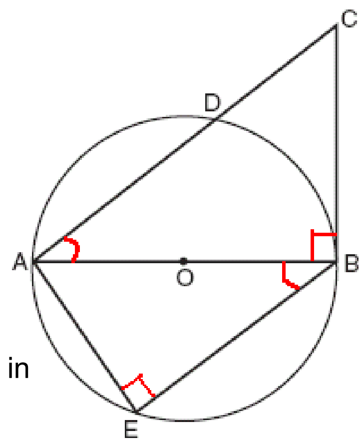
.

Here is the rule for a reflecting a point under the x-axis:

$(x,y) \xrightarrow{r_{x\text{-axis}}} (x,-y)$ That is why A(-2,3) becomes A'(-2,-3).

$T_{-3,-4}$ means a translation where -3 is added to the x coordinate, and -4 is added to the y-coordinate. So A'(-2,-3) would become A''(-2 -3, -3-4) or A''(-5, -7).

27) You are presented with the diagram at the right and asked to prove that ∆ ABE is similar to ∆ CAB. In order to prove two triangles similar, you are required to show that two angles in one triangle are congruent to two angles in the other triangle. This is shown symbolically as a.a. ≅ a.a. What you see at the right marked in red is what you can figure out based on the given. What you see in black is the diagram as presented on the Regents.



Here is the plan. You are given that AOB is a diameter. Angle AEB is an inscribed angle and it intersects arc AOB. Arc AOB is a semi circle and its measure is therefore 180 degrees. An inscribed angle is equal in measure to 1/2 the measure of its intercepted arc. Angle AEB is therefore equal to 90 degrees. In simpler language, an angle inscribed in a semicircle is a right angle.

You are also told that CB is tangent to the circle at B. OB is a radius. A line tangent to the point of the radius of a circle will be perpendicular to the radius at the point of tangency. If a line is tangent to a circle, the line is perpendicular to the radius drawn to the point of tangency. Angle ABC is therefore also a right angle. You have now proven that one angle in one triangle is congruent to the corresponding angle in another triangle.

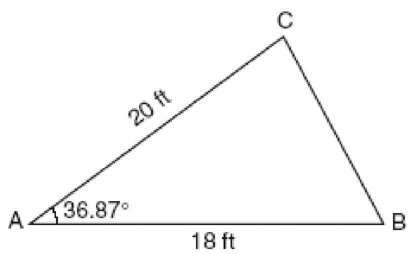
∠AEB ≅ ∠ABC.

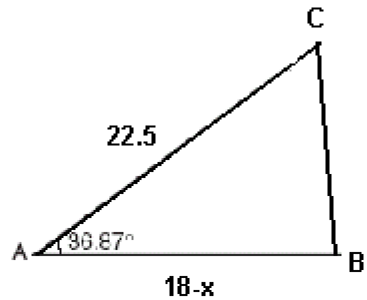
Next, you are told that line AC and line EB are parallel. Line AB is a transversal. It intersects the two parallel lines. Angle EBA and angle CAB are alternate interior angles. When two parallel lines are cut by a transversal, the alternate interior angles formed are congruent. Therefore:

∠EBA ≅∠CAB.

STATEMENTS	REASONS
1. AOB is a diameter	1. Given
2. ∠AEB is a right angle.	2. An angle inscribed in a semicircle is a right angle.
3. $\overline{OB}$ is a radius	3. Definition of radius
4. CB is tangent to the circle at B.	4. Given
5. ∠CBA is a right angle.	5. A line tangent to a circle will be perpendicular to the radius at the point of tangency.
6. ∠AEB ≅ ∠CBA a ≅ a	6. All right angles are congruent.
7. AC is parallel to EB.	7. Given
8. ∠EBA and ∠CAB are alternate interior angles.	8. Definition
9. ∠EBA ≅∠CAB a ≅ a	9. If two parallel lines are cut by a transversal, the alternate interior angles formed are congruent.
10. ∆ ABE ~ ∆ CAB	10. a.a. ≅ a.a.

(28) One of the formulas given on the formula page of your Regent's booklet is for finding the area of a triangle when all that is known are the two sides and the included angle of that triangle. The formula is presented as K = 1/2 ab sin C, where C is the included angle between sides a and b. The first triangle pictured below is the one you are presented with. Fran wants to increase AC to 22.5, keep angle A unchanged, and keep the area unchanged. By how much will AB have to be decreased to keep the area of the new triangle equal to the area of the old triangle.



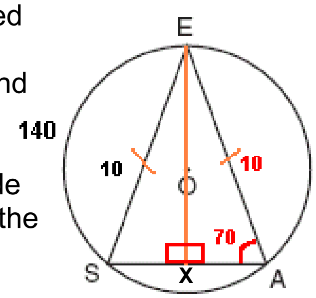


Set up your equation by using the area formula mentioned above. For the first triangle it will be 1/2 (20)(18) sin 36.87. For the second triangle it will be 1/2(22.5)(18-x) sin 36.87. Since we want the areas to be equal, we set these two areas equal to each other.

1/2 (20)(18)sin 36.87 = 1/2 (22.5)(18-x)sin 36.87	Divide both sides by 1/2 (sin36.87)
(20)(18) = (22.5)(18-x)	Simplify
360 = 405 - 22.5x	Subtract 405 from both sides.
-45 = -22.5x	Divide both sides by -22.5.
2 = x	

ANSWER: If AB is decreased by 2 ft. the area remains unchanged.

(29) You are presented with the diagram at the right. What you see highlighted in red is what I have entered based on the given information. You are told that arc SE measures 140. Angle A is an inscribed angle and will therefore be equal to one-half the arc it intercepts. It intercepts the arc of 140 and therefore measures 70 degrees. Since SE and AE are congruent, triangle SEA is isosceles. I drew altitude EX to base SA. The altitude to the base of an isosceles triangle bisects the the base. XA will therefore equal half of SA. You can easily determine the length of XA using the cosine ratio on right triangle AXE. The cosine of an angle is the ratio of adjacent side divided by the hypotenuse. Using right triangle AXE and angle A, the adjacent side is XA and the hypotenuse, EA, is 10.



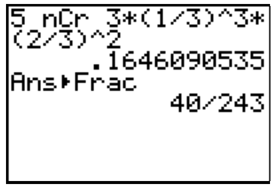
$\cos A = \dfrac{XA}{EA}$	Substitute the known values.
$\cos 70 = \dfrac{XA}{10}$	Multiply both sides by 10.
XA = 10 (cos 70)	Divide both sides by cos 70
XA = 3.420201433	XA is half of SA, so now double XA

SA = 2(3.420201433) = 6.840402867 **To the nearest tenth that is 6.8**

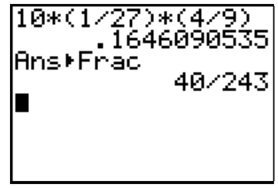
ANSWER: SA = 6.8

30) You are told that the probability that the bus will be **late** is **1/3**.
Therefore, the probability that it will **not** be **late** is **2/3**.
Being late 3 times, 4 times or even 5 times during a 5-day school week all satisfy the condition of being late **at least 3 times.** Your job is therefore to figure out each one of these probabilities and then **add** them together.

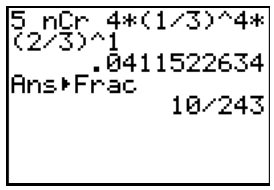
P on 5 spins (3 lates, 2 not late)
$_5C_3$ P(3 lates) • P(2 not late)
$_5C_3\ (1/3)^3\ (2/3)^2$
10 (1/27)(4/9) = **40/243**



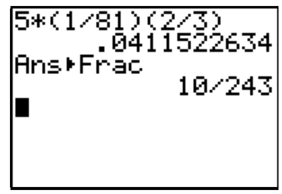
or



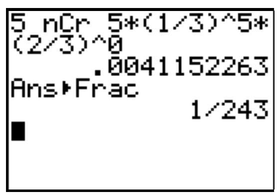
P on 5 spins (4 lates, 1 not late)
$_5C_4$ P(4 lates) • P(1 not late)
$_5C_4\ (1/3)^4\ (2/3)^1$
5 (1/81) (2/3) = **10/243**



or



P on 5 spins (5 lates, 0 not late)
$_5C_5$ P(5 lates) • P(0 not late)
$_5C_5\ (1/3)^5\ (2/3)^0$
1 (1/243) (1) = **1/243**

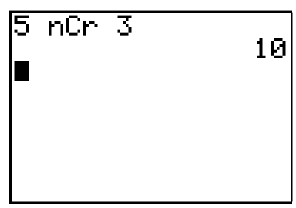


NOW ADD THE 3 PROBABILITIES

ANSWER: Probability = 51/243

PLEASE NOTE:
Here is how you can do $_5C_3$ on your calculator. To continue with $(1/3)^3\ (2/3)^2$ just follow the appropriate key strokes.

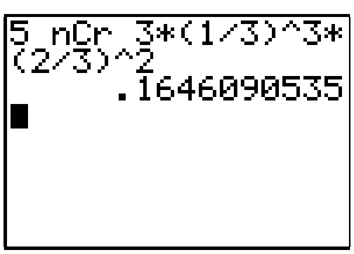
5 MATH ▶ ▶ ▶ 3 3 ENTER



To multiply the fractions by the answer 10, continue inputting the following:

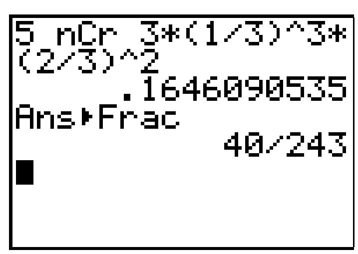
X (1 ÷ 3) ^ 3 X (2 ÷ 3) ^ 2 ENTER

Your screen will look like the one below to the left. To change .1646090535 to a fraction, input the following:

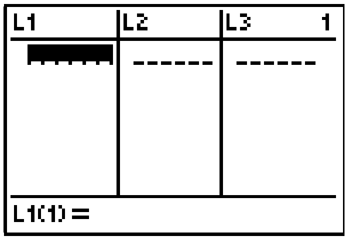


MATH ENTER ENTER

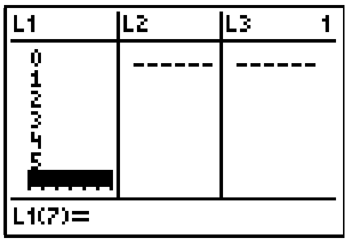
The screen will now look like the one at the right.



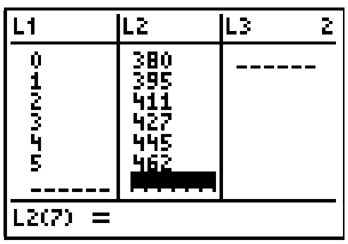
(31) You are being asked for the exponential regression equation based on the data presented in the table, rounding all values to two decimal places. Your first step involves the entering of the data into two tables. Begin by hitting **STAT** **ENTER** Your screen should look like the one below to the left.



Make sure there is no information in any of the columns. The next part is easy. Enter into the **L1** column the numbers 0 thru 5. Simply hit the first number 0 followed by ENTER. Then 1 followed by ENTER, and so on until you have entered all the data from 0 thru 5 that appear the Years Since Investment (x) column. Your screen will now look like the second one to the left.



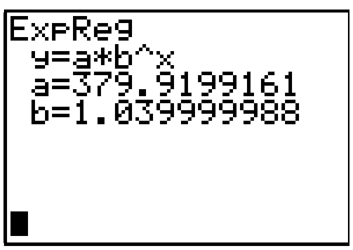
Now use your cursor key ▶ to move into the **L2** column. Enter the data found in the Value of Stock in Dollars (y) column of the table. Begin with 380 followed by ENTER, and so on, until you have entered them all thru 462. Don't forget to hit ENTER. You will now have completed the L1 and L2 column's with the data that was presented in the table. For each L1 you should have an accompanying L2. Check the screen at the left.



You now need to find the exponential regression equation for this data. Hit the following keys:

STAT ▶ **0** **ENTER**

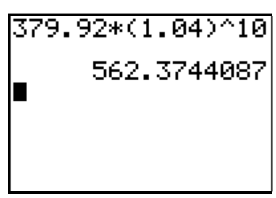
The problem states that coefficient and the base should be rounded to the nearest hundredth. a=379.92 b= 1.04



<u>ANSWER</u>: y=379.92(1.04)^x

Next you are being asked to figure out the value of the stock 10 years after the initial purchase. This 10 is represented by x. All you have to do now is solve the above exponential equation when x equals 10.

Use your calculator: **379.92(1.04)^{10}** = 562.374487



To the nearest dollar: **<u>ANSWER</u>: $562**

(32) $T = 400 - 350(3.2)^{-0.1m}$

You are told that the temperature, T, is 300. Your equation now becomes:

$300 = 400 - 350(3.2)^{-0.1m}$

You will be able to solve for the number of minutes, m, by using the Power Law of Logarithms. First let's do a bit of simplifying:

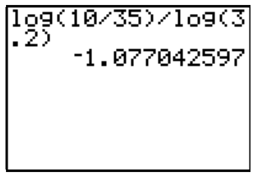
$300 = 400 - 350(3.2)^{-0.1m}$ Subtract 400 from both sides.

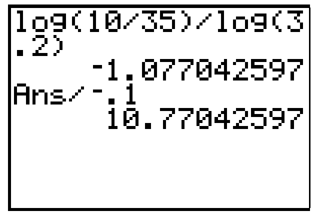
$-100 = -350(3.2)^{-0.1m}$ Divide both sides by -350.

$\frac{10}{35} = (3.2)^{-0.1m}$ Use Power Law of Logarithms

$\log\left(\frac{10}{35}\right) = -.01m \ \log 3.2$ Divide both sides by log 3.2

$$\frac{\log\left(\frac{10}{35}\right)}{\log 3.2} = -0.1m$$





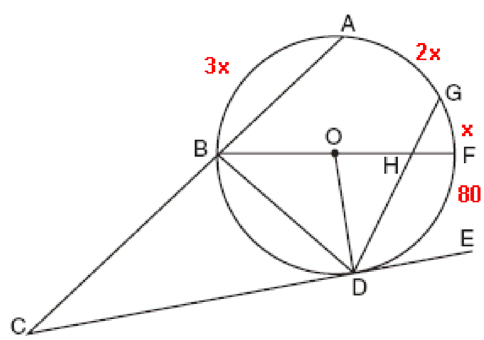
$-1.077042597 = -0.1m$ Divide both sides by -0.1

$10.77042597 = m$

ANSWER: To the nearest minute m=11

33) To the right is the diagram presented to you. What you see marked in red is what I have added based on the givens. As you can see, you are given that the measure of arc FD is 80. In addition, you are told that arcs AB, AG, and GF, are in a ratio of 3:2:1. That is why I have marked them 3x, 2x, and x.

You now have enough information to calculate the measure of arc GF. Here is how. Notice that you are told that BOHF is a diameter. The measure of the arc subtended by a diameter is 180 degrees (semicircle). Set up your equation to solve for arc GF or x.



$3x + 2x + x = 180$ Combine like terms.

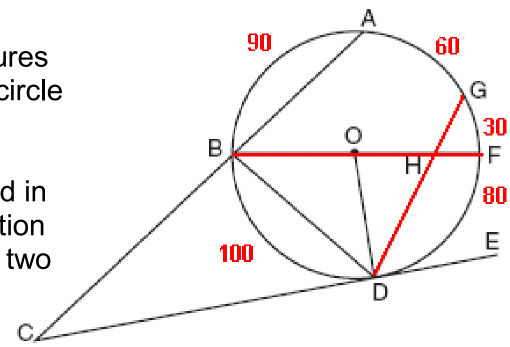
$6x = 180$ Divide both sides by 6.

$x = 30$

You now know that **arc GF**, represented by x, **equals 30 degrees**.

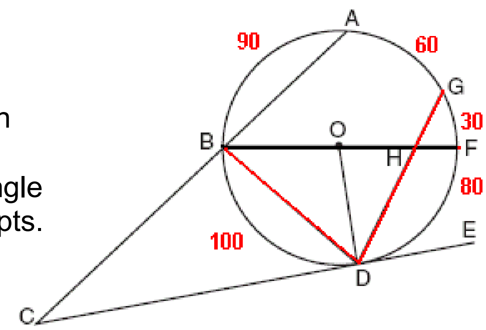
The problem continues on the next page with the information you now know as a result of x = 30. Arc AB will equal 3(30) or 90, arc AG = 2(30) = 60, while arc GF = 30.

ARC GF = 30



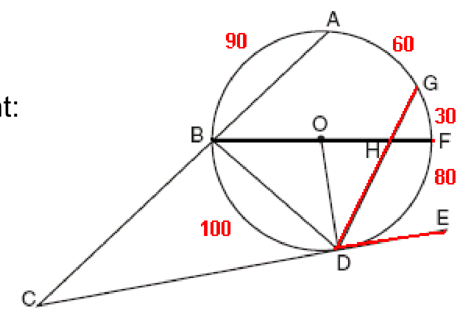
You may have noticed that we also know that arc BD measures 100, since the sum of that arc and arc FD also form a semicircle and therefore together measure 180 degrees.
Next you are asked for the measure of angle BHD.
Notice at the right that chords BOHF and DG are highlighted in red. Angle BHD is one of the angles formed by the intersection of these two chords. An angle formed by the intersection of two chords will equal 1/2 the sum of the measures of the arcs intersected by these two chords. In our case at the right:
m∠BHD = 1/2 (measure arc BD + measure arc GF)
m∠BHD = 1/2 (100+30) = 1/2 (130) = 65

m∠BHD = 65



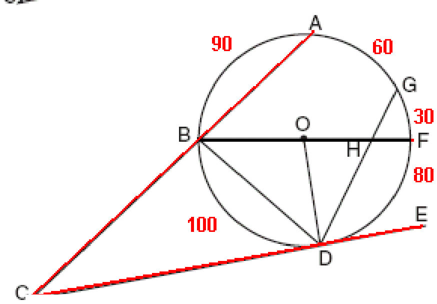
Next you are asked for the measure of ∠BDG. As you can see at the right, this angle is an inscribed angle which intercepts an arc of 90+60 or 150 degrees. An inscribed angle is equal in measure to 1/2 the measure of the arc it intercepts. In other words,
m∠BDG = 1/2(arc AB + AG)= 1/2(90+60) = 1/2(150)= 75

m∠BDG = 75



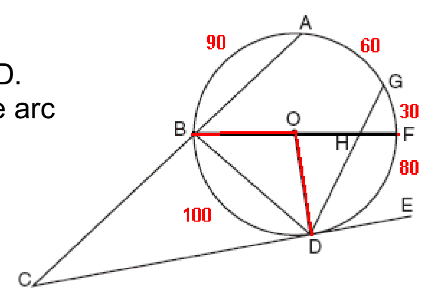
The next angle you are asked for is ∠GDE, formed by a tangent and a chord. It intercepts arc GFD. It will equal 1/2 the measure of the arc it intercepts. In our case at the right:
m∠GDE = 1/2 (arc GF + FD) = 1/2 (80+30) =1/2(110)= 55.

∠GDE = 55



Your next task is to calculate the measure of ∠ C.
As you see at the right, it is formed by secant CBA and tangent CE. Such an angle will be equal in measure to 1/2 the difference of the arcs it intercepts. As you can see, in our case, angle C intercepts arc BD which equals 100. It also intercepts arc AGFD, which is 170, the sum of the three smaller arcs.
m∠ C = 1\2 (170 - 100) = 1/2 (70)= 35

∠C = 35



Finally, you are asked for the measure of angle BOD.
At the right you can see that it is formed by two radii, OB and OD.
It is therefore a central angle and will be equal in measure to the arc it intercepts. It intercepts arc BD which equals 100 degrees.

m∠BOD = 100

(34) $L = -5t^2 - 8t + 120$ is the equation given as representing the amount of water in a bathtub as it drains. L represents the amount of water, in liters, contained in the bathtub, while t represents time in minutes since the drain plug was pulled. The first question asked is how many liters of water were in the bathtub when the plug was pulled?

To answer this question simply set the time, t, equal to 0 and you will calculate how much water was was in the tub before the plug was pulled.

$L = -5t^2 - 8t + 120$ Substitute 0 for t.

$L = -5(0)^2 - 8(0) + 120$ Simplify

$L = 0 - 0 + 120$

$L = 120$

There were 120 liters in the tub.

The next part of the question asks how long it takes for all the water to drain.

That will be the point where there are 0 liters left in the tub. So all you have to do now is set L equal to 0 and solve for the value of t . (You are asked to find t to the nearest tenth of a minute.)

$-5t^2 - 8t + 120 = 0$

This is a quadratic equation in the form of $ax^2 + bx + c = 0$

$-5t^2 - 8t + 120 = 0$ $a = -5$ $b = -8$ $c = 120$ Use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Substitute the values for a, b, and c.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(-5)(120)}}{2(-5)} = \frac{8 \pm \sqrt{64 + 20(120)}}{-10} = \frac{8 \pm \sqrt{64 + 2400}}{-10} = \frac{8 \pm \sqrt{2464}}{-10}$$ = calculator time…

At this point it looks like there are two answers: $\frac{8+\sqrt{2464}}{-10}$ and $\frac{8-\sqrt{2464}}{-10}$ Let's try each one.

$\frac{8+\sqrt{2464}}{-10} = -5.77$ to nearest tenth

(8+√(2464))/-10
-5.763869458

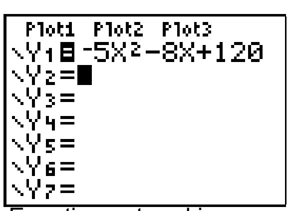
Reject negative answer as time in this problem cannot be negative.

$\frac{8-\sqrt{2464}}{-10} = 4.2$ to nearest tenth

(8-√(2464))/-10
4.163869458

ANSWER: 4.2 minutes to nearest tenth.

ALTERNATE METHOD: You can always solve a quadratic by entering it into the y=editor window, and then search the table for the value of X that will make y equal to 0. I will quickly show you some screen captures. In addition, since they want the answer to the nearest tenth, I set the table to move in increments of .01. That will be the first window you see. Table start makes no difference. All you have to do is scroll by using either the up or down arrow to see what x is when y equals 0. To make life easier in this problem, imagine that L is my y, while t (time) is my x.



Equation entered in y=editor.

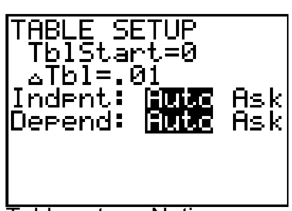


Table set up. Notice the .01 to make rounding to tenths easier.

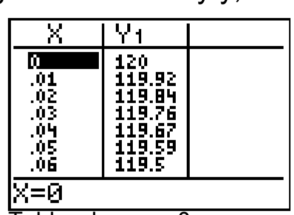
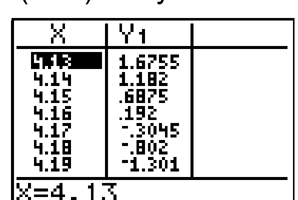


Table when x = 0 We are looking for x when y = 0



Somewhere between x = 4.16 and 4.17, y will = 0 **4.2 to nearest tenth**