Home Instruction Schools **ANSWERS MATH B – August 16th, 2007**

1) You are presented with $f(x) = x^{-x} - x^0 + 2^x$ and asked to find f(3).
Step one involves substituting 3 for x.
$3^{-3} - 3^0 + 2^3$ A number to a negative exponent equals to 1 over that number to the + exponent.
$\frac{1}{3^3} - 3^0 + 8$ Any number (except 0) to the 0 equals 1.
$\frac{1}{27} - 1 + 8$ Simplify
$\mathbf{7\frac{1}{27}}$ **ANSWER: (2)**

2) You are asked to find the equivalent of $3i(2i^2 - 5i)$
Use the distributive property to simplify the above.
You should also know the powers of i. $i^2 = -1$ and $i^3 = -i$

$3i(2i^2 - 5i)$ Use the distributive property.
$6i^3 - 15i^2$ Simplify
$6(-i) -15 (-1)$ Continue simplifying.
$-6i + 15$ or **15 - 6i** **ANSWER: (1)**

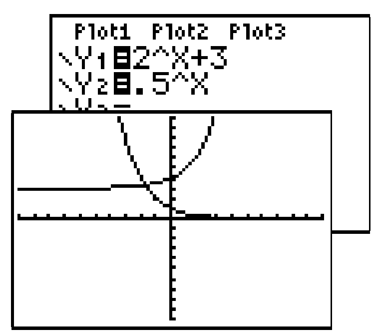
3) If csc θ = -2, what is sin θ ?
Sine and cosecant are reciprocal functions. This means that sin θ = 1 / csc θ .
In our case above, then:
$\sin\theta = \frac{1}{-2}$ or $\mathbf{-\frac{1}{2}}$ **ANSWER: (3)**

4) Express 235° in radian measure.
When converting from degree to radian measure, simply divide the degree measure by 180 and reduce. (Remember the **B**).
$\frac{235}{180}$ **B** Divide numerator and denominator by 5
$\mathbf{\frac{47}{36}}$ **B** **ANSWER: (4)**

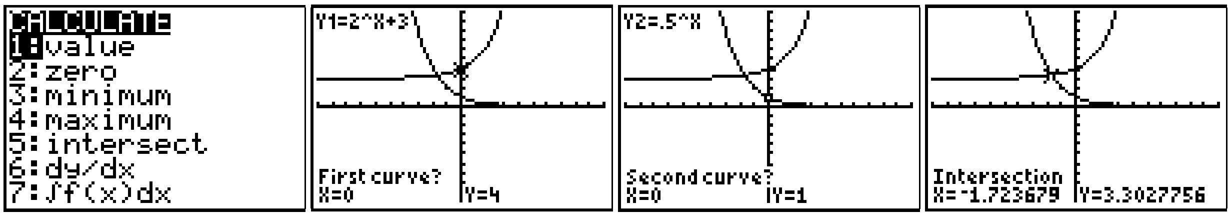
5) You are given the flight paths of two jets. They are represented by the following two equations $y = 2^x + 3$ and $y = 0.5^x$.
What is the best approximation of the intersection of their flight paths?

Enter both equations into the **Y=** editor of your calculator.
Next hit GRAPH and graph them in a standard window.

On the next page I will continue with the instructions of how to use the calculator to find the point of intersection.



Hit the 2nd key followed by **TRACE**. (Remember that you are really accessing the **CALC** key as indicated above the TRACE key in yellowish letters).
Below you see the CALCULATE menu. Select item 5.and you will see the second screen capture below. Hit ENTER and you will see the third screen capture. Finally hit ENTER twice and you will see the final screen capture that shows you the point of intersection.



Look closely at the bottom of that last screen capture and you will see the x and y coordinates of the point of intersection. It is closest to choice 1. **ANSWER: (1)**

6) You are presented with a complex fraction: $\dfrac{\frac{1}{a} - a}{\frac{1}{a} + 1}$

Its **numerator is** $\mathbf{\frac{1}{a} - a}$ and its **denominator is** $\mathbf{\frac{1}{a} + 1}$

Switch the numerator to $-a + \frac{1}{a}$ and treat it as if you were changing a mixed numeral to an improper fraction. (-a times a, plus 1 becomes the numerator and the denominator stays a.)

In other words, your new numerator becomes $\frac{-a^2+1}{a}$ which we will rewrite as $\mathbf{\frac{1 - a^2}{a}}$.

In the same manner, switch the denominator to $1 + \frac{1}{a}$ and change it to an improper fraction.

$\mathbf{\frac{a+1}{a}}$ becomes the new denominator. The original problem has now become:

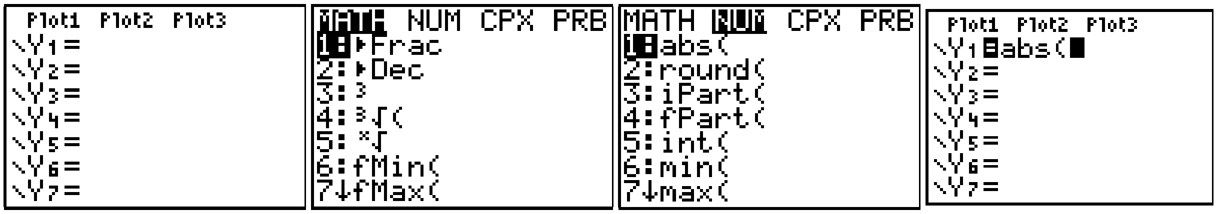
$\mathbf{\frac{1 - a^2}{a} \div \frac{a+1}{a}}$ Change to multiplication of reciprocal.

$\frac{1 - a^2}{a} \bullet \frac{a}{a+1}$ Factor (the difference of two squares).

$\frac{(1+a)(1-a)}{a} \bullet \frac{a}{a+1} = \frac{\cancel{(1+a)}(1-a)}{\cancel{a}} \bullet \frac{\cancel{a}}{\cancel{a+1}} = 1 - a$ **ANSWER: (3)**

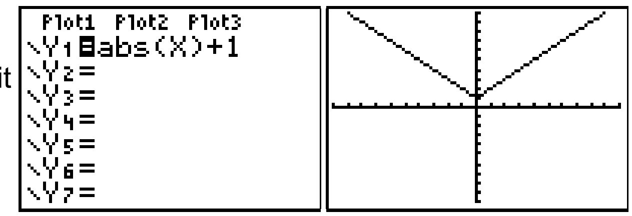
7) Graph each choice and you will see which graph matches the one shown.
Actually, choice 1 will be a perfect match. On the next page are the instructions for graphing an absolute value equation.

Hit the **Y=** key. Now hit **MATH** followed by ► to access the NUM menu. Your first option on that menu is abs, which stands for absolute. Now hit **ENTER**. and you will see the last screen below.



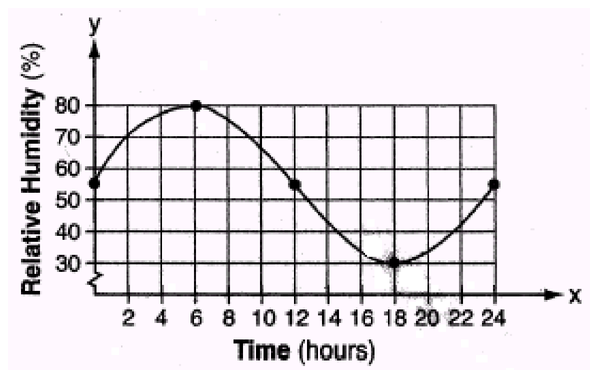
At this point simply enter the rest of the equation given for choice 1. Your screen should look like the screen capture at the right. Now when you hit **GRAPH**. you will see the graph of $f(x) = |x| + 1$

ANSWER: (1)



8) On the coordinate plane, the domain represents the x-values, and the range represents the y-values. You are presented with the graph at the right and asked for the **range** of the set of data shown. The range will be from the lowest y-value thru the highest. At the right, you see the lowest y is 30, and the highest y is 80. This makes the range30 thru 80 (including 30 and 80).
This translates as 30 ≤ y ≤ 80.

ANSWER: (3)



9) You are asked to represent the following equation in logarithmic form: $T = 2\pi\sqrt{\frac{\ell}{g}}$
This problem requires you to know some of the rules of logarithms.
For example, log (abc) = log a + log b + log c

In our example therefore, log T = log 2 + log **B** + log $\sqrt{\frac{l}{g}}$

Next step is as follows. $\sqrt{x}$ can be written as $x^{½}$

Now, using the power rule, $\log x^{½} = ½ \log x$.

In the same vein, $\log\sqrt{\frac{l}{g}} = ½ \log \frac{l}{g}$.

One more rule now. $\text{Log} \frac{l}{g} = \log l - \log g$, Therefore, $½ \log \frac{l}{g} = ½ \log l - ½ \log g$

Putting all of the above together we get log T = log 2 + log **B** + ½ log l - ½ log g
Almost forgot!….you were told that g = 32 so your real answer is:

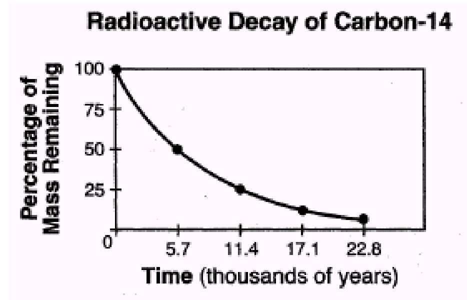
log T = log 2 + log B + ½ log l - ½ log 32 **ANSWER: (2)**

10) The graph shown at the right does not represent a quadratic function because it is not a parabola.

It also does not represent a trigonometric function because it is not one of your familiar sin, cos, or tan curves.

Finally, it is not a straight line so it does not represent a linear function.

Choice 1, exponential, is left as your answer.

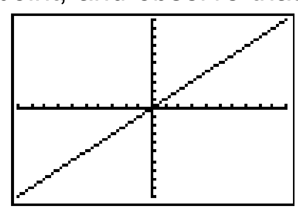


ANSWER: (1)

11) You are told that, under a dialation, the image of P(-15,6) is P'(-5,2). What is the constant of dilation? Under a dilation, the original coordinates are multiplied by a constant. If the constant is greater than 1, the image will be enlarged. If the constant is between 0 and 1 (a fraction), the image will be shrunk. You immediately see that to get from -15 to -5 you will be multiplying by a fraction. -5 is ⅓ of -15, and 2 is ⅓ of 6. This means that the constant of dilation is ⅓.

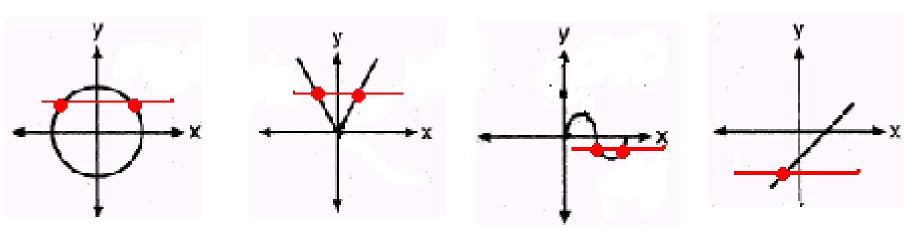
ANSWER: (2)

12) You are presented with four graphs and asked to select the one whose inverse is a function. A relation is a function if each element in the domain has a unique element in the range. What this means is that each x will yield a unique y. You wll never end up having two different values for the same x. The easiest way to ascertain whether a relation is a function is when you are looking at the graph of the relation. If you can drop a vertical line at any point, and observe that the line passes through more than one point on the graph, then the graph does not represent a function. This is known as the vertical line test. Something else that you should know is that a relation and its inverse will be symmetric to the line y = x. The graph of the line y = x is the diagonal line you see graphed at the right.



Now to digress back to our problem for a bit. You can look at each choice and draw its inverse, and then check whether or not it is a function by using the vertical line test.

There is one way of doing our problem that is easier. You save the time it takes to draw or imagine the inverse and then check whether or not it is a function. Simply look at the original graph. Imagine drawing a horizontal line. If that horizontal line goes through two points, then the inverse of that relation will not be a function, because its inverse will end up having a vertical line that will go through two points. Look below and you see a red horizontal line going through the first 3 choices. Only in the last choice will a horizontal line go through only 1 point.



ANSWER: (4)

13) You are asked for the solution set of the following inequality: $x^2 + 4x - 5 < 0$
Step number 1-- make believe it is an equation and solve it.
$x^2 + 4x - 5 = 0$ Factor.
$(x - 1)(x + 5) = 0$ Set both factors equal to 0 and solve for x in each.

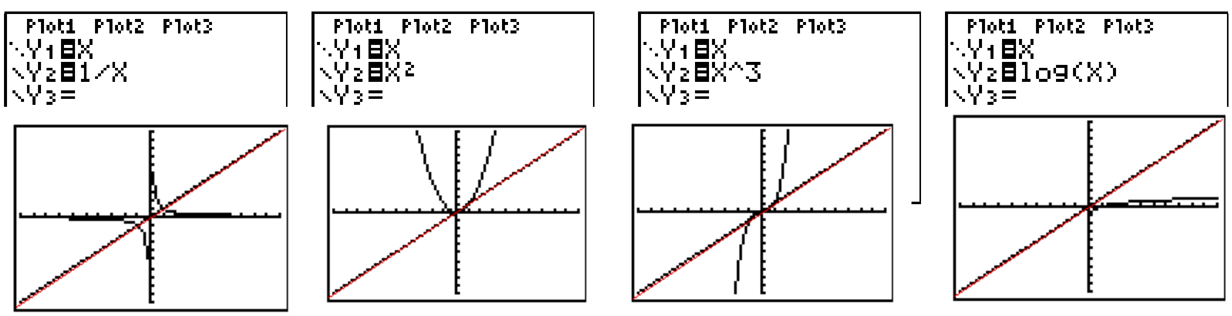
$x - 1 = 0$ Add 1 to both sides. $x + 5 = 0$ Subtract 5 from both sides.
$x =$ **1** $x =$ **-5**

Now recall that the original problem was an inequality. It's answer will still however contain a 1 and -5. The original inequality was a less than (<). Now this will always work. Take your right thumb and face it to the left, the direction of the inequality symbol Take your other thumb and face it in the opposite directions. If the symbols are your thumbs they will be facing as follows: > <. The left thumb is now indicating greater than, and the right thumb is indicating less then. Now let's place the numbers where they belong. -5 is to the left of 1, 1 is to the right of -5. Your solution set for this inequality is x > -5 and x < 1. This is exactly what choice 4 states: **{xI -5 < x < 1}** Negative 5 is less than x which is less than 1. In other words, x is a number between -5 and 1.

(Had the original inequality been a greater than (>), then that would have been the direction of your right thumb, and your left thumb would have been in the opposite direction (<). The symbols in your solution set would therefore be as follows: < >. You would be using the same -5 and 1. Now your solution would be x < -5 or x > 1 which would match choice 2.)

ANSWER: (4)

14) You are presented with four functiions and asked which one is symmetric with respect to the graph of the line y=x. Below are the four functions and their graph. I have also entered the graph for the line y = x. It is the diagonal line you saw in problem number 12.



Above you see the functions entered into the calculator and their corresponding graphs. The first one is symmetric with respect to the line y = x **ANSWER: (1)**

15) The coordinates of ΔJRB are J(1,-2) R(-3,6) B(4,5). You are asked for the coordinates of the vertices of its image after the following transformation:

$T_{2,-1} \circ r_{y\text{-axis}}$

The above can be read as a Translation of (2,-1) following a reflection in the y-axis.
This means that the reflection is done first, and then the translation is done on that reflection.

First let's reflect the original points in the y-axis.

Here is the rule for a reflection in the y-axis: P(x,y) becomes P'(-x,y)
The sign of the x-coordinate will change.

	$r_{\text{y-axis}}$
J(1,-2)	J'(-1,-2)
R(-3,6)	R'(3,6)
B(4,5)	B'(-4,5)

The next step is to translate the new points using the following rule $T_{2,-1}$
This means that you will add 2 to each x-coordinate and add -1 to each y-coordinate.

	$T_{2,-1}$
J'(-1,-2)	J"(1,-3)
R'(3,6)	R"(5,5)
B'(-4,5)	B"(-2,4)

The new coordinates will be (1,-3) (5,5) (-2,4) **ANSWER: (3)**

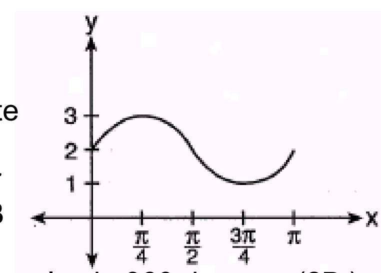
16) You are presented with four choices and asked which one is equivalent to: $\frac{2}{1-\sqrt{3}}$?

This problem requires you to rationalize the denominator. This is done by multiplying the numerator and the denominator by the conjugate of the denominator. The conjugate of 1 - $\sqrt{3}$ is 1 + $\sqrt{3}$. To multiply them you can use FOIL.

$$\frac{2}{1-\sqrt{3}}\left(\frac{1+\sqrt{3}}{1+\sqrt{3}}\right) = \frac{2+2\sqrt{3}}{1-3} = \frac{2+2\sqrt{3}}{-2} = \frac{\cancel{2}+\cancel{2}\sqrt{3}}{\cancel{-2}} = \mathbf{-1-\sqrt{3}}$$

ANSWER: (4)

17) You are presented with the graph at the right and asked for the equation that best represents it. You should immediately notice that it is a sine curve that has been translated two units upwards. Only choices 2 and 4 indicate a sine cure with a translation of +2. You also notice that the sine curve at the right completes one cycle in **B**, rather than 2**B**. This means that it would complete 2 cycles in 2**B** degrees. That is why your answer is choice 4.



The sin 2x indicates that this sine curve would complete 2 cycles in 360 degrees (2**B**.), and the +2 indicates the translation upwards of 2 units. **ANSWER: (4)**

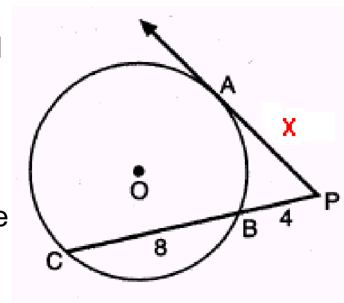
18) You are given four equations and asked to select the one that has **4 - 3i** as one of its roots. You can at this point use the quadratic formula on each choice and see which one is correct. What you should realize, however, is that whenever the roots of a quadratic are imaginary they will always be a conjugate pair. This means that you know the other root will be **4 + 3i**. You also learned that, given a quadratic equation in the form $ax^2 + bx + c = 0$, the sum of the roots can be represented by -b/a and their product by c/a. Now look at the four choices. The product of the roots would not immediately give you an answer as two of the equations end in -25 and two in +25. So let's immediately see what the sum of the roots equal.
The roots are **4 - 3i** and **4 + 3i**. Their sum is represented by -b/a. In each of our equations a=1, so let us now solve for b.

Sum of the roots = $\frac{-b}{a} = \frac{-b}{1} = -b$ Now let's get the actual sum of our roots: 4 - 3i + 4 + 3i = **8**
Therefore, -b = 8. This means that b= -8. **Only the equation for choice 4 has b equal to -8**.
To make sure that it is the correct answer let us make sure that the product of the roots is correct as well.

Product of the roots = $\frac{c}{a}$ Since a is again 1, the product of the roots in our case will equal c.

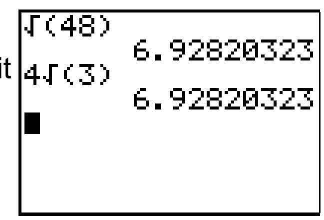
Let us get the actual product. (4 - 3i) (4-3i) = $16 - 9i^2$ = 16 -9(-1) =**25**
You can now see that in the equation for choice 4, b= -8 and c = 25 **ANSWER: (4)**

19) You are given the diagram at the right and are asked for the length of line segment PA which I have marked in red with an X.



You are told that line PA is tangent to the circle at A, and line segment PBC is a secant. Line segment PB = 4, and line segment BC = 8.
The theorem used to solve this problem is that the square of the measure of the tangent segment, in this case X^2, will equal the product of the measures of the secant segment, in this case PC, and its external segment, in this case PB.
In otherwords,

$(PA)^2 = (PC)(PB)$	Substitute the given values.
$X^2 = (4+8)(4)$	Simplify.
$X^2 = (12)(4)$	Continue simplifying.
$X^2 = 48$	Take square root of both sides.
$X = \sqrt{48}$	



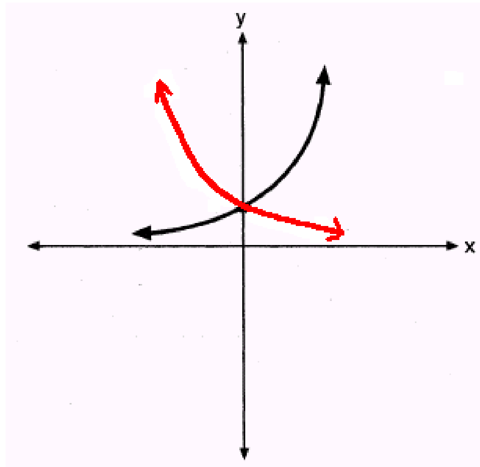
At this point you can simply enter $\sqrt{48}$ in your calculator and hit ENTER, and then do the same for all the choices. The answer you will get for $\sqrt{48}$ will match the one for choice 3.
Or you can get your answer the old-fashioned way:

$\sqrt{48}$	Factor using greatest perfect square
$\sqrt{16}\sqrt{3}$	Simplify
$\mathbf{4\sqrt{3}}$	

ANSWER: (3)

20) If $\log_2 a = \log_3 a$, what is the value of a?
Since both logs are equal to each other, they are equal to the same element. Let's call that element x. In otherwords, if $\log_2 a = x$ and $\log_3 a = x$, we can say $\log_2 a = \log_3 a$.
Now let's rewrite each logarithmic equation in exponential form. Here is a simple example that you can use as a template: $\log_{10}100 = 2$ $10^2 = 100$ or $100 = 10^2$
In the same manner: $\log_2 a = x$ becomes $2^x = a$ and $\log_3 a = x$ becomes $3^x = a$.
This means that we are looking for a case where $2^x = 3^x$ This can only be true when x=0.
Any number to the 0 equals 1. so 2^x and 3^x would become 2^0 and 3^0 and be equal to each other.
They would equal 1. **ANSWER: (1)**

21) The graph shown is an exponential graph. You are first asked to draw its reflection in the y-axis. You see it, more or less, to the right in red.



The next question is what is their point of intersection. The reflection of F(x)= a^x becomes a^{-x} because when reflecting in the y-axis, the sign of the x-coordinate changes while the y remains the same. Regardless of that, you are now looking for the one point that both of these graphs have in common. This now becomes similar to the problem right before this one--number 20. You are looking for the x-value where both of these functions will equal to each other. This will be when x=0, because any number raised to the 0 will equal 1. So in actuality, that is your answer: (0,1). The point of intersection will be the point (0,1).

ANSWER: The point of intersection is (0,1).

22) You are asked to solve the following for all values of x:

$\frac{2}{x+1} = x$ Multiply both sides by x + 1

$x(x + 1) = 2$ Use distributive property.

$x^2 + x = 2$ Subtract 2 from both sides.

$x^2 + x - 2 = 0$ Factor and set each factor equal to 0.

$(x + 2)(x - 1) = 0$

$x + 2 = 0$ Subtract 2 from both sides. $x - 1 = 0$ Add 1 to both sides.

x = -2 **x = 1**

(When solving fractional equations and multiplying both sides by the denominator, the derived equation will not necessarily be equivalent to the original equation. In such cases, one or both of the roots may be extraneous. This means that the root may be "extra." It will work in the derived equation but not in the original.)

Check: x = -2

$\frac{2}{x+1} = x$

$\frac{2}{-2+1} ? -2$

$\frac{2}{-1} ? -2$

$-2 = -2$

x = 1

$\frac{2}{x+1} = x$

$\frac{2}{1+1} ? 1$

$\frac{2}{2} ? 1$

$1 = 1$

ANSWER: x = -2 x = 1

23) The probability that a child will be born with a certain trait is 1/8. The family in question has 4 children. What is the probability that **exactly** 3 of the 4 children will have that trait?
What you should immediately realize is that if the probability for being born with the trait is 1/8, then the probability of not being born with that trait is 7/8.
The number of ways that exactly 3 of the 4 have the trait translates to $_4C_3$ (order does not count).
3 children being born with that trait translates as $\mathbf{(1/8)^3}$.
1 will be born not having the trait translates as $\mathbf{(7/8)^1}$.
Putting this all together, the answer to the question is:
$_4C_3\,(1/8)^3(7/8)^1$ Use your calculator
Here are the instructions for finding $_4C_3$ on your calculator.
First enter the 4. Next hit **MATH** and follow it with ► ► ► . You will now be in the PRB menu.
Select item **3** which reads nCr. Now you can enter the 3, and your screen will show $_4C_3$.
Continue entering the remainder of your problem the way you see in the screen capture at the right. You will notice that your answer shows up as a decimal. If you want to change it to a fraction, here are the steps.
At the point after you have your decimal, hit **MATH**, and then hit the **ENTER** key two times.

```
4 nCr 3(1/8)^3(7
/8)^1
          .0068359375
Ans▸Frac
              7/1024
```

24) You are told that the ratio of the length to the width of a particular painting is $\mathbf{(1+\sqrt{5}):2}$
In addition you are told that the width of the painting is 14 feet. What is the length?
Step number 1…Set up a proportion using length to width. Let the length equal x.

$\frac{\text{Length}}{\text{Width}} \quad \frac{1+\sqrt{5}}{2} = \frac{x}{14}$ The usual step is to now crossmultiply, but here is something simpler

Notice that the denominator 14 is 7 times the first denominator of 2.

$7(1+\sqrt{5})$ This means that the numerator of x will be 7 times the numerator $1+\sqrt{5}$

$7+7\sqrt{5}$ So all that is necessary to find the width is to multiply $1+\sqrt{5}$ by 7.

ANSWER: The width is $7+7\sqrt{5}$ feet.

25) You are given the formula below and asked to solve for v in terms of l and **B**,

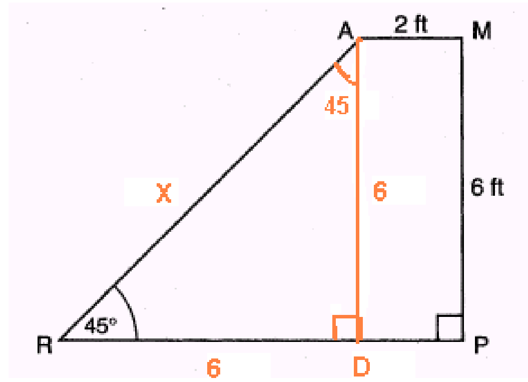
$\ell = \sqrt[3]{\frac{8v}{\pi}}$ Cube both sides.

$\ell^3 = \frac{8v}{\pi}$ Multiply both sides by **B**

$\ell^3\pi = 8v$ Divide both sides by 8.

ANSWER: $\frac{\ell^3\pi}{8} = v$

26) You are presented with the diagram at the right. All the givens are in black. Everything I added is in red. I dropped line AD perpendicular to line RP. This makes triangle RAD an isosceles right triangle. (45-45-90) That is how you know that both of its legs equal 6. (The lengths of AD and MP are equal). At this point you can either recall that in an isosceles right triangle, the hypotenuse will equal the length of a leg times the square root of 2. **In otherwords, the hypotenuse which is represented by RA =** $\mathbf{6\sqrt{2}}$



Or you can use the Pythagorean Theorem to solve for X

$x^2 = 6^2 + 6^2$.

$x^2 = 36 + 36$

$x^2 = 72$

$x = \sqrt{72} = \sqrt{36}\,\sqrt{2} = \mathbf{6\sqrt{2}}$

27) You have a rectangular patio that measures 6 by 8. You want to increase both sides by the same amount so that the new area will equal 150 square meters. To the nearest tenth, what is the number of meters that you will increase each dimension?

One way to do this problem is to set up an equation and solve it. As you will see, it is a quadratic equation that will require the quadratic formula to be solved.

Let x = the amount each dimension will be increased.

The new width will therefore become **6+x** and the new length will become **8+x**. You want the new area to equal 150, which means that (6+x)(8+x) will equal 150. Set up your equation:

$(6+x)(8+x) = 150$	Use FOIL to multiply.
$48 + 6x + 8x + x^2 = 150$	Combine like terms
$48 + 14x + x^2 = 150$	Subtract 150 from both sides.
$-102 + 14x + x^2 = 0$	Rewrite in standard form.
$x^2 + 14x - 102 = 0$	Standard form is $ax^2 + bx + c = 0$

You can now use the quadratic formula to solve the above quadratic equation. Substitute the values for a,b,c from the quadratic equation above **a = 1 b = 14 c = -102**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-14 \pm \sqrt{14^2 - 4(1)(-102)}}{2(1)} = \frac{-14 \pm \sqrt{196 + 408}}{2} = \frac{-14 \pm \sqrt{604}}{2}$$

There are two answers for x one is $x = \frac{-14 + \sqrt{604}}{2}$ and the other $x = \frac{-14 - \sqrt{604}}{2}$.

Use your calculator to solve for x.

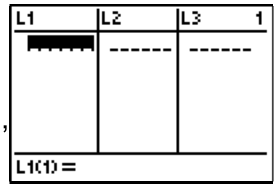
$x = \frac{-14 + \sqrt{604}}{2} =$ (-14+√(604))/2 5.288205727 ≈ **5.3** $x = \frac{-14 - \sqrt{604}}{2} =$ (-14-√(604))/2 -19.28820573

The answer is 5.3 to the nearest tenth. The other answer is rejected, not only because it would decrease the dimensions, but because it would also cause the dimensions to be negative.

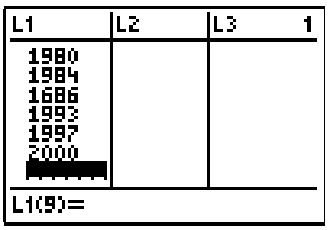
28) You are presented with table at the right. You are asked to find the linear regression equation based on that table, rounding the regression coefficients to the nearest hundreth.

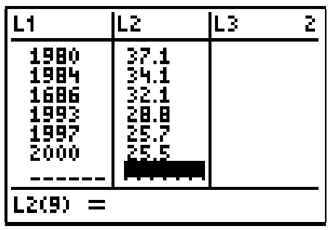
Year (x)	Percent (y)
1971	42.4
1976	37.4
1980	37.1
1984	34.1
1989	32.1
1993	28.8
1997	25.7
2000	25.5

Then you are to estimate the percent for the year 2009.
Step one requires you to enter the information from the table into your calculator.
First hit **STAT** followed by **ENTER**, Your screen will look like the one at the right.

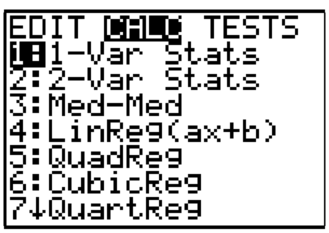


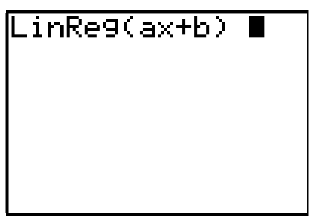
Next you will enter the years into column L1, and the percents into column L2. I find it easiest to first type in one column, and then the next one. Simply type in a number, hit enter and continue until you are done with the first column. Then hit the ► key to move to the L2 column and continue entering the percents. Below you see column L1 complete, followed by L2. You can scroll up and down at any point to see what you have already entered.

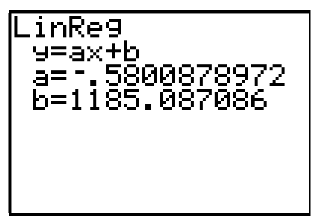




Once the data is entered, you can find the linear regression equation as follows:
Hit **STAT** followed by ► to enter the **CALC** menu as seen below. Choice 4 will generate the required linear regression information. So scroll down to number 4 and hit **ENTER** , or simply hit the number 4 key and you will see the second screen below. At that point hit ENTER and you will see the finall screen below.



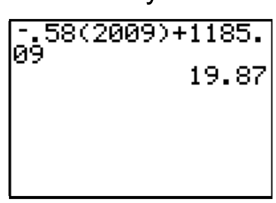




The last screen above contains your answer. First round the value of a and b to the nearest hundredth as requested. **a = -.58 b= 1185.09**
Finally substitute the above values for a and be into the linear equation format of y = ax + b and you have your answer of **y = -.58x + 1185.09**

Now for part 2 of the question. What would be the percent in the year 2009?
Simply substitute 2009 for x in the equation and solve.

y = -.58x + 1185.09 Substitute.
y = -.58(2009) + 1185.09 Use calculator.
y = 19.9 to the nearest tenth.



29) You are given the equation $V = 1500(2)^{\frac{t}{7}}$ V, you are told, represents the value of the investment, and t represents the number of years. In how many years will it take the value of the investment to reach $1,000,000? You were just told that V= 1,000,000, and that you are looking to find the value of t, which represents the number of years. Set up your equation.

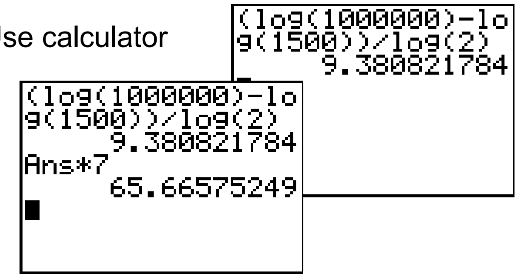
$1{,}000{,}000 = 1500(2)^{\frac{t}{7}}$ Use logarithmic laws to solve for t. You will use the laws used in problem 9

log 1,000,000 = log 1500 + (t/7) log 2 — Subtract log 150 from both sides.

log 1,000,000 - log 1500 = (t/7) log 2 — Divide both sides by log 2

$\frac{\log 1000000 - \log 1500}{\log 2} = \frac{t}{7}$ Use calculator

$9.380821784 = \frac{t}{7}$ Multiply by 7



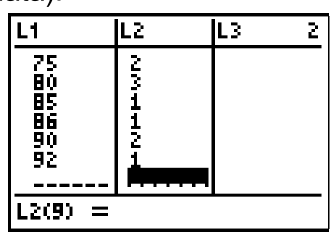
t = 65.7 to the nearest tenth.

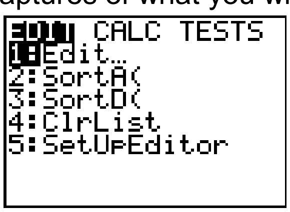
(Make sure to enter expressions properly taking care to use parentheses where required).

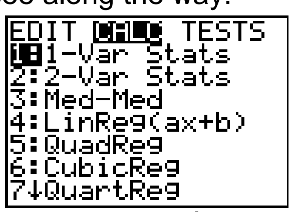
30) You are presented with the table at the right. Begin the problem the way you did number 28 entering the data into L1 and L2. (Make sure you delete the previous data). Now comes the tricky part--letting the calculator know that the data in L2 is the frequency of L1, and not another variable. If you add up the frequency you will see you are really finding the standard deviation of 17 data items. Here's how it's done:

Hit **STAT** ► **ENTER** Below are the screen captures of what you will see along the way.

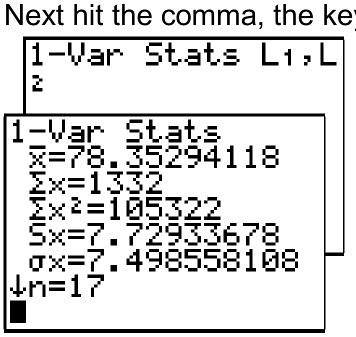
Score	Frequency
70	4
73	3
75	2
80	3
85	1
86	1
90	2
92	1





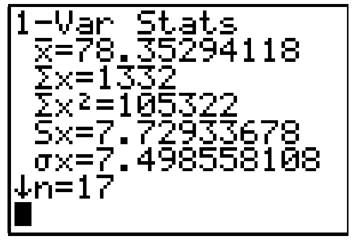


Now at the last screen above: Hit the **2nd** key followed by the the number **1.** Remember when you hit the **2nd** key, the next key you hit will access what is printed above it, to t he left in yellowish print. As you can see, on your calculator, an L1, not the number 1, appears. Next hit the comma, the key above the 7. And follow again with the **2nd** key and the number **2**.



The whole purpose of the above is the screen you see at the left. The calculator now knows that L2 is the frequency of L1. Now hit **ENTER**. The final screen shows you that **the population standard deviation (Fx) to the nearest tenth is 7.5**

The next part of this question continues on the next page.



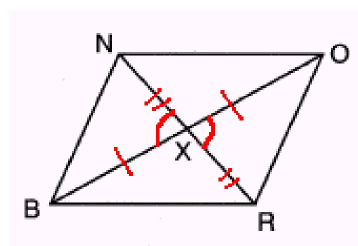
Here is a copy of your current screen capture. In addition to showing the standard deviation, it also shows the mean (**ü)** as being 78.4 to the nearest tenth.
The question you are asked is how many scores fall within 1 population standard deviation of the mean. In order to answer this question you first have to add 7.5 to the mean. The answer will be your upper bound. Then subtract 7.5 from the mean to get your lower bound.
78.4 + 7.5 = **85.9** and 78.4 - 7.5 = **70.9**

Score	Frequency
70	4
73	3
75	2
80	3
85	1
86	1
90	2
92	1

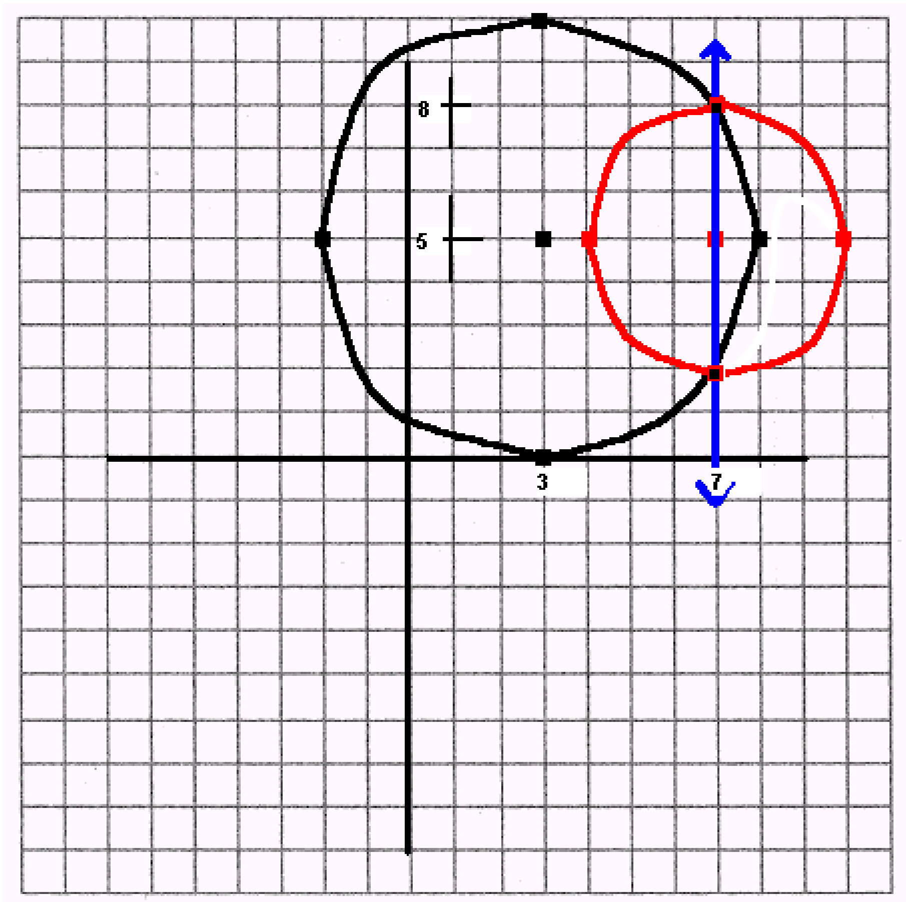
All you need to do now is count how many scores fall within that range: 70.9 - 85.9? As you can see at the right, there are 9 scores.

In other words, **there are 9 scores that fall within one population standard deviation of the mean.**

31) You are presented with quadrilateral BRAN and are told that its diagonals bisect each other at X. The quadrilateral is shown at the right in black. What you can figure out based on the given, to help you do the proof, I have marked in red.
You are asked to prove ΔBNX ≈ ΔORX
The plan you will use is to prove two sides and the included angle of one triangle congruent to two sides and the included angle of the other triangle.



STATEMENTS	REASONS
1. Diagonals OB and NR bisect each other.	1. GIVEN
2. $\overline{OX} \approx \overline{BX}$ **s≈s**	2. Definition of bisector
3. ∠OXR and ∠BXN are vertical angles.	3. When 2 lines intersect, vertical angles are formed.
4. ∠OXR **≈** ∠BXN **a≈a**	4. Vertical angles are congruent to each other.
5. $\overline{RX} \approx \overline{NX}$ **s≈s**	5. Definitiion of bisector
6. ΔBNX ≈ ΔORX	6. SAS ≈ SAS



32) You are given the equations of two circles, and are asked for the equation of the line that passes through their points of intersection.

Equation 1:
$(x-3)^2 + (y - 5)^2 = 25$
This circle is easily sketched at the right. It is the black circle. Based on the above equation, you know that its center is (3,5), and its radius is 5.

Equation 2:
$(x - 7)^2 + (y - 5)^2 = 9$
This circle is easily sketched as well. It is the red circle at the right. Based on its equation, its center is (7,5), and its radius is 3.

You clearly see the points of intersection of these two circles. They intersect at the points (7,2) and (7,8). The bold blue line goes through both of these points. Its equation is **x = 7**

ANSWER: The equation of the line going through the points of intersection of both circles is x = 7.

(The general form of the equation whose graph represents a circle can be written as $(x - x_c)^2 + (y - y_c)^2 = r^2$ where x_c represents the x-coordinate of its center, and y_c represents the y-coordinate of its center, and r represents its radius. Therefore, in our first equation above, **$(x - 7)^2 + (y - 5)^2 = 9$** the x-coordinate at the center is 7, and the y-coordinate at the center is 5. The radius which is r^2 is 3, because 3^2 is 9. The same process is used with our second equation. Keep in mind, though, if the equation had been **$(x + 7)^2 + (y + 5)^2 = 9$** then the radius would still have been 3, but the center of the circle would change. It would now be the point (-7, -5). This is because the real equation would actually be **$(x - (-7))^2 + (y -(- 5))^2 = 9$**)

33) Express in simplest form:

$$\frac{2x}{x^2-4} \div \frac{4}{x^2-4x+4} + \frac{12}{x^2-4} \bullet \frac{2-x}{3}$$

Using the order of operations, let's break the problem down to two individual problem, and then we will combine our results. The first problems will be

$\frac{2x}{x^2-4} \div \frac{4}{x^2-4x+4}$ and the second will be $\frac{12}{x^2-4} \bullet \frac{2-x}{3}$

Then we will add together the answers we get for these two problems.
We will doing lots of factoring and canceling so follow along carefully.

$$\frac{2x}{x^2-4} \div \frac{4}{x^2-4x+4} = \frac{2x}{x^2-4} \bullet \frac{x^2-4x+4}{4} = \frac{2x}{(x+2)(x-2)} \bullet \frac{(x-2)(x-2)}{(2)(2)} = \frac{\cancel{2}x}{(x+2)\cancel{(x-2)}} \bullet \frac{\cancel{(x-2)}(x-2)}{\cancel{(2)}(2)}$$

$$= \mathbf{\frac{x(x-2)}{2(x+2)}}$$

$$\frac{12}{x^2-4} \bullet \frac{2-x}{3} = \frac{(4)(3)}{(x+2)(x-2)} \bullet \frac{2-x}{3} = \frac{(4)\cancel{(3)}}{(x+2)\cancel{(x-2)}} \bullet \frac{\cancel{2-x}^{\,-1}}{\cancel{3}} = \mathbf{\frac{-4}{(x+2)}}$$ Now add these two expressions:

$$\mathbf{\frac{x(x-2)}{2(x+2)}} + \mathbf{\frac{-4}{(x+2)}}$$

We need a common denominator.. If we multiply the numerator and denominator of the second fraction by 2, we will have it.

$$\mathbf{\frac{x(x-2)}{2(x+2)}} + \mathbf{\frac{(2)(-4)}{2(x+2)}} = \frac{x^2-2x}{2(x+2)} + \frac{-8}{2(x+2)} = \frac{x^2-2x-8}{2(x+2)} = \frac{(x-4)(x+2)}{2(x+2)} = \frac{(x-4)\cancel{(x+2)}}{2\cancel{(x+2)}} = \mathbf{\frac{x-4}{2}}$$

ANSWER: $\mathbf{\frac{x-4}{2}}$

(Remember that $\frac{a-b}{b-a} = -1$. That is why in the first part of the problem, $\frac{2-x}{x-2}$ became -1.)

34) You are told the dimensions of the three sides of a triangular field. They are 240, 300, and 360. In addition you are told that one-40 pound bag of fertilizer can cover 6,000 square feet. How many bags should he buy so that he has enough to cover his field? To answer this question all we have to do is figure out the area of the field, and then divide this area by 6,000 to determine how many bags of fertilizer will be necessary.

Let's do this problem using Heron's formula. This formula allows you to calculate the area of a triangle when its three sides are known. Here is Heron's formula:

$A = \sqrt{s(s-a)(s-b)(s-c)}$ Little a, b, and c are the three sides of the triangle, and s equals the semiperimeter. The semiperimeter is one-half of the perimeter.

Let us first find the semipermeter. The perimeter equals the sum of the sides.

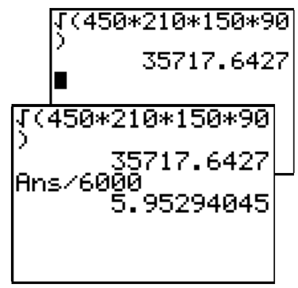
p = a + b + c or **240**+**300**+**360** = 900. **s =** one-half of that or **450**. Now let's find the area

$A = \sqrt{s(s-a)(s-b)(s-c)}$ Substitute for a,b,c, and s.

$A = \sqrt{450(450-240)(450-300)(450-360)}$ Simplify

$A = \sqrt{450(210)(150)(90)}$ Use your calculator.

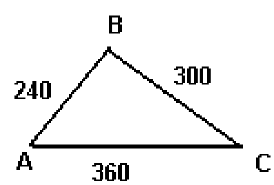
A= 35717.6427 square feet. Divide by 6000



ANSWER: 6 bags will be required to cover the complete field.

Alternative method:

Sketch the triangular field as you see at the right (not drawn to scale).
Use the Law of Cosines to find one angle, for example angle A.



$a^2 = b^2 + c^2 - 2bc \cos A$ Substitute known values.
$(300)^2 = (240)^2 + (360)^2 - 2(240)(360) \cos A$ Use calculator.
90000 = 57600 + 129600 - 172800 cos A Simplify.
90000 = 187200 - 172800 cos A Subtract 187200 from both sides.
-97200 = -172800 cos A Divide both sides by -172800.
.5625 = cos A Find angle A by using second function cosine. ($\cos^{-1}$)
∠A = 55.7711°

Now once you know the angle, you can use the formula K = 1/2 bc sin A to find the area of the triangle. (You are using two sides and the included angle). Now substitute.

K = 1/2 (360)(240) sin 55.7711 Simplify.
K = 43200 sin 55.7711 Continue simplifying.
K = 35717 This is the area. Now divide by 6000 to find the number of bags.

35717/6000 = 5.9

6 bags are required.