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1) SSA≅SSA does not prove that two triangles are congruent.

ANSWER: (2)

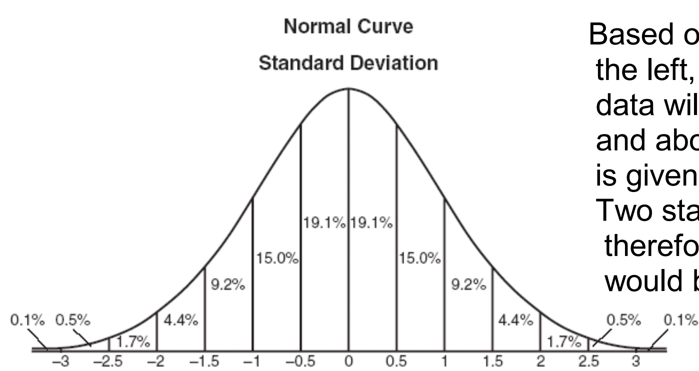
2) When two variables vary inversely, their product will equal a constant. In this case the speed and time are the variables. Their product is (50)(3) or 150. Given a speed of 60 what will the time be? We need to solve the equation 60 times what will equal 150.
60x=150 Divide both sides by 60
x = 2.5 or $\mathbf{2\frac{1}{2}}$

ANSWER: (3)

3) For a set of ordered pairs to be a function, there can be only one y value associated with a specific x value. Choice number 1 contains the pairs (3,1) and (3,2). This cannot name a function because when x=3 we see two different possible y values. **ANSWER: (1)**

4) Given the circle whose equation is $(x+1)^2 + (y-3)^2 = 16$**,** you know that its center is the point (-1,3) and its radius is 4. In general, the equation of a circle is given as:
$(x-h)^2 + (y-k)^2 = r^2$ where h and k are the x- and y- coordinates of the center of the circle, and the radius is r. **ANSWER: (1)**

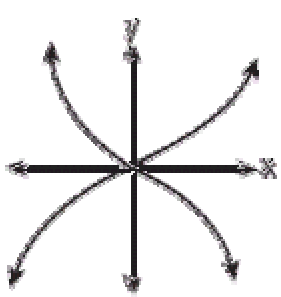
5)



Based on the standard normal curve pictured at the left, you can see that approximately 95.4% of data will fall within 2 standard deviations below and above the mean. In this problem, the mean is given as 56 and the standard deviation as 5. Two standard deviations below 56 would therefore be 56 – 5 – 5 or **46**, and two above would be 56 +5 +5 or **66**. **The answer is** therefore **46-66**.

ANSWER: (2)

6)



You are presented with the graph of f(x), and then asked to select which one of four given graphs best represents f(-x). When you are given f(x) and you replace x with –x then you end up with a reflection in the y-axis. Your f(x) will become f(-x). At the left you see the graph of f(x), and superimposed on it is the graph pictured in choice 4. You see that they are symmetric with respect to the y-axis. Choice 4 therefore best represents the graph for f(-x)

ANSWER: (4)

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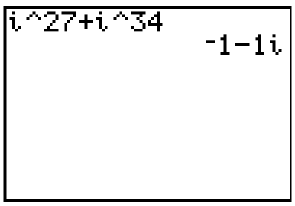
7) In order to answer this question you should know the following:

$i^0=1$ $i^1= i$ $i^2= (\sqrt{-1})(\sqrt{-1}) = -1$ $i^3=(i^2)(i)= -i$ $i^4=(i^2)(i^2)= 1$

Now, in order to simplify powers of i, simply divide by 4 and keep the remainder.
In our problem you are given i^{27}. Divide the 27 by 4. Your answer is 6 with a **remainder of 3**.
This means that i^{27} is equal to i^3. And as you recall, i^3 is equal to **–i**.
You are also presented with i^{34}. Divide 34 by 4 and your **remainder is 2**. This means that i^{34} is equal to i^2 which as shown above is equal to **–1**. So now, in order to simplify:
$i^{27} + i^{34} = -i -1$
This problem can also easily be done using your graphing calculator. Enter the following:

2nd . ^ 2 7 + 2nd . ^ 3 4 ENTER



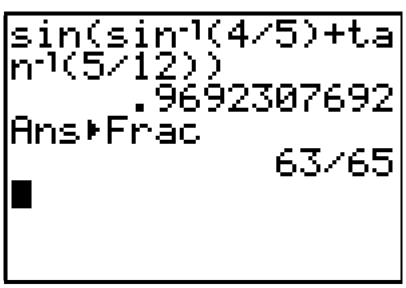
To the left is what your screen will look like after you enter the above. Remember that after hitting the yellow 2nd key, you are accessing the symbol to the upper left of the key you are actually hitting. For example, in the above, you are really accessing the **×** and not the . (decimal point).

ANSWER: (3)

8) The circumference of a circle forms a rotation of 360°. You are told that the larger curved edge is ¼ of the circumference of the circle or 90°. The smaller shaded section is 1/5 the circumference of the circle or 72°. The vertical angles formed by the intersection of two chords in a circle is equal to ½ the degree measure of the two arcs opposite these angles. In our case x equals ½ (90 +72) or 81°.

ANSWER: (3)

9) This problem is easily done using a calculator. What it entails is finding the angle whose sine is 4/5, and the angle whose tangent is 5/12, and then finding the sine of the sum of these two angles. Your calculator will give you the answer as a decimal which you will convert into a fraction. Follow these key strokes to arrive at the screen capture below to your left:



SIN 2nd SIN 4 ÷ 5) + 2nd TAN 5 ÷ 12

)) ENTER

This will yield the decimal answer at the left.
To change it to a fraction continue with the following keys:
MATH ENTER ENTER

And now for solving this problem without the calculator. You are asked for the value of sin (A+B). Look at your formula sheet and you will see that sin (A+B) = sin A cos B + cos A sin B. You are give that sin A = 4/5, and tan B = 5/12. You do have enough information to figure out the value of sin B, as well as the values for cos A and cos B. Both angles are given as being in quadrant I.

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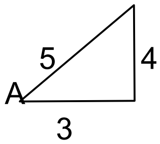
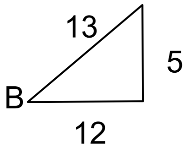


Picture the triangle at the left. Its sin is 4/5. It is obviously a 3, 4,5 right triangle. As such, its base is 3. Based on the diagram you see that a triangle whose **sin A** is **4/5** will have a **cos A** of **3/5**.



You are given that tanB is 5/12. This makes the triangle a 5,12,13 right triangle. **sin B** is therefore **5/13**, and **cos B** is **12/13**.

Now substitute these values into the formula:
sin (A+B) = sin A cos B + cos A sin B

$$\left(\frac{4}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) = \frac{48}{65} + \frac{15}{65} = \frac{63}{65}$$

ANSWER: (1)

10) Tangent is negative in quadrants 2 and 4. Secant is the reciprocal of cosine and will match its sign in all quadrants. Cosine is positive in quadrants 1 and 4. Which means that secant will also be positive in quadrants 1 and 4. Quadrant 4 is the answer, as this is the quadrant where the tangent of an angle will be negative while its secant will be positive. **ANSWER: (4)**

11) $2x^2+8x+n=0$ When the discriminant b^2-4ac is negative, then the equation will have imaginary roots. In this equation, a=2, b= 8, and c=n and is unknown. So all you have to now do is set b^2-4ac equal to less than 0, which in essence makes it negative.

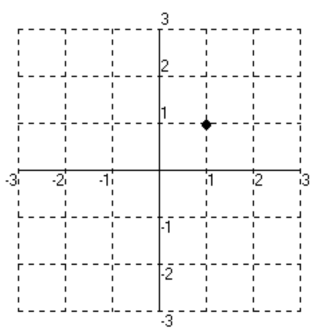
$b^2-4ac < 0$ Substitute
$8^2 - 4(2)n < 0$
$64 - 8n < 0$ Subtract 64 from both sides.
$-8n < -64$ Divide both sides by –8 (Remember to switch inequality symbol)
$n > 8$

Choice number 1 is greater than 8. **ANSWER: (1)**

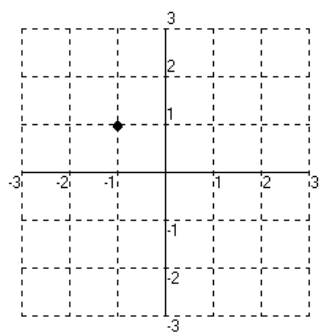
12) When raising a binomial to a power n, realize that your resulting answer will contain n+1 terms. In this problem, $(x+y)^4$ will therefore result in a polynomial of 5 terms. This problem is asking you for the middle term—in other words, term number 3. Each term can be represented as the product of a numerical coefficient, an x term, and a y term.
Here is how to determine the numerical coefficient. The numerical coefficient of term 1 will be represented by ${}_nC_0$, followed by ${}_nC_1$ for term 2, and so on. Term number 3's numerical coefficient will be represented by $\mathbf{{}_nC_2}$. In our case where n is 4, the numerical coefficient of the 3^{rd} term will be:
$\mathbf{{}_4C_2}$, which is equal to **6**
Following the numerical coefficient will be the variables x and y raised to some power. Realize that if they are raised to the 0 power they will be equal to one and not appear. All that remains now is how to determine these two variables—the x and y.
The y term's exponent will always be one less then whichever term you are working on. The 1^{st} term will have y^0, the 2^{nd} will have y^1, and the 3^{rd} which is our case will have y^2.
Once you know the y exponent, it is easy to determine the x exponent. Both exponents always have to add up to that exponent to which you are raising the binomial—in our case 4. So as

long as as our y is raised to the 2nd power, x will be raised to the 2nd as well, since 2+2=4. This is what our 3rd term will look like:
$\mathbf{{}_4C_2\,(x)^2(y)^2}$ or $\mathbf{6x^2y^2}$.
If you are curious, this is what each term of $(x+y)^4$ would look like:
${}_4C_0(x)^4(y)^0 + {}_4C_1(x)^3(y)^1 + {}_4C_2(x)^2(y)^2 + {}_4C_3(x)^1(y)^3 + {}_4C_4(x)^0(y)^4$ **ANSWER: (3)**

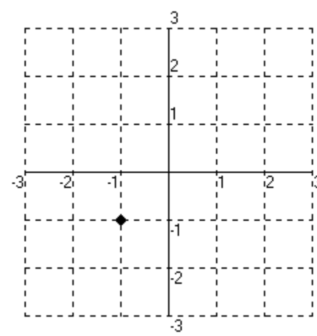
13) This problem is asking you to do 2 transformations on 1 point. You are given the point (1,1) and asked what its image will be under a composite transformation. You are asked to reflect it in the x-axis **following** a rotation of 90 about the origin. In other words, what is the image of **(1,1) under** $\mathbf{r_{x\text{-axis}} \circ R_{0,90^o}}$. You first rotate (1,1) 90º about the origin, and then reflect it under the x-axis. As you can see in the diagrams below, the final image is **(-1,-1)**.



Above you see the point (1,1)



Above is (1,1) after a 90º rotation



(1,1) under $\mathbf{r_{x\text{-axis}} \circ R_{0,90^o}}$

ANSWER: (4)

14) For all problems of this type use the law of sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
In this problem you are given 2 sides of a triangle and the measure of angle A which is opposite the given side A. Set up your ratio:

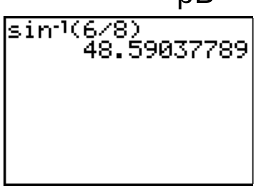
Given: a=8 mpA = 30º b= 12 $\frac{8}{\sin 30} = \frac{12}{\sin B}$ Cross multiply

8 (sinB) = 12 (sin 30) Divide both sides by 8

$\sin B = \frac{12 \sin 30}{8}$ Sin 30 = .5

$\sin B = \frac{12(.5)}{8} = \frac{6}{8}$ Now use your $\sin^{-1}$ key to find angle B

pB ≈ 49°



You now know that pB=49 and that pA = 30. This leaves 101º for the measure of the 3rd angle,or pC of the triangle as all 3 angles have to add up to 180º. Now let's see if an other triangle can exist.
The sine of a 49° angle will be equal to the sine of a (180-49)° angle. This means that angle B can also equal 131° and still have a sine of 6/8. But let us see if such a triangle is possible using the information given in our problem.
pA = 30 pB=131 Now pC will equal 180-(30+131) or 19. That is good. Had the sum of the measures of angles A, B, and C been greater than180, then no 2nd triangle would have been possible. So now, **2 triangles are possible**. **ANSWER: (2)**

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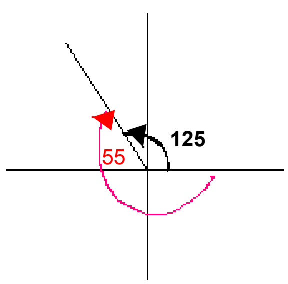
15) In essence, you are being asked to simplify the following: $\frac{(b^{2n+1})^3}{b^n \bullet b^{4n+3}}$
There are three rules you have to remember for this question. The first is that when you raise a power to a power, you multiply the powers. So that $(x^2)^3$ will equal x^6. But when you multiply $x^2(x^3)$ your answer will be x^5 (the exponents are actually added.
Following the above rules, let us first simplify the numerator. It is a power raised to a power, so we multiply the exponents 3(2n+1) = 6n+3. Your new numerator is now $\mathbf{b^{6n+3}}$.
The denominator on the other hand is the product of two powers with the same base, so we add the exponents. $b^n(b^{4n+3}) = b^{n+4n+3} = \mathbf{b^{5n+3}}$.

Your fraction now looks as follows: $\frac{b^{6n+3}}{b^{5n+3}}$

When dividing powers with the same base, the powers are subtracted:
6n+3-(5n+3) = 6n+3-5n-3 = n
Your final answer will be $\mathbf{b^n}$. **ANSWER: (2)**

16) Try to remember this when dealing with logarithms:
$\log_{10}100 = 2$ because $10^2 = 100$ or $2 = \log_{10}100$
Once you can recall the above then you will be able to switch back and forth from logarithmic form to standard form. In this problem you are presented with $\mathbf{y = \log_4 x}$ Using the format shown above, this is the same as $\mathbf{4^y = x}$.
Now, you are being asked for the inverse of the above function. One way of finding the inverse of a function is to switch the x and y. In the above it becomes: $\mathbf{4^x = y}$ **ANSWER: (3)**

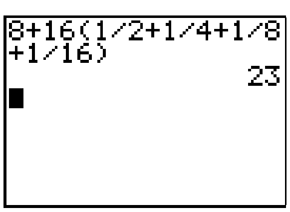
17)



At the left you see the black arc which indicates an angle of approximately 125°. Now, moving clockwise, as indicated by the red arc, you have an angle of –(180+55)° or -235° which is coterminal to the angle of 125°. (Also remember that counter-clockwise rotations produce positive angles, while clockwise rotations produce negative angles.) **ANSWER: (2)**

18) $d = 8 + 16\sum_{k=1}^{n}\left(\frac{1}{2}\right)^k$ This a simple problem requiring you to know what the summation symbol means. The problem states that n, the number of bounces is 4. You are therefore being asked to calculate the following.
(Remember that k is going from 1 to 4).

$$d = 8 + 16\left(\left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4\right) \quad d = 8 + 16\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right) \quad d=23$$



The answer is easily arrived at using your calculator. The key strokes are simple. At the left is a screen capture of the results. **ANSWER: (4)**

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19) Just for the sake of knowledge (or fun), compare y= 27 sin 13x + 30 to the following:

y = a sin bx + c

a = amplitude In the above equation the amplitude would be 27.

b = frequency In the above equation the frequency is 13. this means that the graph of the above sine curve will complete itself 13 time in an interval of 2pi radians or (360°). Its period would therefore be 360/13. That would be the number of degrees for the above graph to complete one "sine" cycle.

c= vertical shift In the above the vertical shift would be 30. This would be the value arrived at when x equals 0°. In general, this value can also be arrived at by dividing by 2 the sum of the greatest y-value and the smallest y-value. Finally, here is what you are asked for in this problem. What is the maximum altitude of the graph represented by the above equation? Simply add the a and c values:
27 + 30 = 57 **ANSWER: (4)**

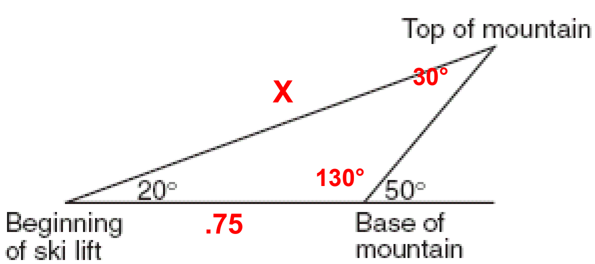
20) What this problem really wants you do is to simplify or rather rationalize this expression:

$\frac{11}{\sqrt{3}-5}$ To do this you have to multiply both the numerator and denominator by the conjugate of the denominator. $\sqrt{3} – 5$ and $\sqrt{3} + 5$ are conjugates.

$$\frac{11}{\sqrt{3}-5}\left(\frac{\sqrt{3}+5}{\sqrt{3}+5}\right) = \frac{11\sqrt{3}+55}{3-25} = \frac{11\sqrt{3}+55}{-22} = \frac{11(\sqrt{3}+5)}{11(-2)} = \frac{\sqrt{3}+5}{-2} \text{ or } \frac{\mathbf{-\sqrt{3}-5}}{\mathbf{2}}$$

ANSWER: (1)

21) In this problem you are given 2 sides of a triangle and an angle opposite one of the sides. For all problems of this type, use the Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



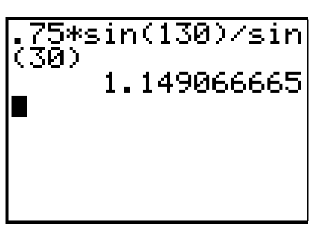
At the left is the diagram presented as part of this question. The 20° and 50° angles are given. You are also told that the ski lift begins .75 mile from the base of the mountain. You are asked to find the distance from the beginning of the ski lift to the top of the mountain.

Using the above information set up the following proportion using the law of sines:

$\frac{x}{\sin 130} = \frac{.75}{\sin 30}$ Cross multiply

x (sin 30) = .75 (sin 130) Divide both sides by (sin 30)

$x = \frac{.75(\sin 130)}{\sin 30}$ Now use your calculator to multiply .75 by the sine of 130° and divide that answer by the sine of 30°. **1.15 miles** to the nearest hundredth.



ANSWER 1.15

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22) This question asks you to express the following in simplest *a+bi* form:

$\sqrt{-48} + 3.5 + \sqrt{25} + \sqrt{-27}$ Recall that $i = \sqrt{-1}$

$\sqrt{16}\sqrt{3}\,i + 3.5 + 5 + \sqrt{9}\sqrt{3}\,i$ Simplify

$4\sqrt{3}\,I + 8.5 + 3\sqrt{3}\,i$ Continue simplifying

$\mathbf{8.5 + 7\sqrt{3}\,i}$ **ANSWER:** $\mathbf{8.5 + 7\sqrt{3}\,i}$

23) You are presented with the following : $x^{-3} = \frac{27}{64}$ In order to solve for x, step number one involves raising both sides to the –(1/3) power (the reciprocal of –3). When powers are raised to a power the exponents are actually multiplied. This means that (x^{-3}) raised to the – (1/3) will equal x, and we are being asked to solve for x.

We now also have to raise $\frac{27}{64}$ to the negative 1/3 power as well. Remember that any number raised to a negative exponent will equal 1 divided by that number to the positive exponent. For example, $x^{-3} = \frac{1}{x^3}$. So $\frac{27}{64}$ raised to the –1/3 will be the same as 1 over 27/64 raised to the +1/3. $\frac{1}{\frac{27}{64}}$ is simply the reciprocal of $\frac{27}{64}$ which equals $\frac{64}{27}$. All that remains now is to raise $\frac{64}{27}$ to the 1/3 power. Raising a number to the 1/3 power is the equivalent of finding its cube root. The cube root of the numerator 64 is 4, and the cube root of the denominator 27 is 3. Your final answer will be $\mathbf{\frac{4}{3}}$. **ANSWER:** $\mathbf{\frac{4}{3}}$

24) You are presented with the following equation: $P(x) = -x^2 + 120x - 2000$

It should remind you of a quadratic equation in the form of $ax^2 + bx + c = y$

Such an equation when graphed will form a parabola. In our case, since the numerical coefficient represented by "a" is negative, the parabola will open downward and have a maximum turning point. Based on the given information, it will hit the x-axis at two points. At those 2 points, P(x), or the profit will be 0. So in essence we have to solve the following equation: $-x^2 + 120x - 2000 = 0$. There are many ways to do this. This equation, though, can easily be solved by factoring. You can make life a bit easier by first multiplying both sides by -1. This will result in the following equation:

$x^2 - 120x + 2000 = 0$ Factor.

$(x - 20)(x - 100) = 0$ Set each factor equal to 0, and solve for x.

$x - 20 = 0$ $x - 100 = 0$

$x = 20$ $x = 100$

This information tells you that the graph will hit the x-axis at 20, and at that point y will start being positive. What this means in our problem is that when x, which is the price for each coat, goes above 20, then there will be a profit. As x continues to increase so will the profit. At some point, though, there will be a maximum profit, and then the profits will begin to decrease. The profit will again hit 0 when x is equal to 100. The answer to "for what values of x does the company make a profit is: $\mathbf{20 < x < 100}$ **ANSWER:** $\mathbf{20 < x < 100}$

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25) You are presented with a complex fraction whose numerator is $\frac{1}{r} - \frac{1}{s}$. Let's simplify this numerator. Our common denominator will be (rs). We will have to multiply the first term by s/s and the second term by r/r so that both will end up with a denominator of rs.

$\frac{1}{r}\left(\frac{s}{s}\right) - \frac{1}{s}\left(\frac{r}{r}\right) = \frac{s}{rs} - \frac{r}{rs} = \frac{s-r}{rs}$ Your new **numerator** will therefore be: $\mathbf{\frac{s-r}{rs}}$

Now let's simplify the denominator of $\frac{r^2}{s^2}$ - 1. To make life easier, instead of a 1, let's substitute something with a denominator of s^2. How about $\frac{s^2}{s^2}$ which equals 1. So our **denominator** is now $\frac{r^2}{s^2} - \frac{s^2}{s^2}$ which is equal to $\mathbf{\frac{r^2-s^2}{s^2}}$. This complex fraction can now be rewritten as the numerator divide by the denominator or:

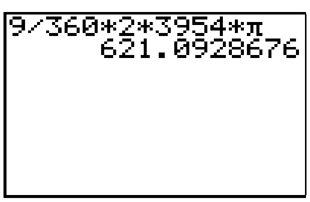
$$\frac{\frac{s-r}{rs}}{\frac{r^2-s^2}{s^2}} = \frac{s-r}{rs} \div \frac{r^2-s^2}{s^2} = \frac{s-r}{rs} \cdot \frac{s^2}{r^2-s^2} = \frac{s-r}{rs} \cdot \frac{s^2}{(r+s)(r-s)}$$

Now the s from the denominator rs will cancel with an s from the s^2. The s-r will cancel with the r-s and become -1. So what you now have will look as follows:

$$\frac{-1}{r} \cdot \frac{s}{r+s} = -\frac{s}{r(r+s)} \text{ or } -\frac{s}{r^2+rs}$$

ANSWER: $\mathbf{-\frac{s}{r(r+s)}}$ **or** $\mathbf{-\frac{s}{r^2+rs}}$

26) In this problem you are told that the Earth's radius is 3,954 miles. If the radius of a circle is known, its circumference can easily be calculated by using the formula C= **B**d or 2**B**r. In our problem you are presented with arc HK which contains an angle of 9°. Its length will therefore be $\frac{9}{360}$ of the total circumference. So let's figure out the circumference and multiply it by $\frac{9}{360}$



HK = $\frac{9}{360}$ (2**B**r) Substitute 3954 for r, and use the pi key on your calculator for **B**. HK = $\frac{9}{360}$ (2)(3954)(**B**) = 621.0928676

621.1 to the nearest tenth. **ANSWER: 621.1**

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27) This problem requires you to know how to solve an absolute inequality. When presented with an absolute inequality, you can always set up two inequalities which will allow you to remove the absolute value sign. In this case you are presented with the following absolute inequality:

$|d - 620| \le .05d$ On the next line are the two inequalities this one yields.
$d - 620 \le .05d$ and **$-(d - 620) \le .05d$** Now let's solve each inequality:

$d - 620 \le .05d$ Multiply each term by 100 to get rid of decimal.
$100d - 62000 \le 5d$ Subtract 5d from both sides.
$95d - 62000 \le 0$ Add 62000 to both sides.
$95d \le 62000$ Divide both sides by 95.
$d \le 652.6315789$ **$d \le 652.6$** to the nearest tenth.

$-(d - 620) \le .05d$ Get rid of parenthesis (multiplying by a -1 inside)
$-d + 620 \le .05d$ As before, multiply each term by 100.
$-100d + 62000 \le 5d$ Add 100d to both sides.
$62000 \le 105d$ Divide both sides by 105.
$590.4761905 \le d$ **$590.5 \le d$** or **$d \ge 590.5$** to nearest tenth.

ANSWER: minimum of 590.5 and maximum of 652.6

28) You are presented with the following equation $A = P\left(1+\frac{r}{n}\right)^{nt}$

A is the amount of money in the account. The problem states that you want this amount to be **$2500**. **P** represents the amount deposited. It is given as being **$1,000**. The interest rate is given as **r** and is 2.8% which as a decimal is **.028** (decimal is moved to places to the left). The number of times the interest is compound per year is **n** and given as monthly or **12**. The amount of years is given as "t". And that is what this problem wants you to figure out. In how many years will the individual in question have $2500 in her account. Let's begin by substituting all known values.

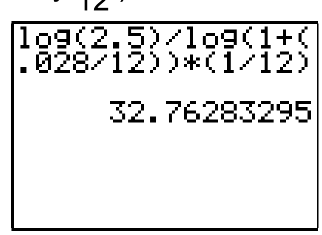
$A = P\left(1+\frac{r}{n}\right)^{nt}$ $2500 = 1000\left(1+\frac{.028}{12}\right)^{12t}$. First we can simplify the equation a bit by dividing both sides by 1000. This results in:

$2.5 = \left(1+\frac{.028}{12}\right)^{12t}$ Use the laws of logarithms (power rule)

$\log 2.5 = 12t \cdot \log\left(1+\frac{.028}{12}\right)$ Divide both sides by $\log\left(1+\frac{.028}{12}\right)$

$12t = \frac{\log 2.5}{\log\left(1+\frac{.028}{12}\right)}$ Divide both sides by 12 (same as multiplying fraction by $\frac{1}{12}$).

$t = \frac{\log 2.5}{\log\left(1+\frac{.028}{12}\right)} \cdot \left(\frac{1}{12}\right)$ Screen capture required to calculate t:



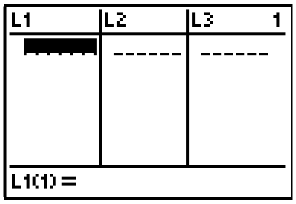
(Be careful with the parenthesis!)

To the nearest tenth the answer is **32.8**. **Answer: 32.8**

29) You are presented with the following table of values and asked to write an exponential regression equation:

Trial	0	1	3	4	6
Coins Returned	1,000	610	220	132	45

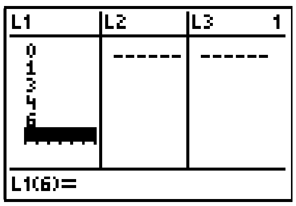
The first step consists of entering this data into your calculator.



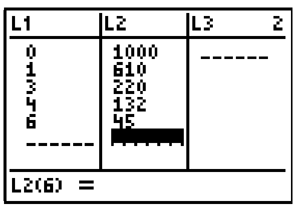
Hit the STAT key followed by the ENTER key and your screen will look like the one at the left.

STAT **ENTER**

Entering the values under L1 and L2 are easy. Let's enter the Trial values under L1 and the number of Coins Returned under L2. Simply enter each value starting with 0 and ending with 6. Hit **ENTER** after entering each value. Your screen should look like the one on the left that has values in the column labeled L1. To enter the corresponding Coins Returned values, first hit the ▶ (right arrow key) and your cursor will automatically move to the top of the L2 column. At this point simply add the proper values hitting ENTER after each one.

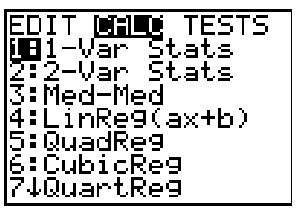


At the left is a screen capture of what your screen should look like after you have entered your data.

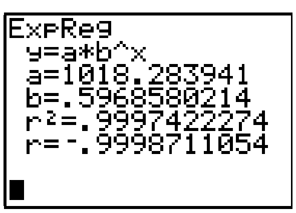


At this point you are ready to have your calculator calculate the the exponential regression equation based on the data you have entered. Hit the following keys:

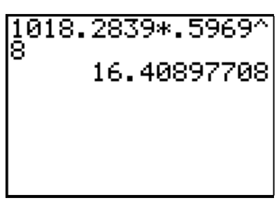
STAT ▶ Your screen will now look like the one at the left with the cursor over the middle column which reads CALC.



Now hit the **0** key, and at the top of your screen you will see: **ExpReg** Now Hit **ENTER** and your screen will look like the one below and to the left. Based on this last screen, the equation is in the form of **$y = a(b)^x$** Rounding to the nearest ten-thousandth, your equation would be: **$y = 1018.2839(.5969)^x$**



The final question asks you to predict how many coins would be returned after the eight trial. Simply solve the above equation substituting 8 for x:



The answer is **16** coins.

Answer: $y = 1018.2839(.5969)^x$ and 16

30) Based on the given information, you know that the probability that Tim will hit a home run is 1 out of every 10 times at bat or $\frac{1}{10}$. This means that the probability that he will not hit a home run is $\frac{9}{10}$. You are asked for the probability that he will hit a home run **at least** four times.

At least 4 times means that you will have to find the probability for 4 home runs and also the probability for 5 times. And then you have to add these probabilities together because both of them satisfy **at least 4 times**. The probability for 4 home runs out of 5 times at bat will be:

$$_5C_4\left(\frac{1}{10}\right)^4\left(\frac{9}{10}\right)^1 = 5\left(\frac{1}{10000}\right)\left(\frac{9}{10}\right) = \frac{45}{100000}$$

The probability for a home run each of the 5 times is easily gotten by the counting principle:

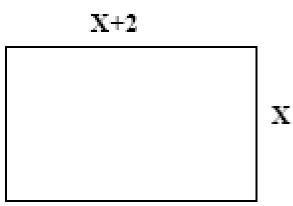
$$\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \text{ or } \left(\frac{1}{10}\right)^5 = \frac{1}{100000}$$

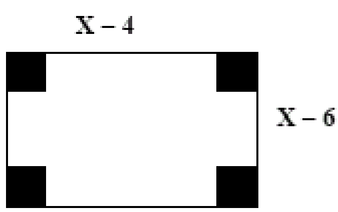
The answer will therefore be:

$$\frac{45}{100000} + \frac{1}{100000} = \frac{46}{100000}$$

ANSWER: $\frac{46}{100000}$

31)





You are told that the piece of cardboard is 2 centimeters longer than it is wide. So **X** can represent the width while **X+2** can represent the length. The diagram at the left represents this piece of cardboard. The second diagram at the left represents the cardboard box after a square measuring 3 centimeters on each side is cut from each corner.

The original side that was x+2 is now 6 less or x+2-6= x –4, and the side of x is also 6 less or x – 6. These new sides are turned up to fold a box whose volume is given as 765.

Basically we now know that the height of the box is 3 (because each cut out square has sides of 3).

The formula for the volume of this cardboard box is :

V= l w h (length, times width, times height)

In our case substituting what is given, we have:

$3(x-4)(x-6) = 765$ Multiply the binomials.

$3(x^2 - 10x + 24) = 765$ Divide both sides by 3.

$x^2 - 10x + 24 = 255$ Subtract 255 from both sides.

$x^2 - 10x - 231 = 0$ Factor and solve.

$(x + 11)(x - 21) = 0$

$x+11=0$ $x - 21 = 0$

$x = -11$ reject $x = 21$ good

The original sides were x and x+2. Using 21 for x, the length and width were 21 and 23.

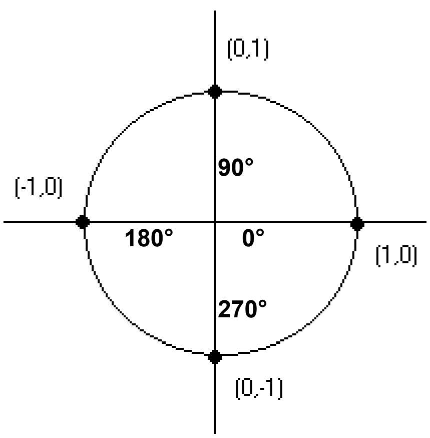
ANSWER: 21 and 23 centimeters

32) You are presented with the following equation and asked to solve algebraically for all values of **2** in the interval 0° ≤ **2** < 360°.

$\frac{\sin^2\theta}{1+\cos\theta} = 1$ Multiply both sides by 1 + cos**2**

sin^2**2** = 1 + cos**2** Substitute 1 – cos^2**2** for sin^2**2** (since sin^2**2** + cos^2**2** = 1)
1 – cos^2**2** = 1 + cos**2** Subtract cos**2** from both sides.
1 – cos^2**2** – cos**2** = 1 Subtract 1 from both sides.
– cos^2**2** – cos**2** = 0 Multiply both sides by –1.
cos^2**2** + cos**2** = 0 Factor and solve.
cos**2** (cos**2** + 1) = 0
cos2 = 0 cos**2** + 1 = 0
cos2 = -1

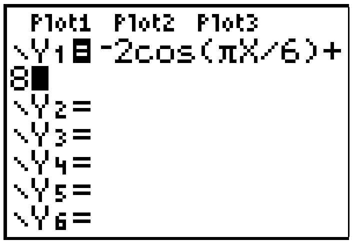
You are now looking for all the values of **2** so that cos**2**=0, and all values of **2** so that cos**2**= -1.



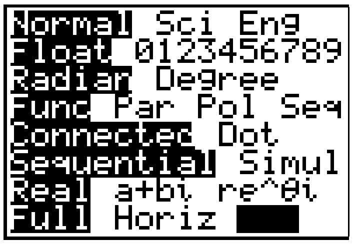
Here is one of many ways to remember the values of sine, cosine, and tangent for the quadrantal angles. At the left you see a coordinate plane with points marked off one unit from the center (a unit circle). Also indicated are the degree value of each quadrantal angle. Now, recall that each point on the axis consists of an x-value and y-value. For our purposes here, the y-value will be the sine of the particular quadrantal angle, while the x-value will be the cosine of that angle (tangent will be sine/cosine). For a 90° angle we see the (x,y) as being (0,1). This means that sin90°=1 and cos90°=0.
Now back to our problem **we are looking for the angle whose cosine is 0**. Which angles on the coordinate plane have x-values of 0? **90°** and **270°**

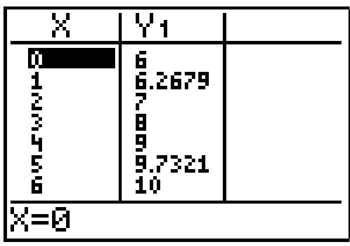
We are also looking for the angle whose cosine is –1. But wait! In our problem, cos**2** can not equal –1, for if it did then our denominator would equal 0. The denominator in the original problem is 1+cos **2**. If cos**2** were –1 it would equal 1 – 1 or 0, and the fraction would be undefined. **ANSWER: 90°, 270°**

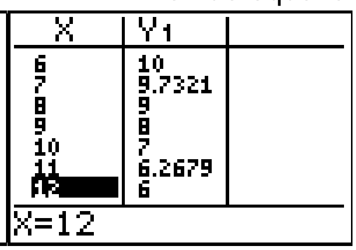
33) For this problem simply type the given equation into your calculator:



Before you can graph any equation you first input it using the Y= key. At the left you see what your screen should look like after you have entered your equation. Notice that instead of a t I've used an X. There is one more thing that you have to do so that you answer this question. You have to put your calculator into "radian" mode. Simply hit the MODE key and make sure that Radian rather than Degree is shaded on your screen. You see this screen below to the left. Now all that remains is to access the TABLE function of your calculator by hitting 2nd GRAPH. Now you may have to use up and down keys to scroll through the values of X from 0 thru 12 to see the number of times the Y value will equal at least 7. The Y value represents the height of the tide in this equation, while the X are the hours. You are looking for how many hours between t=0 and t=12 will the tide equal at least 7 feet. Remember we are using X instead of t. At the left you will see two screen captures showing the Y values as X is increasing from 0 thru 12.



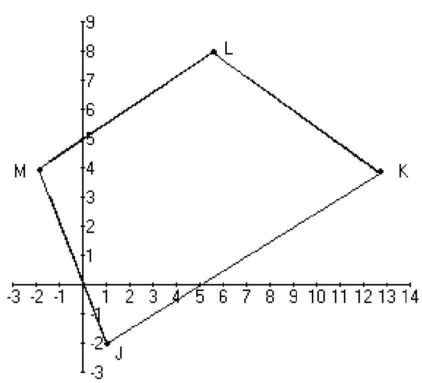




You can see that between the hours of 2 and 10 the tide will be at least 7. That is a total of 8 hours

ANSWER: 8 hours

34) More or less here is what the trapezoid would look like when graphed. (I am not an artist.)



You are given quadrilateral JKLM with the following points: J(1,-2), K(13,4), L(6,8), M(-2,4). You are really being asked two tasks:

(1) Prove JKLM a trapezoid.

(2) Prove that JKLM is not an isosceles trapezoid.

A trapezoid is a quadrilateral that has ONLY one pair of parallel sides. We know that parallel lines have equal slopes. So let us begin by proving that two of the sides of JKLM are parallel, and that the other two are not.

Slope LM $= \frac{\text{Change in Y}}{\text{Change in X}} = \frac{8-4}{6-(-2)} = \frac{4}{8} = \frac{1}{2}$

Slope KJ $= \frac{\text{Change in Y}}{\text{Change in X}} = \frac{4-(-2)}{13-1} = \frac{6}{12} = \frac{1}{2}$

LM is parallel to KJ since their slopes are equal.

Now we continue to prove that the other two sides, LK and MJ are not parallel.
The proof continues on the next page.

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Slope LK = $\frac{\text{Change in Y}}{\text{Change in X}} = \frac{8-4}{6-13} = \frac{4}{-7}$

Slope MJ = $\frac{\text{Change in Y}}{\text{Change in X}} = \frac{4-(-2)}{-2-1} = \frac{6}{-3} = -2$

The slopes are not equal so these 2 sides are not parallel. JKLM is therefore a trapezoid because it has only one pair of parallel sides.

Now we have to prove the second requirement that this trapezoid is not isosceles. What this means is that we have to prove that the non-parallel sides, LK and MJ are not equal in length. This can be done by using the distance formula.

$d = \sqrt{(x1-x2)^2+(y1-y2)^2}$ Let's first find the distance of LK L(6,8) K(13,4)

distance LK = $\sqrt{(6-13)^2 + (8-4)^2} = \sqrt{(-7)^2 + (4)^2} = \sqrt{\mathbf{49 + 16}}$ **=** $\sqrt{\mathbf{65}}$

Now let's find the distance of MJ M(-2,4) J(1,-2)

distance MJ = $\sqrt{(-2-1)^2 + (4-(-2))^2} = \sqrt{(-3)^2 + (6)^2} = \sqrt{\mathbf{9 + 36}}$**) =** $\sqrt{\mathbf{45}}$

LK and MJ are not congruent. The trapezoid is therefore not isosceles!