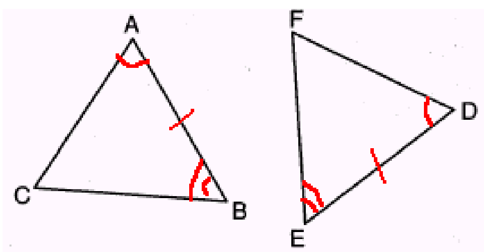
Home Instruction Schools ANSWERS Geometry June 16, 2009

PART 1

1) ***Juliann plans on drawing ΔABC, where the measure of ∠A can range from 50° to 60° and the measure of ∠B can range from 90° to 100°, Given these conditions, what is the correct range of measures possible for ∠C?***
(1) 20° to 40° (2) 30° to 50° (3) 80° to 90° (4) 120° to 130°

The solution to this problem is based on the theorem that the sum of the measures of a triangle will always equal 180°.
Select lowest angle measures: ∠A = 50 ∠B = 90 ∠**C = 40** (50+90+40 =180)
Select greatest angle measures: ∠A = 60 ∠B = 100 ∠**C = 20** (60+100+20=180)
Angle C can range from 20 to 40 degrees. **ANSWER: (1)**

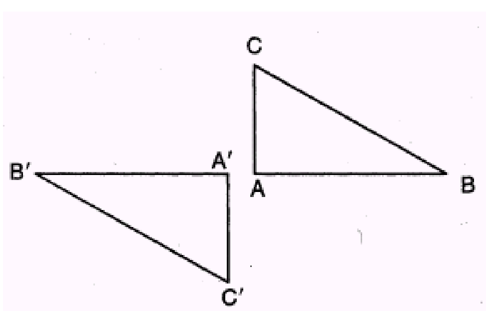
2) ***In the diagram of ΔABC and ΔDEF below,*** $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\angle B \cong \angle E$.



Which method can be used to prove △ABC ≅ △DEF?
(1) SSS (3) ASA
(2) SAS (4) HL

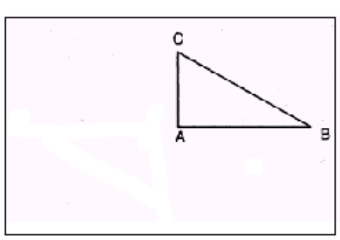
In the diagram above, you see marked in red what is given in the problem.
You see two angles and the included side of one triangle congruent to two angles and the included side of the other triangle. This is the definition of ASA. **ANSWER: (3)**

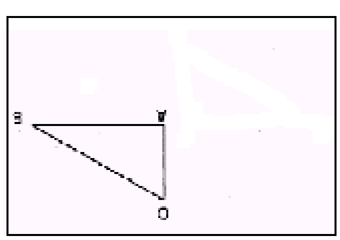
3) ***In the diagram below, under which transformation will ΔA'B'C' be the image of ΔABC?***



(1) rotation
(2) dilation
(3) translation
(4) glide reflection

The transformation shown looks like a rotation of 180 degrees



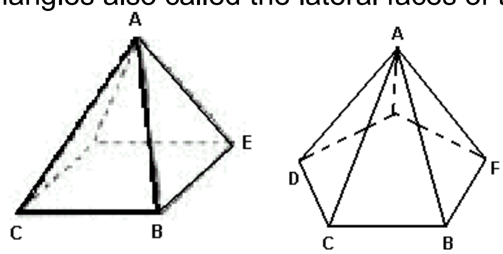


Here they are side by side after a rotation of 180 degrees. Put the diagrams one on top of the other and you end up with the original diagram.
ANSWER: (1)

4) ***The lateral faces of a regular pyramid are composed of***
(1) squares (3) congruent right triangles
(2) rectangles (4) congruent isosceles triangles

A regular pyramid is a pyramid whose base is a regular polygon, and whose lateral edges are congruent. Seen below are two examples of regular pyramids.

$\overline{AB}$ and $\overline{AC}$ in each pyramid below are examples of lateral edges. In a regular pyramid lateral edges will always be congruent. The lateral edges are the intersection of the triangles also called the lateral faces of the pyramid.

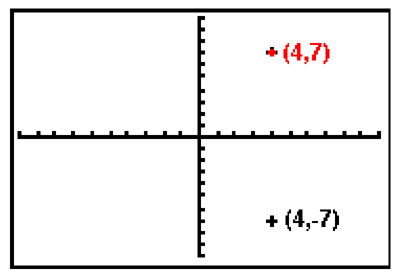


The lateral faces of a regular pyramid will therefore be congruent isosceles triangles.

ANSWER: (4)

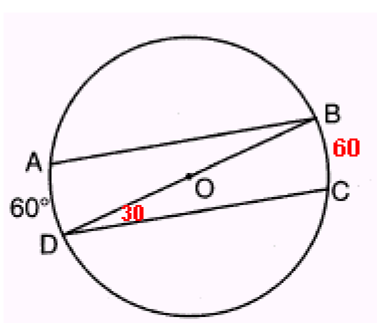
5) ***Point A is located at (4,-7). The point is reflected in the x-axis. Its image is located at***
(1) (-4,7) (2) (-4-7) (3) (4,7) (4) (7,-4)

The rule for reflecting a point in the x-axis is:
$r_{x\text{-axis}}$ P(x,y) → P'(x, -y)
This means that the P', the image of P when reflected in the x-axis, will keep the same x-coordinate but will have a negated y-coordinate. As you see at the right, the original point (4,-7) has as its image the point (4,7). The x-coordinates are the same, but the y-coordinates are negatives (opposites) of each other.



ANSWER: (3)

6) In the diagram of circle O below, chords $\overline{AB}$ and $\overline{CD}$ are parallel, and $\overline{BD}$ is a diameter of the circle.



If m$\overset{\frown}{AD}$ = 60, what is m∠CDB?
(1) 20 (3) 60
(2) 30 (4) 120

(What you see in red at the left has been added by me.)

Two concepts are required for this problem.
1) You have to know that parallel chords intercept equal arcs. This means, that in the above diagram, arcs AD and BC are equal. You are told that the measure of arc AD is 60, so you now know that the measure of arc BC is 60 degrees as well.

2) You also have to know that an inscribed angle, (angle CDB is an inscribed angle), will be equal in measure to one-half the degree measure of the arc it intercepts. In the case above it intercepts an arc of 60°. **m∠CDB is therefore 30°.** **ANSWER: (2)**

7) ***What is an equation of the line that passes through the point (-2,5) and is perpendicular to the line whose equation is*** $y = \frac{1}{2}x + 5$***?***

(1) $y = 2x + 1$ (3) $y = 2x + 9$

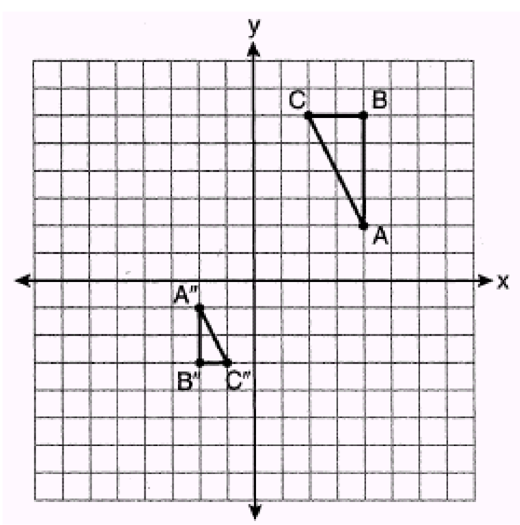
(2) $y = -2x + 1$ (4) $y = -2x - 9$

Perpendicular lines have slopes that are negative reciprocals. When the equation of a line is represented in the form of y = mx + b, m represents the slope. The equation given in this problem therefore has a slope of 1/2. The negative reciprocal of 1/2 is -2. You immediately know that choices 1 and 3 are out of the running as they indicate slopes of +2. All you really have to do now is substitute the given point (-2,5) into the two remaining equations and you will see that choice 2 is the answer.

y = -2x +1	y = -2x - 9
5 = -2(-2) + 1	5 = -2(-2) - 9
5 = 4 + 1	5 = 4 - 9
5 = 5	5 = -5
TRUE	FALSE

Here's is the regular method of solving this problem.
You are at the point where you determined that the slope of the line will be 2.
Substituting into the slope-intercept form of the line we get:
y = mx + b Slope is -2
y = -2x + b Now to solve for b, the y-intercept. Substitute (-2,5) x= -2 y= 5.
5 = -2(-2) +b
5 = 4 + b Subtract 4 from both sides.
1 = b
So you know that the slope, m, is -2 and the y-intercept, b, is 1.
y = mx + b Substitute for m and b.
y = -2x + 1 **ANSWER: (2)**

8) ***After a composition of transformations, the coordinates A(4,2), B(4,6), and C(2,6) become A"(-2,-1), B"(-2,-3), and C"(-l,-3), as shown on the set of axes below.***



Which composition of transformations was used?

(1) $R_{180°} \circ D_2$ (3) $D_{\frac{1}{2}} \circ R_{180°}$

(2) $R_{90°} \circ D_2$ (4) $D_{\frac{1}{2}} \circ R_{90°}$

The answer appears on the next page.

You see that the image is not the same size as the original figure. The only transformation that has that characteristic is a dilation. Since the image is smaller, it had to be the result of a dilation between 0 and 1. A dilation greater than 1 would result in a larger image. Choices 1 and 2 are therefore out of the running.
In addition, the image appears in quadrant III, which means it has been rotated 180 degrees (resulting in a reflection through the origin).
Choice 3 is the correct answer. It states that a Dilation of 1/2 took place following a Rotation of 180°. This means that we first do the rotation and then the dilation.
The rule for a 180 degree rotation is: $R_{180°}\ P(x,y) \rightarrow P'(-x,-y)$
Using this rule here is what would result in our figures original coordinates:
A(4,2) →(-4,-2) B(4,6) →(-4,-6) C(2,6) →(-2,-6)
A dilation of 1/2 means that each coordinate would now be multiplied by 1/2 resulting in:
(-4,-2) →A"(-2,-1) (-4,-6) →B"(-2,-3) (-2,-6) →C"(-1,-3)
Look back at the problem and you will see that these are the coordinates given for the resulting image A"B"C". **ANSWER: (3)**

9) ***In an equilateral triangle, what is the difference between the sum of the exterior angles and the sum of the interior angles?***
(1) 180° (2) 120° (3) 90° (4) 60°

The sum of the exterior angles of any polygon will be 360°. The sum of the interior angles will depend on the polygon in question. For an equilateral triangle, or any triangle, the sum of the interior angles will be 180. 360 -180 = 180 **ANSWER: (1)**

10) ***What is an equation of a circle with its center at (-3,5) and a radius of 4?***
(1) $(x - 3)^2 + (y + 5)^2 = 16$
(2) $(x + 3)^2 + (y - 5)^2 = 16$
(3) $(x - 3)^2 + (y + 5)^2 = 4$
(4) $(x + 3)^2 + (y - 5)^2 = 4$

Here's a little explanation or rather example. If Xc will be the x-coordinate at the center of a circle, and Yc will be the y-coordinate at the center of the circle, then the equation of the circle will be :
$(x - Xc)^2 + (y - Yc)^2 = r^2$

Here are some examples:
$(x + 3)^2 + (x + 5)^2 = 4^2$ center: (-3, -5); radius = 4
$(x - 3)^2 + (x + 5)^2 = 16$ center: (+3, -5); radius = $\sqrt{16}$ or 4
$(x + 3)^2 + (x - 5)^2 = 16$ center: (-3, +5); radius = $\sqrt{16}$ or 4 **ANSWER: (2)**

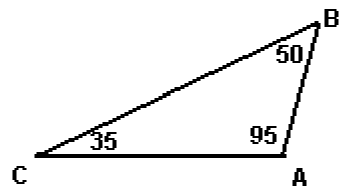
11) ***In ΔABC, m∠A = 95, m∠B = 50, and m∠C = 35. Which expression correctly relates the lengths of the sides of this triangle?***

(1) $AB < BC < CA$ (3) $AC < BC < AB$
(2) $AB < AC < BC$ (4) $BC < AC < AB$

The relationship between the angles of a triangle and its sides is quite simple.

Answer continues on next page…

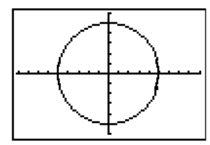
Below is a diagram of the triangle (not drawn to scale) indicating the given information.



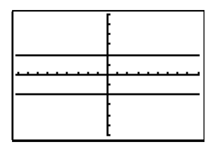
What you should know is that the greater an angle, the greater the opposite side.
$\angle A$ is the greatest angle. making $\overline{BC}$ the greatest side.
$\angle B$ is less than $\angle A$, so $\overline{AC}$, the side opposite $\angle B$ will be less than $\overline{BC}$, the side opposite $\angle A$.
$\angle C$ is the smallest angle, so $\overline{AB}$ will be the smallest side.

he relationship between the sides is therefore BC > AC > AB which is the same as:
AB< AC< BC **ANSWER: (2)**

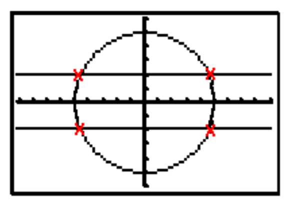
12) ***In a coordinate plane, how many points are both 5 units from the origin and 2 units from the x-axis?***
(1) 1 (2) 2 (3) 3 (4) 4



The locus of points 5 units from the origin is a circle whose center is the origin, and whose radius is 5.



The locus of points 2 units from the x-axis would be a pair of parallel lines -- one, two units above the x-axis, and the other two units below the x-axis.



Now imagine both diagrams on one coordinate axis. I've marked with an X **four points**. Each one of those points satisfy two conditions. Each point 5 units from the origin, and at the same time 2 units from the x-axis.

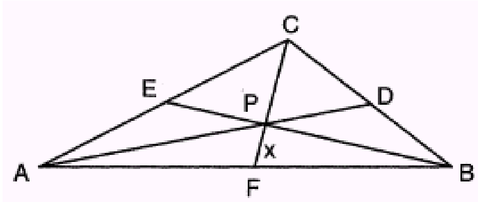
ANSWER: (4)

13) ***What is the contrapositive of the statement, "If I am tall, then I will bump my head"?***
(1) If I bump my head, then I am tall.
(2) If I do not bump my head, then I am tall.
(3) If I am tall, then I will not bump my head.
(4) If I do not bump my head, then I am not tall.

One way of finding the contrapositive, is to first find the inverse of the original statement, and then find the converse of that inverse.

Original statement: If I am tall, then I will bump my head.
Inverse: If I am not tall, then I will not bump my head.
Converse of above inverse: If I do not bump my head, then I am not tall.
Contrapositive of original statement: **If I do not bump my head, then I am not tall.**
ANSWER: (4)

14) In the diagram of $\triangle ABC$ below, Jose found centroid P by constructing the three medians. He measured $\overline{CF}$ and found it to be 6 inches.



If $PF = x$, which equation can be used to find x?

(1) $x + x = 6$
(2) $2x + x = 6$
(3) $3x + 2x = 6$
(4) $x + \frac{2}{3}x = 6$

The distance from any vertex to the centroid of a triangle, will be 2/3 the distance from that vertex to the midpoint of the opposite side.

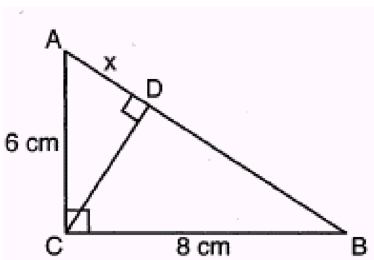
This means that $CP = \frac{2}{3}CF$. Since $CP = \frac{2}{3}$ of the median, $PF = \frac{1}{3}$ of the median.

PF will therefore equal one-half of CP, (1/3 is half of 2/3).

You are told that PF is x. CP is therefore 2x. Together they comprise the median which equals 6.

In other words, $2x + x = 6$ **ANSWER: (2)**

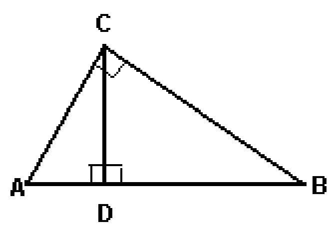
15) ***In the diagram below, the length of the legs AC and BC of right triangle ABC are 6 cm and 8 cm, respectively. Altitude CD is drawn to the hypotenuse of ΔABC.***



What is the length of $\overline{AD}$ to the *nearest tenth of a centimeter*?

(1) 3.6
(2) 6.0
(3) 6.4
(4) 4.0

This problem is new to the curriculum. When presented with an altitude drawn to the hypotenuse of a right triangle, three proportions can be set up. Each one contains a segment that is a mean proportional. At the right you see the triangle above with the hypotenuse AB as its base. Altitude CD is drawn to that hypotenuse. Here are the three proportions.

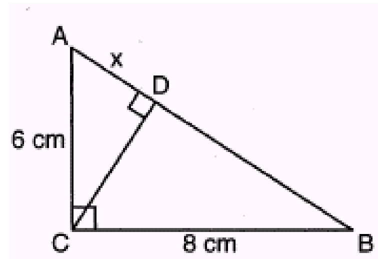


$\frac{AD}{CD} = \frac{CD}{BD}$ Altitude is mean proportional between the segments of the hypotenuse.

$\frac{AB}{BC} = \frac{BC}{BD}$ Each leg (BC, AC) is the mean proportional between the hypotenuse

$\frac{AB}{AC} = \frac{AC}{AD}$ and the segment of the hypotenuse adjacent to that leg.

Answer continues on next page



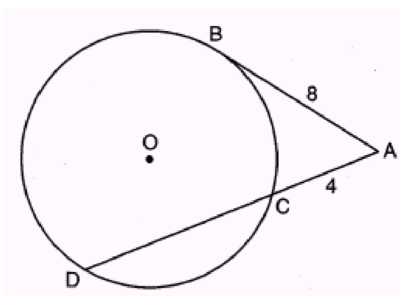
Here is the original diagram again.
You are asked to determine the value of $\overline{AD}$, represent by x.
Step 1 requires you to determine the length of hypotenuse AB. You should know it equals 10.
You can either use the Pythagorean theorem:
$(AB)^2 = 6^2 + 8^2$ or use the fact that the given triangle is a multiple of the 3, 4, 5 Pythagorean triple. Each side has been doubled.
This means that instead of 5, the **hypotenuse will be 10**.
You now use the third proportion listed:

$$\frac{AB}{AC} = \frac{AC}{AD}$$

Cross multiplying you get $(AC)^2 = (AB)(AD)$ Now substitute the known values.
$(6)^2 = 10\,x$ Simplify.
$36 = 10x$ Divide each side by 10.
$3.6 = x$ (Already to tenth.)

x = 3.6 **ANSWER: (1)**

16) In the diagram below, tangent $\overline{AB}$ and secant $\overline{ACD}$ are drawn to circle O from an external point A, $AB = 8$, and $AC = 4$.



What is the length of $\overline{CD}$?

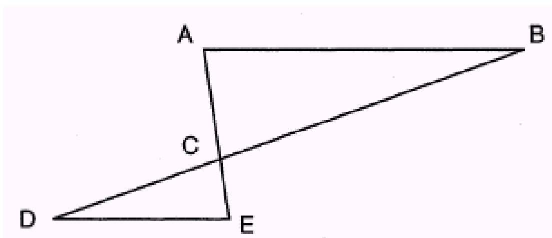
(1) 16
(2) 13
(3) 12
(4) 10

When a tangent and secant are drawn to a circle from a given point, the length of the tangent, in this case AB, will be the mean proportional between the length of the secant, AD, and its external segment. AC.

$\frac{AD}{AB} = \frac{AB}{AC}$ or $(AB)^2 = (AD)(AC)$ Let length of CD= x, so length of AD = x + 4

$(AB)^2 = (AD)(AC)$ Substitute givens.
$8^2 = (AD)\,(4)$ Simplify.
$64 = (AD)\,(4)$ Divide both sides by 4.
$16 = AD$
You now know that AD=16. Since AC = 4, this leaves 16-4, or 12 for CD.
ANSWER: (3)

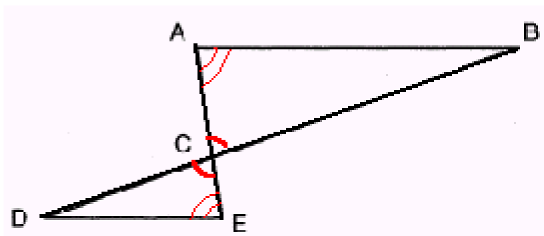
17) ***In the diagram of ΔABC and ΔEDC below, $\overline{AE}$ and $\overline{BD}$ intersect at C, and ∠CAB ≅ ∠CED.***



Which method can be used to show that ΔABC must be similar to ΔEDC?

(1) SAS (3) SSS
(2) AA (4) HL

Here is the diagram again with the given information marked in red.

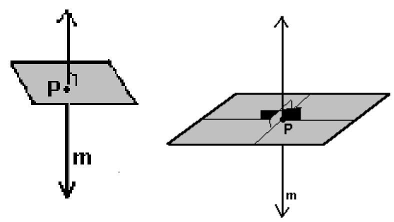


You are told that two line segments intersect at C. As a result of that information you know that a pair of vertical angles are formed. **Vertical angles are congruent.** You are also told that ∠CAB and ∠CED **are congruent.**

You now know that the two triangles are similar, because two angles of one triangle are congruent to two angles of the other. This is what is meant by AA. **ANSWER: (2)**

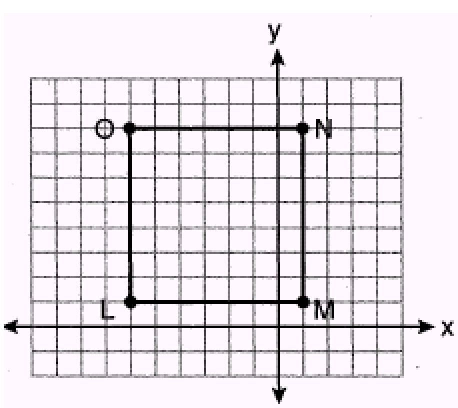
18) ***Point P is on line m. What is the total number of planes that are perpendicular to line m and pass through point P?***
(1) 1 (2) 2 (3) 0 (4) infinite

By definition, the line and plane will be perpendicular if every line in the plane is perpendicular to the line. As can be seen or imagined by the diagram at the right, there can be only one plane perpendicular to a given line at a given point on a given line.



ANSWER: (1)

19) ***Square LMNO is shown in the diagram below.***

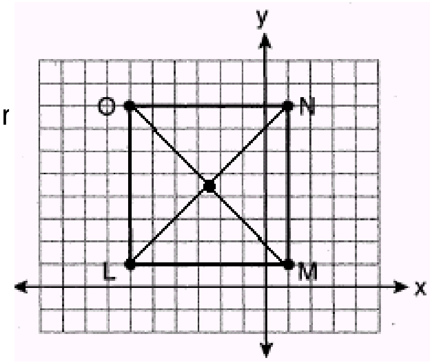


What are the coordinates of the midpoint of diagonal $\overline{LN}$?

(1) $\left(4\frac{1}{2}, -2\frac{1}{2}\right)$ (3) $\left(-2\frac{1}{2}, 3\frac{1}{2}\right)$
(2) $\left(-3\frac{1}{2}, 3\frac{1}{2}\right)$ (4) $\left(-2\frac{1}{2}, 4\frac{1}{2}\right)$

Answer begins on next page

Perhaps the easiest way to do this problem is to draw both diagonals as you see at the right. In a square the diagonals bisect each other. This means that their point of intersection will be their midpoint. As you can see, the midpoint must be choice 4, **(-2½, 4½).**



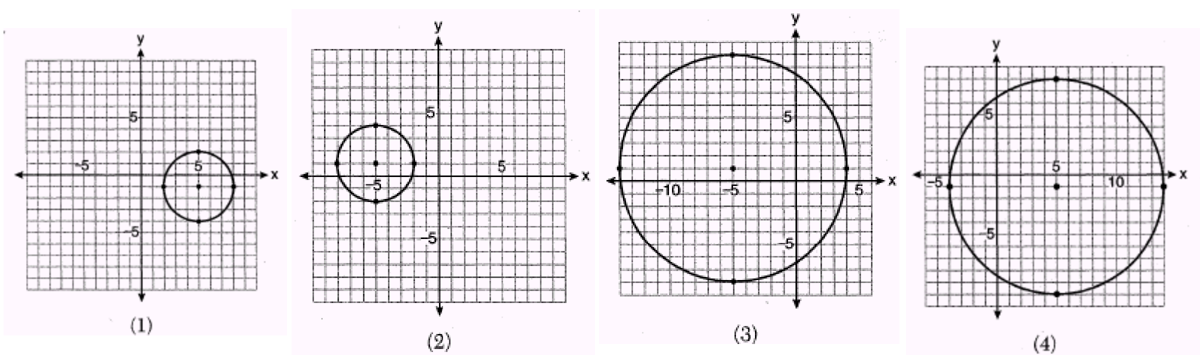
Alternate method using midpoint formula:
You are asked for the midpoint of diagonal $\overline{LN}$

The coordinates of L are (-6,1).
The coordinates of N are (1,8)

To find the x-coordinate of the midpoint, add the x-coordinates of the two given points and divide by 2.
(-6) + (1) = -5; now divide by 2 and your x-coordinate of the midpoint is **-2½.**

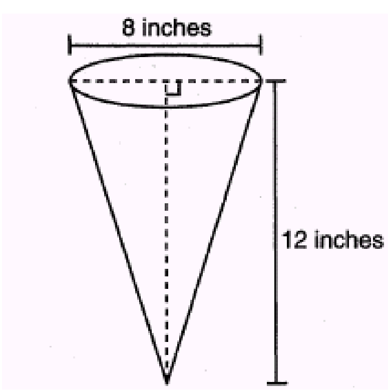
To find the y-coordinate, we follow the same procedure--add the two y-coordinates and divide the sum by 2.
(1) + (8) = 9; now divide by 2 and your y-coordinate is **4½.** **ANSWER: (4)**

20) ***Which graph represents a circle with the equation $(x - 5)^2 + (y + 1)^2 = 9$?***



To refresh your memory, review problem 10 from this regents again.
Based on the equation above, it should be obvious that the graph would be one of a circle with its **center at (5,-1)**, having a **radius of 3**.
This is the circle pictured in choice 1. **ANSWER: (1)**

21) ***In the diagram below, a right circular cone has a diameter of 8 inches and a height of 12 inches.***



What is the volume of the cone to the *nearest cubic inch*?

(1) 201 (3) 603
(2) 481 (4) 804

The formula for finding the volume of a right circular cone is: $V=\frac{1}{3}\pi r^2 h$

(On your reference sheet it appears as $V=\frac{1}{3}Bh$, where B is the area of the base.
The base of the cone is a circle whose area is πr^2).

Now it's simply a matter of substituting what is given.
You are told that the diameter is 8, so you know that the radius, **r**, **equals 4**.
The height, **h**, **equals 12**.

$V=\frac{1}{3}\pi r^2 h$ Substitute.

$v=\frac{1}{3}\pi(4)^2(12)=\frac{1}{3}\pi(16)(12)=\frac{1}{3}\pi(192)=64\pi=201.06$

201.06 to the nearest cubic inch is 201. **ANSWER: (1)**

22) ***A circle is represented by the equation $x^2 + (y + 3)^2 = 13$. What are the coordinates of the center of the circle and the length of the radius?***
(1) (0,3) and 13
(2) (0,3)and $\sqrt{13}$
(3) (0,-3) and 13
(4) (0,-3)and $\sqrt{13}$

Begin by reviewing numbers 10 and 20 on this regents. This equation is quite similar but imagine it written as follows: $(x - 0)^2 + (y + 3)^2 = 13$

It is now obvious that the center of the circle is **(0,-3)** and its radius is $\sqrt{13}$.
(Had the equation been $x^2 + y^2 = 13$,
its center would have been (0,0) with a radius of $\sqrt{13}$). **ANSWER: (4)**

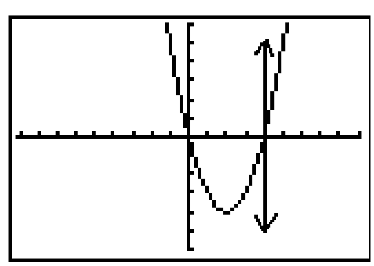
23) ***Given the system of equations:***

$$y = x^2 - 4x$$
$$x = 4$$

The number of points of intersection is
(1) 1 (2) 2 (3) 3 (4) 0

The second equation tells you that x = 4. Substitute 4 for x in the first equation.

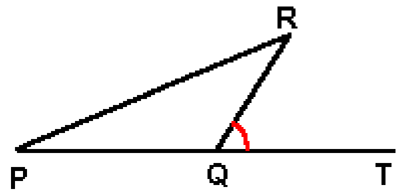
$y = x^2 - 4x$ Substitute 4 for x.
$y = (4)^2 - 4(4)$ Simplify.
$y = 16 - 16$ Simplify.
$y = 0$



This tells you that the point of intersection for these two equations would be only one point -- the point where x = 4 and y = 0 (4,0)
The screen capture at the right shows you the parabola and straight line intersecting at that one point (4,0). **ANSWER: (1)**

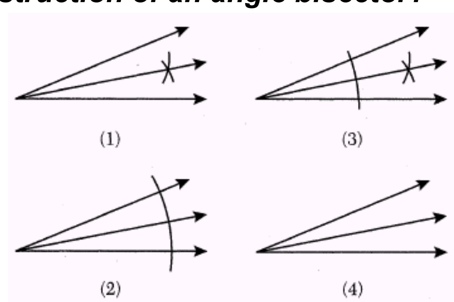
24) Side $\overline{PQ}$ of $\triangle PQR$ is extended through Q to point T. Which statement is *not* always true?

(1) $m\angle RQT > m\angle R$
(2) $m\angle RQT > m\angle P$
(3) $m\angle RQT = m\angle P + m\angle R$
(4) $m\angle RQT > m\angle PQR$



To the right you see one possible triangle.
Let's investigate each choice.
∠RQT pictured at the right in red is an exterior angle. As such, it is equal to the sum of the measures of ∠P and ∠R. In other words,
∠RQT = ∠P + ∠R.
Choices 1 and 2 are therefore always true.
∠RQT will certainly be greater than just ∠R or ∠P alone (since it equals their sum).
Choice 3 is always true as well. The exterior ∠ will always equal the sum of the two remote interior angles.
Choice 4 is not always true as you see in the drawn diagram. **ANSWER: (4)**
(There are two other possibilities for ∠RQT. It could have been a right angle, in which case it would have been congruent to ∠PQR.
It could also have been an obtuse angle, in which case it actually would have been greater than ∠PQR).

25) ***Which illustration shows the correct construction of an angle bisector?***



Choice 3 is the correct answer. The arc is drawn first. The points at which the arc intersects the sides of the angle are then used to draw the two intersecting arcs. The point where these two small arcs intersect, is then used as a guide to draw the angle bisector.

ANSWER: (3)

26) ***Which equation represents a line perpendicular to the line whose equation is 2x + 3y = 12?***

(1) $6y = -4x + 12$
(2) $2y = 3x + 6$
(3) $2y = -3x + 6$
(4) $3y = -2x + 12$

Perpendicular lines have slopes that are negative reciprocals. Let us first determine the slope of the line represented by the equation 2x + 3y =12. One way of doing this is to transform the equation into the form where y = mx + b. In such a case, m represents the slope.

$2x + 3y = 12$ Subtract 2x from both sides.
$3y = -2x + 12$ Divide both sides by 3.
$\frac{3y}{3} = \frac{-2x}{3} + \frac{12}{3}$ Simplify.
$y = -\frac{2}{3}x + 4$ **The slope of this line is $-\frac{2}{3}$.**

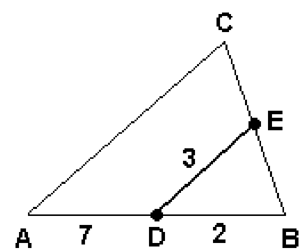
The negative reciprocal of $-\frac{2}{3}$ is $\frac{3}{2}$ (because their product equals -1).

Answer continues on next page…

All that remains now is to figure out which equation represents a line with a slope of 3/2. Instead of transforming each choice into the form y = mx + b, notice choice 2. 2y = 3x + 6. You are looking for "y=" rather than "2y". This means you will have to divide by 2. When you divide "3x" by 2, you will obtain the **3/2** which represents the slope you are looking for. **ANSWER: (2)**

27) In $\triangle ABC$, point D is on $\overline{AB}$, and point E is on $\overline{BC}$ such that $\overline{DE} \parallel \overline{AC}$. If $DB = 2$, $DA = 7$, and $DE = 3$, what is the length of $\overline{AC}$?

(1) 8 (2) 9 (3) 10.5 (4) 13.5



First draw a diagram. (The diagram at the right is not drawn to scale). You are told that $\overline{DE} \parallel \overline{AC}$.

In such a case, where a line parallel to one side of a triangle intersects the other two sides of that triangle, a triangle similar to the given triangle is cut off. In the diagram at the right, therefore, ∆DBE ends up being similar to ∆ABC. As such, many proportions using corresponding sides can be set up.

Here is one:

$$\frac{\text{DE in small triangle}}{\text{AC in original triangle}} = \frac{\text{DB in small triangle}}{\text{AB in original triangle}}$$ Substitute values. Note: AB = 7+ 2 = 9

$\frac{3}{AC} = \frac{2}{9}$ Cross multiply.

2 (AC) = (3) (9) Simplify.

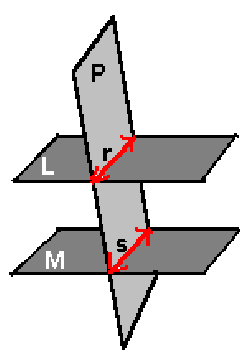
2 (AC) = 27 Divide by 2

AC = 13.5 **ANSWER: (4)**

28) ***In three-dimensional space, two planes are parallel and a third plane intersects both of the parallel planes. The intersection of the planes is a***

(1) plane (3) pair of parallel lines

(2) point (4) pair of intersecting lines



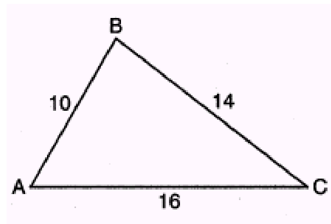
At the right you can see planes L and M parallel to each other. Two planes are parallel if they do not intersect. Plane P intersects these two parallel planes.

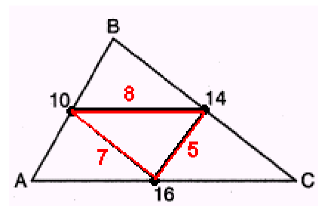
You can see that the lines of intersection, lines r and s, are parallel.

ANSWER: (3)

PART II

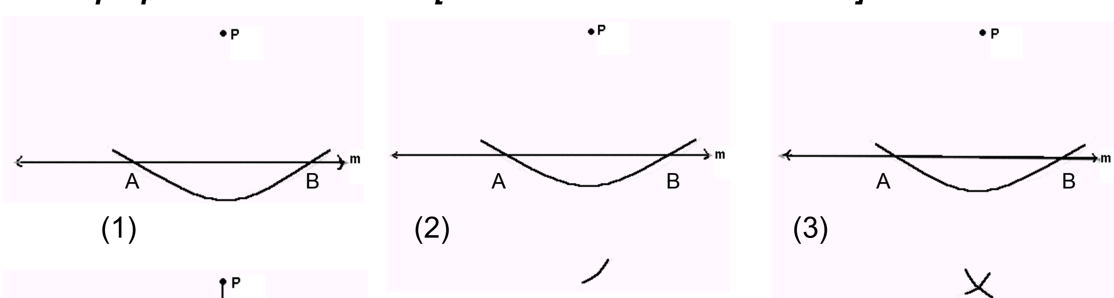
29) ***In the diagram of △ABC below, AB = 10, BC = 14, and AC = 16. Find the perimeter of the triangle formed by connecting the midpoints of the sides of △ABC.***

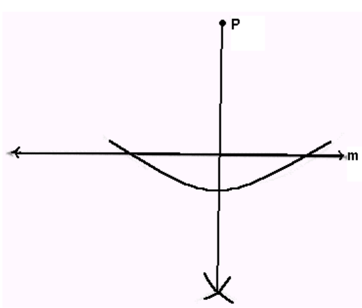




The answer to this question is based on the theorem that: If a line segment joins the midpoint of two sides of a triangle, it will be parallel to the third side and **equal to one-half the length of the third side.**
In other words, the new smaller triangle that is formed will consist of three sides whose lengths will equal one-half the length of the original side to which it is parallel.
Its perimeter will therefore equal half of the original perimeter.
Original perimeter: 10 + 16 + 14 = 40
Smaller triangle: 7 + 5 + 8 = 20 (20 is half of 40).
ANSWER: The new triangle will have a perimeter of 20.

30) ***Using a compass and straightedge, construct a line that passes through point P and is perpendicular to line m. [Leave all construction marks.]***





To the left is the final construction with all construction marks intact.
(1) Place compass point at point P, and draw the arc you see intersecting line m at A and B.
(2) Place compass point at intersection point A, draw an arc.
(3) Place compass at right intersection point B, keeping same radius, and draw second arc intersecting the first.

Lastly connect point P to the intersection of the two small arcs and you have your perpendicular constructed to the line from point P.

31) ***Find an equation of the line passing through the point (5,4) and parallel to the line whose equation is 2x + y = 3.***

Let's use y = mx + b, which known as the slope-intercept form of a line.
"m" represents the slope, and "b" represents the y-intercept.
First find the slope of the line represented by the given equation.

2x + y = 3	Subtract 2x from both sides.
y = -2x + 3	**Slope = -2**

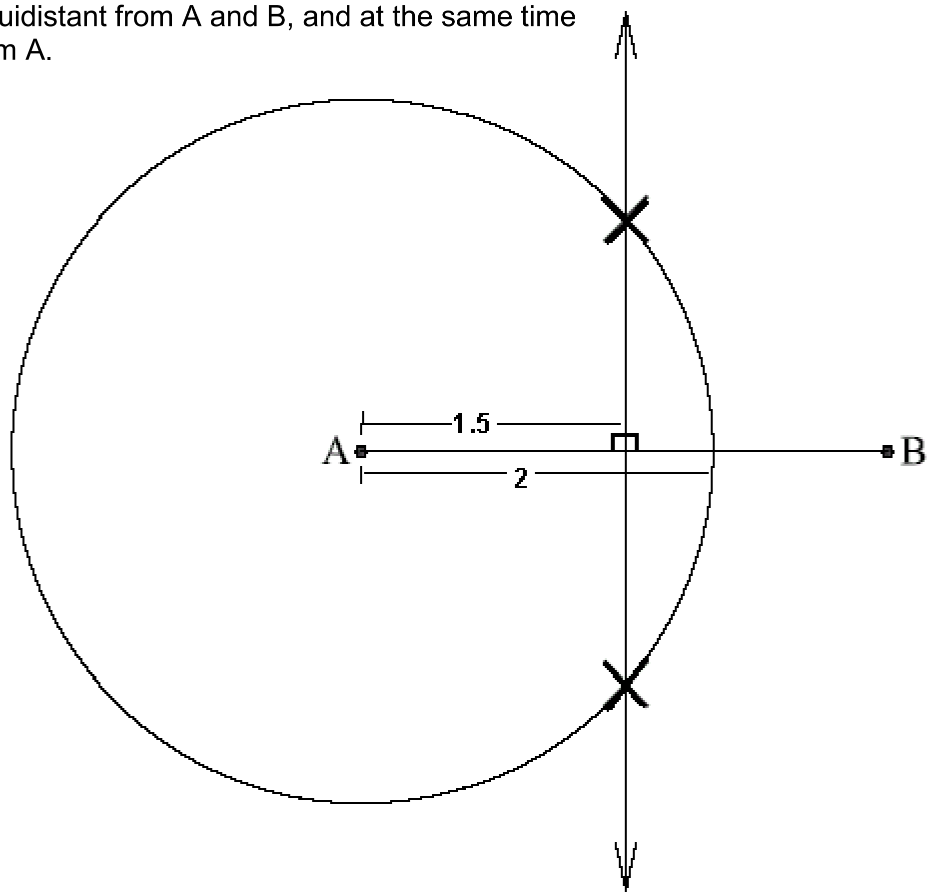
Parallel lines have equal slope so the new equation will look as follows:

y = -2x + b	Now substitute (5.4) to determine the y-intercept, "b".
y = - 2x +b	Substitute 5 for x and 4 for y.
4 = -2(5) + b	Simplify.
4 = -10 + b	Add 10 to both sides.
14 = b	You now know the equation of the new line. Its slope is -2 and its y-intercept is 14.

ANSWER: y = -2x + 14

32) ***The length of $\overline{AB}$ is 3 inches. On the diagram below, sketch the points that are equidistant from A and B and sketch the points that are 2 inches from A. Label with an X all points that satisfy both conditions.***

In the diagram below, the vertical line represents the points equidistant from A and B. It is a sketch of the perpendicular bisector of line AB. It is 1.5 inches from A, and 1.5 inches from B.
The locus of points that are 2 inches from A is the circle whose center is A, and whose radius is 2. Notice that it intersects the vertical line, because the vertical line is only 1.5 inches from A.
The two points marked with an X satisfy both conditions.
They are equidistant from A and B, and at the same time 2 inches from A.



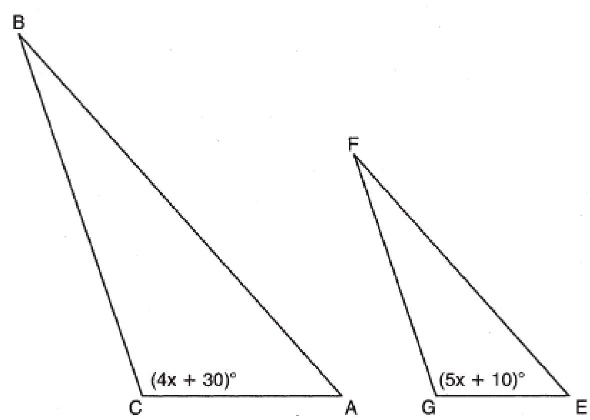
33) ***Given: Two is an even integer or three is an even integer.***
Determine the truth value of this disjunction. Justify your answer.
In logic, a disjunction joins two simple sentences using the word "or".
The sentence given above is a disjunction.
ANSWER: The disjunction is true because at least one of its sentences is true. Two *is* an even integer. There is only one case where a disjunction is false. That is if both of its sentences are false.

You can also use symbolic logic to determine the truth value of the given statement.

Let "w" represent: Two is an even integer.
Let "r" represent: Three is an even integer.
The given sentence can now be represented by:

w V r (The symbol for disjunction is V) Now substitute the truth values.
T V F The first sentence is true, the second is false.
T T V F = T (For disjunction, only F V F = F)

34) ***In the diagram below, △ABC ~ △EFG, m∠C = 4x + 30, and m∠G = 5x + 10.***
Determine the value of x.



The corresponding angles of similar triangles will be congruent.
m∠C therefore = m∠G
Set up your equation and solve for x.

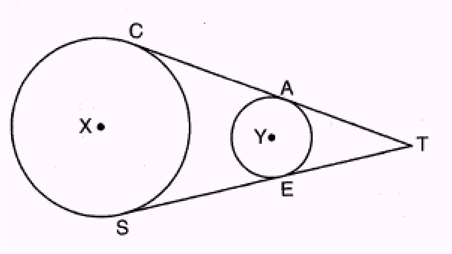
4x + 30 = 5x + 10 Subtract 4x.
30 = x + 10 Subtract 10.
20 = X

ANSWER: x = 20

Part III begins on next page…

Part III

35) ***In the diagram below, circles X and Y have two tangents drawn to them from external point T. The points of tangency are C, A, S, and E. The ratio of TA to AC is 1:3. If TS = 24, find the length of $\overline{SE}$.***

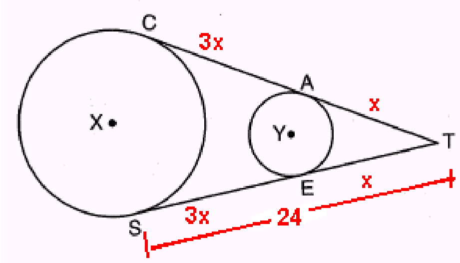


(Not drawn to scale)

When two tangents are drawn to a circle from an external point, the two tangents will be equal in length. Therefore, in the diagram at the left, TC =TS.
You are told that TS = 24, so TC = 24 as well.
For the same reason that TC = TS, TA equals TE.
AC will therefore also equal ES, because equals subtracted from equals are equal.
TC - TA = TS -TE or simply AC = ES

The diagram at the right now contains in red all the information you know. Set up your equation.

$x + 3x = 24$ Combine like terms.
$4x = 24$ Divide both sides by 4.
$x = 6$



(Not drawn to scale)

You are looking for SE which equals 3x.
SE = 3x = 3(6) = 18

ANSWER: SE = 18

36) ***Triangle ABC has coordinates A(-6,2), B(-3,6), and C(5,0). Find the perimeter of the triangle. Express your answer in simplest radical form. [The use of the grid below is optional.]***

To solve this problem you will use the distance formula of a line. To use this formula you need to know the coordinates of two points.

Distance = $\sqrt{(\text{difference of x-coordinates})^2 + (\text{difference of y-coordinates})^2}$ or

$$d = \sqrt{(x1 - x2)^2 + (y1 - y2)^2}$$

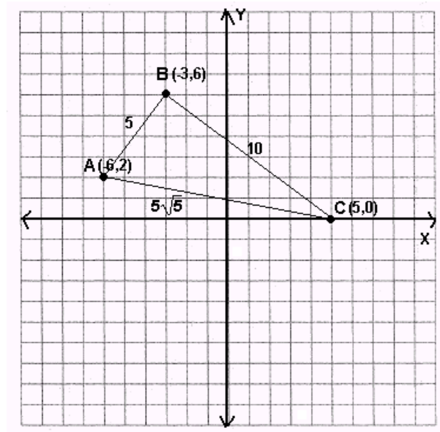
AB = $\sqrt{((-6 - (-3))^2 + (2 - 6)^2} = \sqrt{(-6+3)^2 + (2-6)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{25} =$ **5**

BC = $\sqrt{(-3 - 5)^2 + (6 - 0)^2} = \sqrt{(-8)^2 + (6)^2} = \sqrt{64 + 36} = \sqrt{100} =$ **10**

CA = $\sqrt{(5 - (-6))^2 + (0 - 2)^2} = \sqrt{(5+6)^2 + (-2)^2} = \sqrt{(11)^2 + (-2)^2} = \sqrt{125} =$ **$5\sqrt{5}$**

(Continued on next page)

It was not necessary to graph the triangle, but here it is. Now all that remains is to find its perimeter which is the sum of its sides.



$5 + 10 + 5\sqrt{5} = 15 + 5\sqrt{5}$

ANSWER: Perimeter is $15 + 5\sqrt{5}$

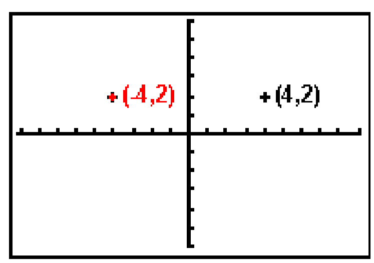
Note: Here are the steps in simplifying $\sqrt{125}$:
$\sqrt{125} = \sqrt{25}\,\sqrt{5} =$ **$5\sqrt{5}$**

37) ***The coordinates of the vertices of parallelogram ABCD are A(-2,2), B(3,5), C(4,2), and D(-1,-1). State the coordinates of the vertices of parallelogram A"B"C"D" that result from the transformation*** **$r_{y\text{-axis}} \circ T_{2,-3}$.**
[The use of the set of axes below is optional.]

Like number 8 on this regents, this problem deals with a composition of transformations. The transformation $r_{y\text{-axis}} \circ T_{2,-3}$ means a reflection in the y-axis, following a translation of (2,-3). This means that the translation is done first, and then the reflection.

The rule for the translation above is simple. You **add 2 to your x-coordinate**, **and -3 to your y-coordinate.** Using this rule,
A(-2,2) becomes A'(-2+2,2-3) or **A'(0,-1).**
B(3,5) becomes B'(3+2,5-3) or **B'(5,2),**
C(4,2) becomes C'(4+2,2-3) or **C'(6,-1).**
D(-1,-1) becomes D'(-1+2,-1-3) or **D'(1,-4).**

Now you have to do the following transformation $r_{y\text{-axis}}$ on the newly found A'B'C'D'. This transformation is a reflection in the y-axis. In problem 5 of this regents you saw the rule for a reflection in the x-axis. Here is the rule for a reflection in the y-axis:



$r_{y\text{-axis}}$ $P(x,y) \rightarrow P'(-x,y)$.
This means the x-coordinate is negated while the y-coordinate remains unchanged. You see an example of a reflection in the y-axis to the right. Now let's use this rule on A'B'C'D'.

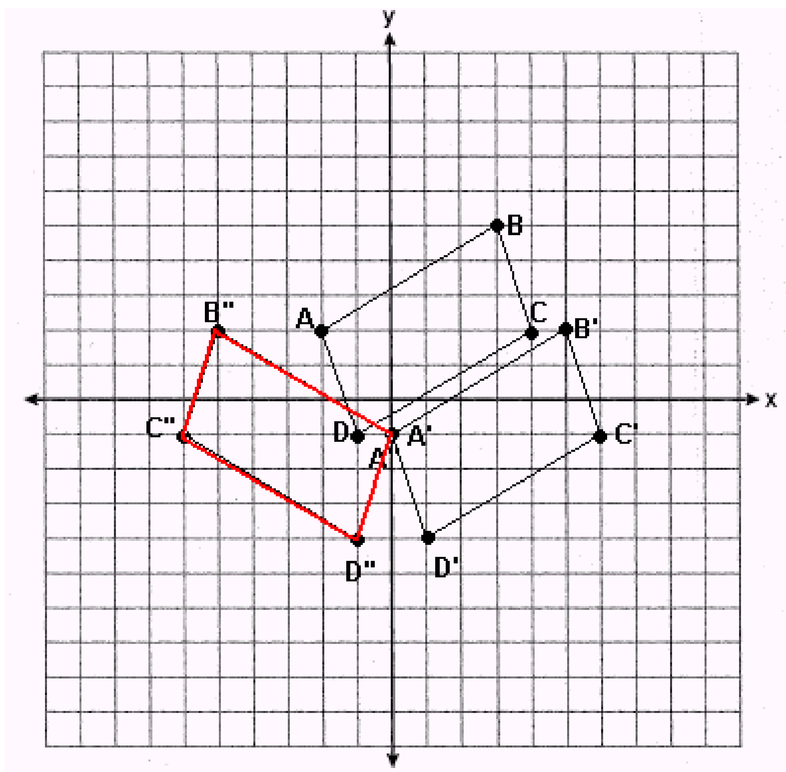
A'(0,-1) becomes **A"(0,-1).**
B'(5,2) becomes **B"(-5,2).**
C'(6,-1) becomes **C"(-6,-1).**
D'(1,-4) becomes **D"(-1,-4).**

Continued on the next page…

A more concise way of showing your work for this problem is as follows:

A(-2,2) $\xrightarrow{T_{2,-3}}$ A'(0,-1) $\xrightarrow{r_{y\text{-axis}}}$ **A"(0,-1)**
B(3,5) $\longrightarrow$ B'(5,2) $\longrightarrow$ **B"(-5,2)**
C(4,2) $\longrightarrow$ C'(6,-1) $\longrightarrow$ **C"(-6,-1)**
D(-1,-1) $\longrightarrow$ D'(1,-4) $\longrightarrow$ **D"(-1,-4)**

Although you were not required to graph this transformation, below is the graphic representation of this composition of transformations.

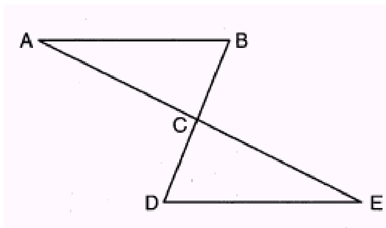


Part IV begins on the next page…

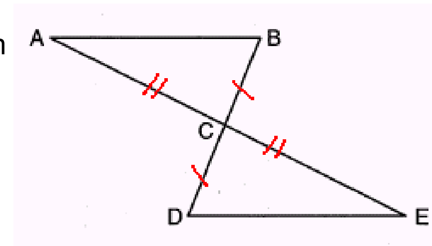
PART IV

38) Given: $\triangle ABC$ and $\triangle EDC$, C is the midpoint of $\overline{BD}$ and $\overline{AE}$

Prove: $\overline{AB} \parallel \overline{DE}$



To the right is the diagram again, with the givens marked in red.



Plan: You will prove the two triangles congruent using SAS. The corresponding line segments are congruent because the midpoint divides each original segment into two congruent parts. The two angles will be congruent because they are vertical angles. Once you know the two triangles are congruent, you know that ∠B is congruent to ∠D because of corresponding parts of congruent triangles. Once you know these two angles are congruent, you know that $\overline{AB} \parallel \overline{DE}$ because the two angles formed are congruent alternate interior angles.

Given: ΔABC and ΔEDC, C is the midpoint of $\overline{BD}$ and $\overline{AE}$,
Prove: $\overline{AB} \parallel \overline{DE}$

Statements	Reasons
1. C is midpoint of $\overline{BD}$ and $\overline{AE}$	1. Given
2. $\overline{BC} \cong \overline{DC}$ (s. ≅ s.) $\overline{AC} \cong \overline{EC}$ (s.≅ s.)	2. The midpoint of a line segment divides the segment into two congruent parts.
3. ∠ACB and ∠ECD are vertical ∠'s	3. Two lines that intersect form vertical angles.
4. ∠ACB ≅ ∠ECD (a.≅ a.)	4. Vertical angles are congruent.
5. ΔABC ≅ ΔEDC	5. s.a.s. ≅ s.a.s.
6. ∠B ≅ ∠D	6. Corresponding parts of congruent triangles are congruent.
7. $\overline{AB} \parallel \overline{DE}$	7. If two lines are cut by a transversal forming a pair of congruent alternate interior angles, the two lines are parallel.

One final time, here is the diagram again, this time with what was proven marked in red.