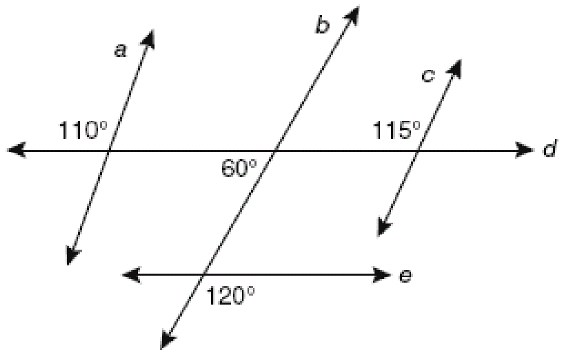
PART 1

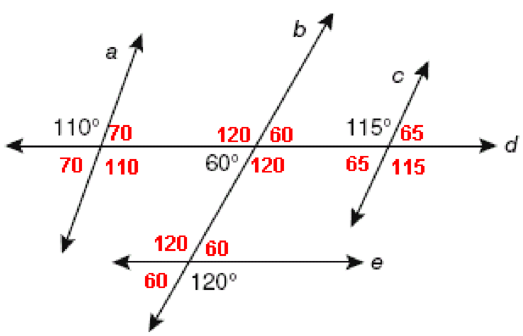
1) ***Based on the diagram below, which statement is true?***



(1) $a \parallel b$ (3) $b \parallel c$

(2) $a \parallel c$ (4) $d \parallel e$

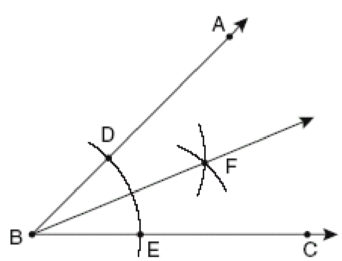
Here is the diagram again. This time all the missing angles are there in red.



The red angles that were missing are easily calculated knowing that about each vertex of two intersecting lines, the angles are either supplementary or vertical. Vertical angles will be congruent, while supplementary angles will have degree measures adding up to 180 degrees.

You can now easily see that lines d and e are parallel because the angles formed by transversal b are congruent alternate interior angles, congruent corresponding angles, and interior angles on the same side of the transversal that are supplementary. Any one of these conditions will only be true when a pair of parallel lines are cut by a transversal, which is the case here. **ANSWER: (4)**

2) ***The diagram below shows the construction of the bisector of ∠ABC.***



Which statement is *not* true?

(1) $m\angle EBF = \frac{1}{2} m\angle ABC$

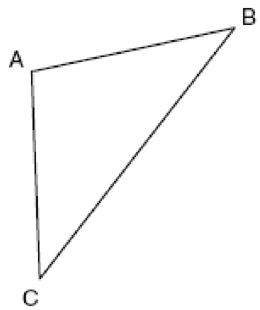
(2) $m\angle DBF = \frac{1}{2} m\angle ABC$

(3) $m\angle EBF = m\angle ABC$

(4) $m\angle DBF = m\angle EBF$

The bisector of an angle divides the angle into two congruent angles.
This means that the resulting two angles will be equal in measure, and each one will equal half the measure of the bisected angle--in our case, angle ABC.
Choice 3 states, in essence, that one of the resulting angles is equal to the original angle. This is a false statement. **ANSWER: (3)**

3) ***In the diagram of ΔABC below,*** $\overline{AB} \cong \overline{AC}$. ***The measure of ∠B is 40°.***

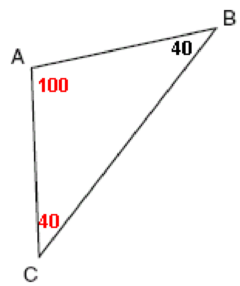


What is the measure of $\angle A$?

(1) 40° (3) 70°

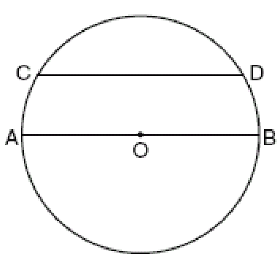
(2) 50° (4) 100°

Here is the diagram again with the angles inserted.



You are told that side AB and side AC are congruent. Therefore, the angles opposite these sides will be congruent as well. So you now know that angles B and C will each measure 40º . The sum of the measures of the three angles of a triangle is 180º. Angles B and C account for 80 of those degrees. This leaves 100º for angle A. **ANSWER: (4)**

4) In the diagram of circle O below, chord $\overline{CD}$ is parallel to diameter $\overline{AOB}$ and $m\widehat{AC} = 30$.

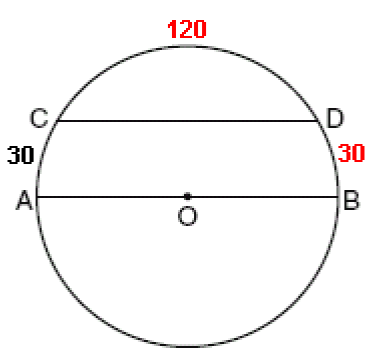


What is $m\widehat{CD}$?

(1) 150 (3) 100

(2) 120 (4) 60

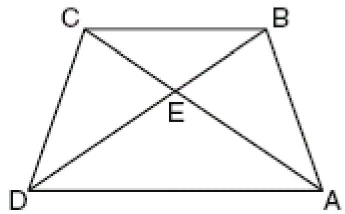
Here's the diagram with some missing information.



The measure of arc AC is given as 30. Since chord CD is parallel to the diameter, arc BD and arc AC will be equal. (Parallel chords intercept equal arcs.) Arc BD will therefore have a measure of 30 as well. A semi circle measures 180. This means that the sum of the measures of arcs AC + CD + BD will equal 180. Arcs AC and BD account for 60, leaving 120 for arc CD.

ANSWER: (2)

5) ***In the diagram of trapezoid ABCD below, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E and ΔABC ≅ ΔDCB.***



Which statement is true based on the given information?

(1) $\overline{AC} \cong \overline{BC}$ (3) $\angle CDE \cong \angle BAD$

(2) $\overline{CD} \cong \overline{AD}$ (4) $\angle CDB \cong \angle BAC$

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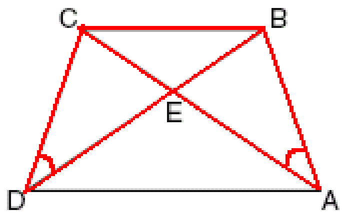


Here is the diagram again with the two congruent triangles highlighted. Also indicated are two of the angles that are congruent (choice 4) due the fact that corresponding parts of congruent triangles are congruent.

ANSWER: (4)

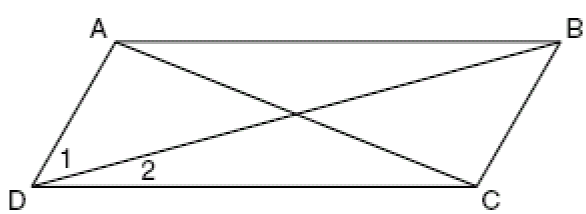
6) ***Which transformation produces a figure similar but not congruent to the original figure?***

(1) $T_{1,3}$ (3) $R_{90°}$

(2) $D_{\frac{1}{2}}$ (4) $r_{y=x}$

Of the transformations presented, choice 1 is a translation, choice 3 is a rotation, and choice 4 is a reflection. The image produced by a translation, rotation or reflection will be congruent to the original image. Choice 2 names a dilation of 1/2. Dilations will produce a figure similar to the original figure. **ANSWER: (2)**

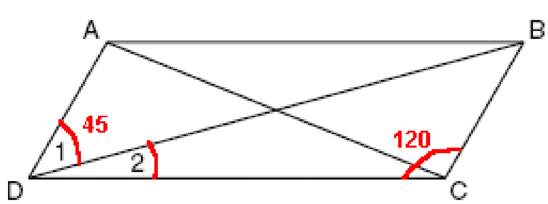
7) ***In the diagram below of parallelogram ABCD with diagonals $\overline{AC}$ and $\overline{BD}$, m∠1 = 45 and m∠DCB = 120.***



What is the measure of ∠2?

(1) 15° (3) 45°

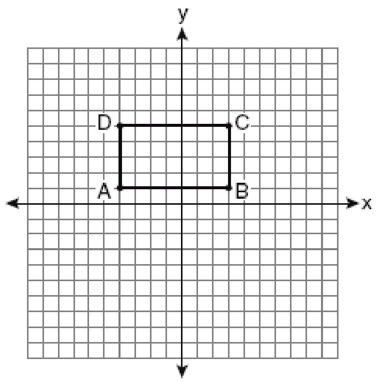
(2) 30° (4) 60°



∠DCB and ∠ADC are consecutive angles of a parallelogram. As such they are supplementary - the sum of their measures is 180˚. You are told that ∠DCB measures 120˚. This means that ∠ADC will measure 60˚. ∠ADC consists of ∠1 and ∠2. ∠1 is given as 45˚. You need 15˚ more to reach 60˚. ∠2 will therefore equal 15˚.

ANSWER: (1)

8) ***On the set of axes below, Geoff drew rectangle ABCD. He will transform the rectangle by using the translation (x,y) → (x + 2,y + 1)and then will reflect the translated rectangle over the x-axis.***



What will be the area of the rectangle after these transformations?

(1) exactly 28 square units
(2) less than 28 square units
(3) greater than 28 square units
(4) It cannot be determined from the information given.

The transformations named are a translation followed by a reflection. As stated in the answer to number 6, the image obtained after translations and reflections will be congruent to the original image. therefore, the area of the rectangle will remain the same. All that remains is to determine the area of rectangle ABCD. From A to D there are 4 units, and from D to C there are 7 units. Area equals length time width.
4 times 7 =28. The new image will retain the same number of square units.

ANSWER: (1)

9) ***What is the equation of a line that is parallel to the line whose equation is y = x + 2?***

(1) x + y = 5 ***(3) y − x = −1***
(2) 2x + y = −2 ***(4) y − 2x = 3***

The slope intercept form of a line is y = mx + b, where m represents the slope of the line, and b represents the y-intercept. The given equation, y = x + 2, is already in the form of y = mx + b. In the given equation m = 1. (The equation is really y = 1x +2.) The slope of the given line is therefore 1.
Parallel lines will have equal slopes. All you need to do now is to check the given choices for an equation whose line will have a slope of 1. Choice 3 will be your answer. The objective is to set each of the equations into the form of y = mx + b, and find the one where m=1.

Here is what each equation will look like once transformed:

(1)	(2)	(3)	(4)
x + y = 6	2x + y = -2	y - x = -1	y - 2x = 3
y = -x +6	y = -2x - 2	y = x - 1	y = 2x + 3
m = -1	m = -2	m = 1	m = 2

Choice 3 has a slope of 1, and will be parallel to the line whose equation is y = x + 2.

ANSWER: (3)

10) ***The endpoints of $\overline{CD}$ are C(−2,−4) and D(6,2). What are the coordinates of the midpoint of $\overline{CD}$?***
(1) (2,3) ***(3) (4,−2)***
(2) (2,−1) ***(4) (4,3)***

You can find your answer using the midpoint formula.

The coordinates of C are (-2,-4).
The coordinates of D are (6,2)

To find the x-coordinate of the midpoint, add the x-coordinates of the two given points and divide by 2.
(-2) + (6) = 4; now divide by 2 and your x-coordinate of the midpoint is **2.**

To find the y-coordinate, we follow the same procedure--add the two y-coordinates and divide the sum by 2.
(-4) + (2) = -2; now divide by 2 and your y-coordinate is **-1.**
The midpoint is (2,-1) **ANSWER: (2)**

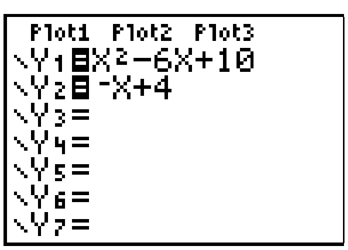
11) ***What are the center and the radius of the circle whose equation is $(x - 3)^2 + (y + 3)^2 = 36$?***
(1) center = (3,−3); radius = 6
(2) center = (−3,3); radius = 6
(3) center = (3,−3); radius = 36
(4) center = (−3,3); radius = 36

Here's a little explanation or rather example. If Xc will be the x-coordinate at the center of a circle, and Yc will be the y-coordinate at the center of the circle, then the equation of the circle will be : $(x - Xc)^2 + (y - Yc)^2 = r^2$
Here are some examples:

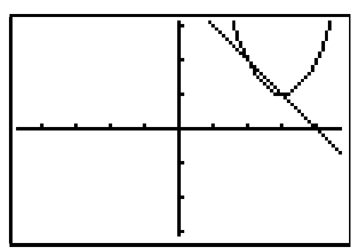
$(x + 3)^2 + (y + 3)^2 = 6^2$ center: (-3, -3); radius = 6
$(x + 3)^2 + (y - 3)^2 = 6$ center: (-3, +3); radius = $\sqrt{6}$
$(x - 3)^2 + (y + 3)^2 = 36$ center: (+3, -3); radius = $\sqrt{36}$ or 6 **ANSWER: (1)**

12) ***Given the equations:***
$y = x^2 - 6x + 10$
$y + x = 4$
What is the solution to the given system of equations?
(1) (2,3) (3) (2,2) and (1,3)
(2) (3,2) (4) (2,2) and (3,1)

There are many ways to do this problem. Here's one method using your calculator. Enter both equations into the Y= editor. Transform the second equation to y = -x + 4.



To the left you see both equations entered. To the right you see their graphs. **They intersect at two points. Those two points are the solution.**



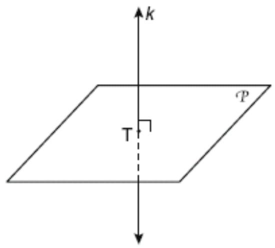
Based on the image of their graphs above to the right, choice 4 is the answer.
ANSWER: (4)

13) ***The diagonal $\overline{AC}$ is drawn in parallelogram ABCD. Which method cannot be used to prove that △ABC ≅ △CDA?***
(1) SSS (3) SSA
(2) SAS (4) ASA

SSA does not prove congruency unless you are dealing with right triangles, and then the conventional method would be referred to as hy. leg (hypotenuse leg) rather than SSA.

ANSWER: (3)

14) ***In the diagram below, line k is perpendicular to plane P at point T. Which statement is true?***

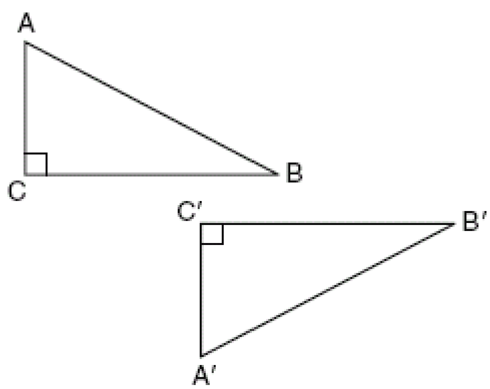


(1) Any point in plane P also will be on line k.
(2) Only one line in plane P will intersect line k.
(3) All planes that intersect plane P will pass through T.
(4) Any plane containing line k is perpendicular to plane P.

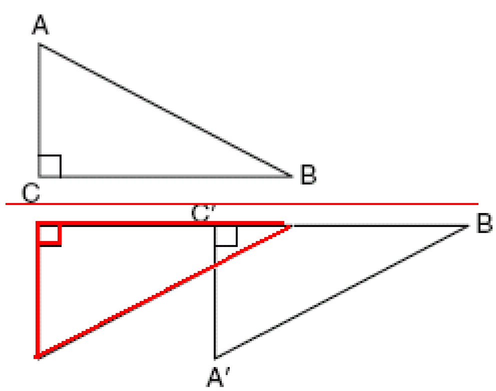
If line k is perpendicular to a plane P, then any plane containing that line will automatically be perpendicular to the plane P.

ANSWER: (4)

15) ***In the diagram below, which transformation was used to map △ABC to △A'B'C'?***



(1) dilation (3) reflection
(2) rotation (4) glide reflection



The transformation used is named a glide reflection. It is a composition of two transformations -- a translation and a reflection. It makes no difference in which order the transformations are completed.

At the left you see a red triangle which is a reflection of ΔABC under the thin red horizontal line of symmetry. The red triangle was then translated a number of units to the right to obtain the image ΔA'B'C'.
(You can use your imagination and see that the same image would have resulted if the original triangle had first been translated to the right and then reflected.)

ANSWER: (4)

16) ***Which set of numbers represents the lengths of the sides of a triangle?***
(1) {5, 18, 13} ***(3) {16, 24, 7}***
(2) {6, 17, 22} ***(4) {26, 8, 15}***

In any triangle, the sum of any two sides has to be greater than the third side.
Keeping this in mind:
Choice 1 is disqualified because 5 +13 is not greater than 18.
Choice 3 is disqualified because 16 + 7 is not greater than 24.
Choice 4 is disqualified because 8 + 15 is not greater than 26.
Choice 2 is your answer. The sum of ANY two of its sides will be greater than the third side. **ANSWER: (2)**

17) ***What is the slope of a line perpendicular to the line whose equation is*** $y = -\frac{2}{3}x - 5$***?***

(1) $-\frac{3}{2}$ (3) $\frac{2}{3}$

(2) $-\frac{2}{3}$ (4) $\frac{3}{2}$

Perpendicular lines have slopes that are negative reciprocals. Let us first determine the slope of the line represented by the given equation. It is already in the slope intercept form where y = mx + b. In such a case, m represents the slope. In the above equation m = $-\frac{2}{3}$. The line represented by the above equation therefore has a slope of $-\frac{2}{3}$.

The negative reciprocal of $-\frac{2}{3}$ is $\frac{3}{2}$ (because their product equal -1).

ANSWER: (4)

18) ***A quadrilateral whose diagonals bisect each other and are perpendicular is a***
(1) rhombus ***(3) trapezoid***
(2) rectangle ***(4) parallelogram***

The diagonals of a parallelogram bisect each other. Since choices 1,2, and 4 are members of the parallelogram family, their diagonals will bisect each other. However, only in a rhombus and square, because their sides are congruent, will the diagonals be perpendicular to each other. Based on that, choice 1 is the answer. **ANSWER: (1)**

19) If the endpoints of $\overline{AB}$ are $A(-4,5)$ and $B(2,-5)$, what is the length of $\overline{AB}$?

(1) $2\sqrt{34}$ (3) $\sqrt{61}$

(2) 2 (4) 8

To solve this problem you will use the distance formula of a line. To use this formula you need to know the coordinates of two points.

Distance = $\sqrt{(\text{difference of x-coordinates})^2 + (\text{difference of y-coordinates})^2}$ or

d = $\sqrt{(x1 - x2)^2 + (y1 - y2)^2}$

Problem continues on next page…

The two given endpoints are A(-4,5) and B(2,-5).
Let the coordinates of A be (x1,y1) and the coordinates of B (x2,y2)
Now let's substitute into the formula:

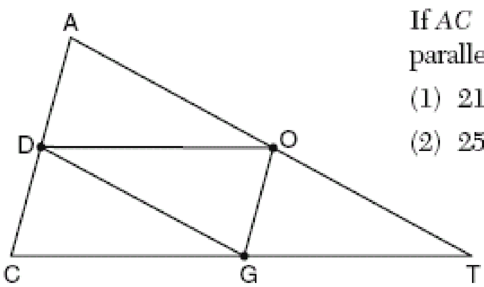
$$d = \sqrt{(x1 - x2)^2 + (y1 - y2)^2}$$

$$d = \sqrt{(-4-2)^2 + (5-(-5))^2} = \sqrt{(-6)^2 + (5+5)^2} = \sqrt{36 + 10^2} = \sqrt{36+100} = \sqrt{136}$$

The length of $\overline{AB}$ is $\sqrt{136}$. You now have to figure out which of the choices is the equivalent of this length. You can use your calculator to find the square root of 136, and then do the same for choices 1 and 2 and see which one matches. Or you can simplify the square root of 136 by factoring out its largest perfect square.

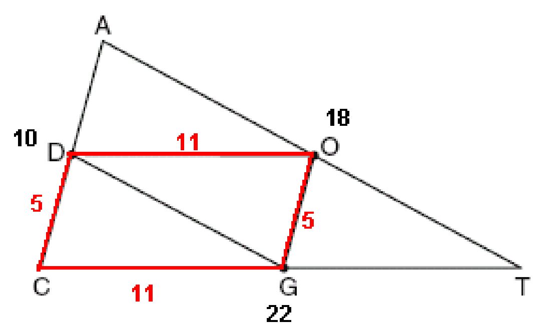
$\sqrt{136} = \sqrt{4} \bullet \sqrt{34} = \mathbf{2\sqrt{34}}$ **ANSWER: (1)**

20) In the diagram below of $\triangle ACT$, D is the midpoint of $\overline{AC}$, O is the midpoint of $\overline{AT}$, and G is the midpoint of $\overline{CT}$.



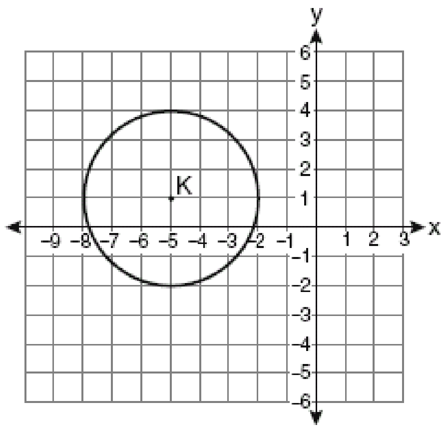
If $AC = 10$, $AT = 18$, and $CT = 22$, what is the perimeter of parallelogram $CDOG$?

(1) 21
(2) 25
(3) 32
(4) 40



The answer to this question is based on the theorem that: If a line segment joins the midpoint of two sides of a triangle, it will be parallel to the third side and **equal to one-half the length of the third side.**
In other words, the new smaller sides will consist of lengths equal to one-half the length of the original side to which it is parallel.
The perimeter of parallelogram CDOG will be
CD=5 because it is half of AC (D is the midpoint).
DO =11 because it is half of CT.
OG = 5 because it is half of AC.
GC = 11 because it is half of CT. (G is the midpoint).
5 + 5 + 11+ 11 = 32
(Actually, once you are told that CDOG is a parallelogram, you do not need to use the theorem. You know that opposite sides of a parallelogram are equal.
So OG will equal CD, and OD will equal CG.) **ANSWER: (3)**

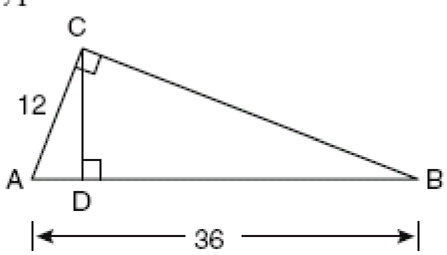
21) ***Which equation represents circle K shown in the graph below?***



(1) $(x + 5)^2 + (y - 1)^2 = 3$
(2) $(x + 5)^2 + (y - 1)^2 = 9$
(3) $(x - 5)^2 + (y + 1)^2 = 3$
(4) $(x - 5)^2 + (y + 1)^2 = 9$

The graph shows circle K with its center at (-5,1) having a radius of 3.
Here's a little explanation or rather example. If Xc will be the x-coordinate at the center of a circle, and Yc will be the y-coordinate at the center of the circle, and r will be the radius, then the equation of the circle will be :
$(x - Xc)^2 + (y - Yc)^2 = r^2$
Here are some examples:
Center (5,1); radius 3 Equation: $(x - 5)^2 + (y - 1)^2 = 3^2$ or 9
Center (-5,-1); radius 3 Equation: $(x + 5)^2 + (y + 1)^2 = 9$
Center (5, -1); radius 3 Equation: $(x - 5)^2 + (y + 1)^2 = 9$
Center (-5, 1); radius 3 Equation: $(x + 5)^2 + (y - 1)^2 = 9$ ANSWER: (2)

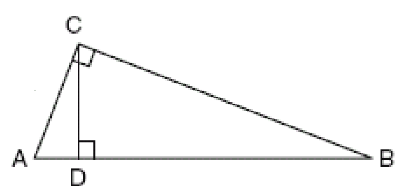
22) In the diagram below of right triangle ACB, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.



If $AB = 36$ and $AC = 12$, what is the length of $\overline{AD}$?

(1) 32 (3) 3
(2) 6 (4) 4

When an altitude is drawn to the hypotenuse of a right triangle, three proportions can be set up. Each one contains a segment that is a mean proportional. At the right you see the triangle above with the hypotenuse AB as its base. Altitude CD is drawn to that hypotenuse. Here are the three proportions.



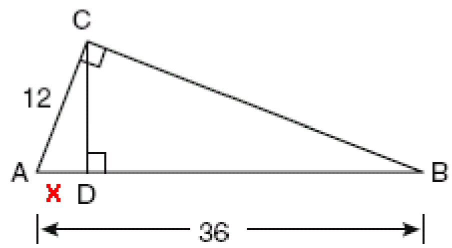
$\frac{AD}{CD} = \frac{CD}{BD}$ Altitude is mean proportional between the segments of the hypotenuse.

$\frac{AB}{BC} = \frac{BC}{BD}$ Each leg (BC, AC) is the mean proportional between the hypotenuse

$\frac{AB}{AC} = \frac{AC}{AD}$ and the segment of the hypotenuse adjacent to that leg.

Answer continues on next page

Here is the diagram again:
Let x represent the length of $\overline{AD}$
You can now use the third proportion listed on the previous page to determine x.

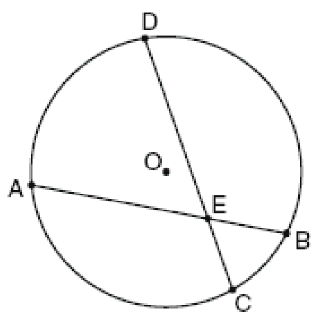


$$\frac{AB}{AC} = \frac{AC}{AD}$$

Cross multiplying you get $(AC)^2 = (AB)(AD)$ Now substitute the known values.
$(12)^2 = 36x$ Simplify.
$144 = 36x$ Divide each side by 36.
$4 = x$

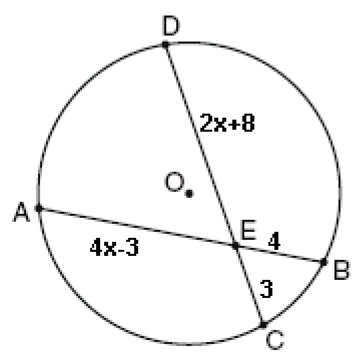
ANSWER: (4)

23) ***In the diagram of circle O below, chord $\overline{AB}$ intersects chord $\overline{CD}$ at E, DE = 2x + 8, EC = 3, AE = 4x − 3, and EB = 4.***



What is the value of x?
(1) 1 (3) 5
(2) 3.6 (4) 10.25

Here is the diagram again with the givens inserted.
You will now use the following theorem:
If two chords that intersect inside a circle, the product of the measures of the segments of one chord will equal the product of the measures of the segments of the other chord.
In our example this translates to:



(DE)(EC) = (AE)(EB) Substitute the givens.
(2x+8)(3) = (4x-3)(4) Use distributive property.
6x + 24 = 16x - 12 Subtract 6x from both sides.
24 = 10x - 12 Add 12 to both sides.
36 = 10x Divide both sides by 10.
3.6 = x

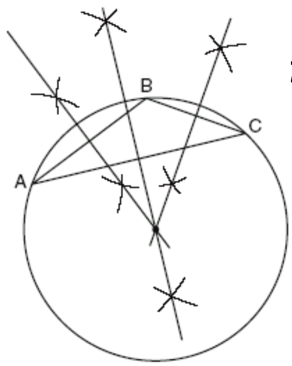
ANSWER: (2)

24) ***What is the negation of the statement "Squares are parallelograms"?***
(1) Parallelograms are squares.
(2) Parallelograms are not squares.
(3) It is not the case that squares are parallelograms.
(4) It is not the case that parallelograms are squares.

The negation of the statement is expressed by prefacing the statement with "It is not the case."

ANSWER: (3)

25) ***The diagram below shows the construction of the center of the circle circumscribed about △ABC.***



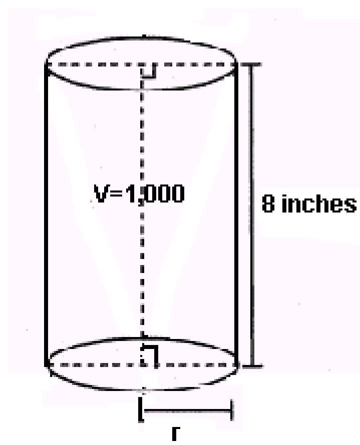
This construction represents how to find the intersection of
(1) the angle bisectors of △ABC
(2) the medians to the sides of △ABC
(3) the altitudes to the sides of △ABC
(4) the perpendicular bisectors of the sides of △ABC

Each of the three sides of the triangle has a perpendicular bisector drawn through it. The center of the circle is their intersection.

ANSWER: (4)

26) ***A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the nearest tenth of an inch?***
(1) 6.3 (3) 19.8
(2) 11.2 (4) 39.8

Pictured at the right is my attempt at a right circular cylinder. Its height is 8 inches, and its volume is 1,000 cubic inches. You are asked to find its radius, which I've labeled r.



Your reference sheet gives the formula for the volume of a right circular cylinder as V=Bh, where B is the area of the base.

The base of a cylinder consists of a circle whose area is πr^2.
So we can rewrite the formula as
$V = \pi r^2 h$ Substitute the givens.
$1000 = \pi r^2(8)$ Divide both sides by 8π.
$39.78873577 = r^2$. Find the square root.
r = 6.3 to nearest tenth.

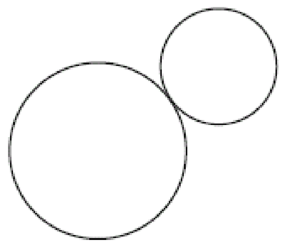
```
1000/(8π)
     39.78873577
√(Ans)
     6.307831305
```

ANSWER: (1)

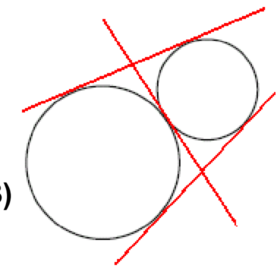
27) ***If two different lines are perpendicular to the same plane, they are***
(1) collinear (3) congruent
(2) coplanar (4) consecutive

Two lines perpendicular to the same plane will be parallel, and two parallel lines determine a plane. It follows, therefore, that these two perpendicular lines will be coplanar. They will lie in the same plane. **ANSWER: (2)**

28) ***How many common tangent lines can be drawn to the two externally tangent circles shown below?***



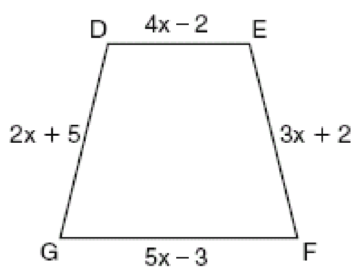
(1) 1 (3) 3
(2) 2 (4) 4



There are 3 common tangent lines possible as seen at the right. **ANSWER: (3)**

PART II

29) ***In the diagram below of isosceles trapezoid DEFG, DE ∥ GF , DE = 4x − 2, EF = 3x + 2, FG = 5x − 3, and GD = 2x + 5. Find the value of x.***

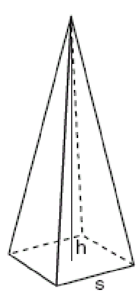


In the trapezoid pictured at the left, the two non-parallel sides DG and EF will be equal in measure. This is true because you are told that the trapezoid is isosceles.
Once you know that DG=EF you can set up your equation and solve for x.

2x + 5 = 3x +2	Subtract 2x from both sides.
5 = x + 2	Subtract 2 from both sides.
3 = x	

ANSWER: X = 3

30) ***A regular pyramid with a square base is shown in the diagram below.***



A side, s, of the base of the pyramid is 12 meters, and the height, h, is 42 meters. What is the volume of the pyramid in cubic meters?

Your reference sheet gives the volume of a pyramid as $V = \frac{1}{3}Bh$ where B is the area of the base.

The base is a square. The area of a square is obtained by squaring its side. You are told that the side of the square is 12 meters. Its **area**, B, is therefore 12^2 or **144** meters squared. The **height** of the pyramid is given as **42** meters.

$V = \frac{1}{3}Bh$ Substitute what is known.

$V = \frac{1}{3}(144)(42)$ Multiply.

(1/3)*144*42
2016

V = 2016 **ANSWER: 2,016 cubic meters**

31) ***Write an equation of the line that passes through the point (6,−5) and is parallel to the line whose equation is 2x − 3y = 11.***

To answer this question you have to know that parallel lines have equal slopes. So let us first find the slope of the line represented by the above equation. To do that you can transform the equation into the slope intercept form of a line: y = mx + b.
In that form m represents the slope and b represents the y-intercept.

$2x - 3y = 11$ Subtract 2x from both sides.
$-3y = -2x + 11$ Divide both sides by -3. (Division of two negatives is a positive).
$y = \frac{2}{3}x - \frac{11}{3}$ The line has a **slope of $\frac{2}{3}$.**

This means that any line with a slope of 2/3 will be parallel to the above line.
So you know the m in y = mx + b. It is 2/3. Let's substitute the 2/3.

y = 2/3 x + b Actually any line whose equation begins with 2/3 x will be parallel to the given line. But you don't want just any line. You want it to pass through the point (6, -5).
So, let's determine the value of b. You are told you want the line to pass through the point (6,-5). This means that x = 6 when y = -5. Substitute these values for x and y.

$y = \frac{2}{3}x + b$ Substitute.

$-5 = \frac{2}{3}(6) + b$ Multiply.

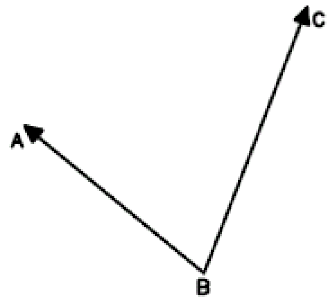
$-5 = 4 + b$ Subtract 4 from both sides.
$-9 = b$ y-intercept equals -9.

Your line will have a slope of 2/3 and a y-intercept of -9.

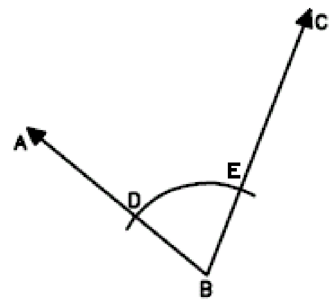
Answer: The equation will be $y = \frac{2}{3}x - 9$

32 begins on the next page…

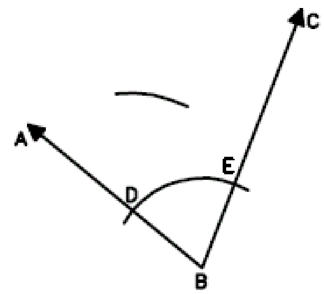
32) ***Using a compass and straightedge, construct the angle bisector of △ABC shown below. [Leave all construction marks.]***



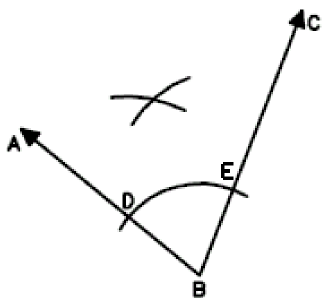
(1)



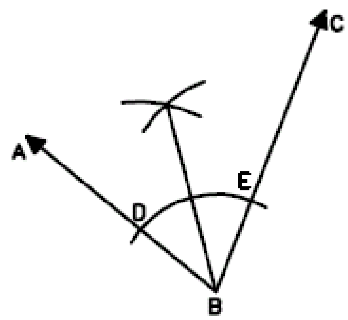
(2)



(3)



(4)



(5)

A. Begin by putting the compass point at B and drawing the arc you see in diagram 2.
B. Keep same radius, put compass point at D and draw the arc you see in diagram 3.
C. Put compass point at E and draw an arc intersecting the previous arc as seen in diagram 4.
D. Finally complete the construction by drawing a line from B through the intersecting arcs as you see in the final diagram 5.

This final line is the angle bisector.

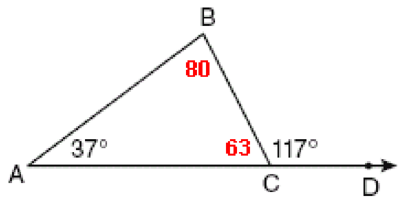
33) ***The degree measures of the angles of △ABC are represented by x, 3x, and 5x − 54. Find the value of x.***

The sum of the measures of the three angles of any triangle will equal 180 degrees. Set up your equation and solve for x.

(x) + (3x) + (5x - 54) = 180	Combine like terms.
9x -54 = 180	Add 54 to both sides.
9x = 234	Divide both sides by 9.
x = 26	

ANSWER: X = 26

34) ***In the diagram below of $\triangle ABC$ with side $\overline{AC}$ extended through D, $m\angle A = 37$ and $m\angle BCD = 117$. Which side of $\triangle ABC$ is the longest side? Justify your answer.***



(Not drawn to scale)

Step number one involves determining the measures of the missing angles.
I have inserted them into the diagram.
Below is an explanation how to determine the missing two angles, and then the answer to the question.

$\angle BCD$ is an exterior angle of the triangle. As such it equals in measure the sum of the measures of $\angle A$ and $\angle B$. So $m\angle B = 117 - 37$ or 80.
$\angle BCA$ and $\angle BCD$ are supplementary (they add up to 180). So $\angle BCA = 180 - 117$ or 63.

Step two is knowing that the longest side of a triangle will be opposite the greatest angle of the triangle.

ANSWER: $\overline{AC}$ is the longest side of the triangle because it is opposite the greatest angle.

35) ***Write an equation of the perpendicular bisector of the line segment whose endpoints are (−1,1) and (7,−5). [The use of the grid below is optional.]***
Here's one algebraic method that can be used to answer this question.
The first step involves your knowing that perpendicular lines will have slopes that are negative reciprocals. Two values are negative reciprocals if their product is -1.
So your first task is to determine the slope of the line whose endpoints are given above.
Given two points, the slope is determined by writing a fraction where the difference in the y-coordinates is the numerator and the difference in the x-coordinates is the denominator. Using m to represent the slope, this would be the formula for determining the slope when two points are given.

$m = \frac{y1 - y2}{x1 - x2}$ Using the given points, let (−1,1) be (x1,y1), and (7,−5) be (x2,y2).

The above formula now becomes $\mathbf{m} = \frac{1 - (-5)}{-1-7} = \frac{1+5}{-8} = \frac{6}{-8} = -\mathbf{\frac{3}{4}}$

The negative reciprocal of $-\frac{3}{4}$ is $\mathbf{\frac{4}{3}}$ because their product equals -1.

You now know that the slope of the perpendicular line is $\frac{4}{3}$.

Now for the next step. The problem does not ask for just any perpendicular line. It asks for the perpendicular bisector. What you now have to do is find the coordinates of the midpoint of the line defined by the two given endpoints.
Simply add their x-coordinates and divide the sum by 2. -1+7= 6 divided by 2 = **3**
Now add their y-coordinates and divide by 2. 1+(-5)= -4 divided by 2 = **-2**.

You now know that the the midpoint is (3, -2).

Continues on next page…

Now you can use the point-slope form of a line to determine the equation of the line going through the given point. The slope you want is **4/3** and the point is **(3.-2)**

Here is the point slope form of a line: $y - y_1 = m(x - x_1)$ where m is the slope and (x_1, y_1) is the given point. Substituting the givens into the above form we get:

$y - y_1 = m(x - x_1)$

$y - (-2) = \frac{4}{3}(x - 3)$ (minus a minus becomes a plus)

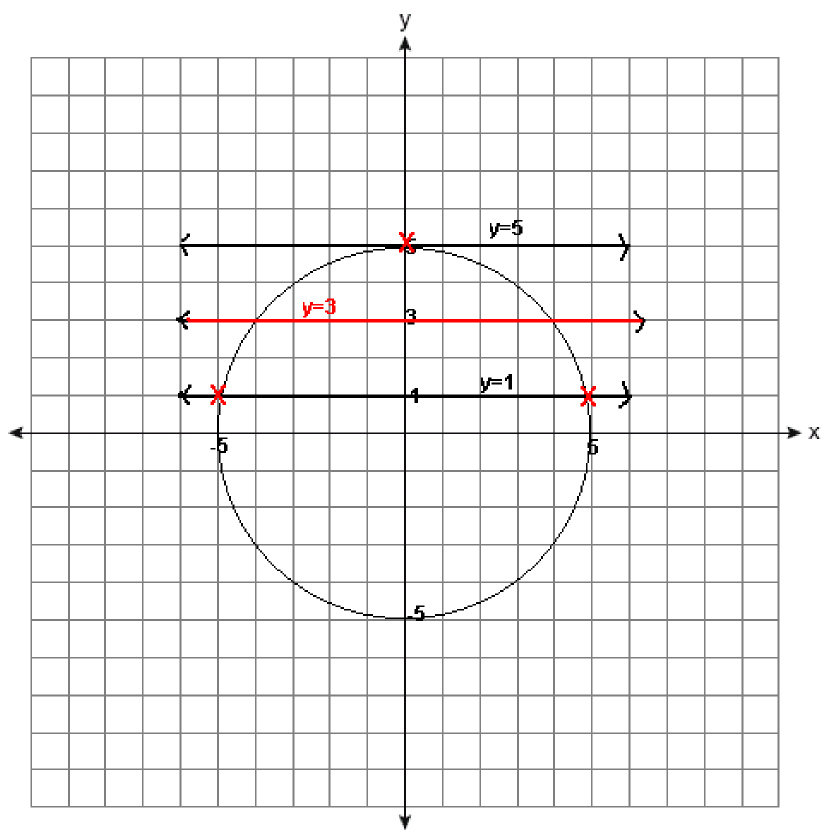
$\mathbf{y + 2 = \frac{4}{3}(x - 3)}$ The answer is in an acceptable form.

The above equation represents a line whose slope is $\frac{4}{3}$, and which passes through the point (3, -2).

ANSWER: $y + 2 = \frac{4}{3}(x - 3)$ or its equivalent $y = \frac{4}{3}x - 6$

36) ***On the set of axes below, sketch the points that are 5 units from the origin and sketch the points that are 2 units from the line y = 3. Label with an X all points that satisfy both conditions.***

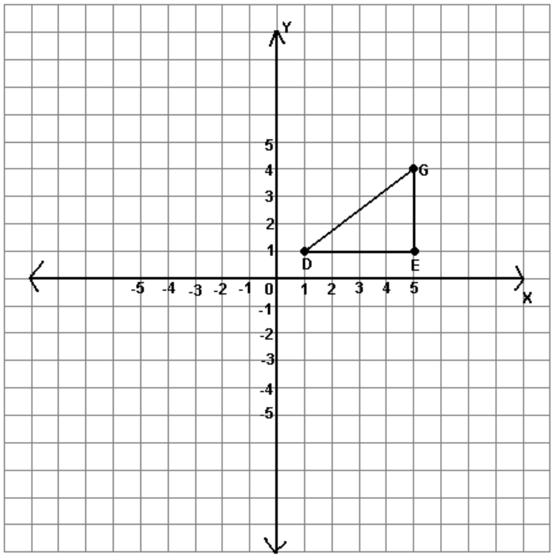
The locus of all points 5 units from the origin will be represented by a circle with its center at the origin having a radius of 5.
The locus of all points 2 units from the line y=3 will be the two lines y=1 and y=5.

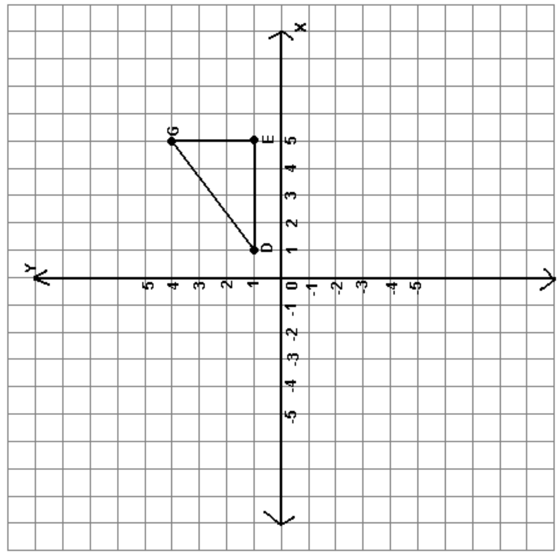


To the left is your answer. The line y=3 is in red. Two units above it is the line y=5, and two units below it is the line y=1.
The circle is the locus of all points 5 units from the origin.
The three X's indicate the points where both conditions are met. Each x is at a point both 5 units from the origin and 2 units from the line y=3.

37) ***Triangle DEG has the coordinates D(1,1), E(5,1), and G(5,4). Triangle DEG is rotated 90° about the origin to form ∆D'E'G'. On the grid below, graph and label ∆DEG and ∆D'E'G'. State the coordinates of the vertices D', E', and G'. Justify that this transformation preserves distance.***

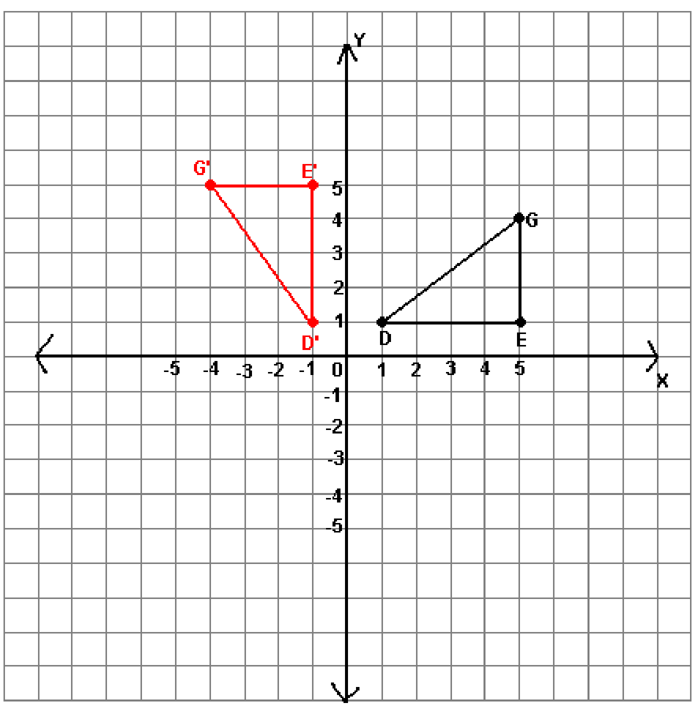
First for a simple way to find the coordinates after a rotation.





Above to the left is the graph of ∆DEG. To its right is what your sheet of graph paper would look like after a 90 degree rotation about the origin. Now for a little imagination. In your mind relabel and renumber the x and y-axis appropriately. D is no longer (1,1). It is now **(-1,1).** E is no longer at (5,1) but rather at **(-1,5)**. And finally, G is no longer (5,4) but **(-4,5)**. Those will be the coordinates for D', E', and G' respectively.

Below is your answer with both graphs above appearing on one coordinate plane.

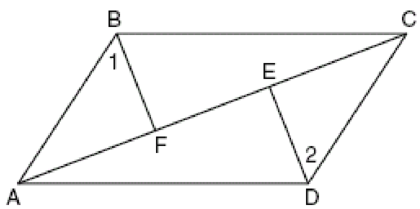


ANSWER: To the left is your graph.

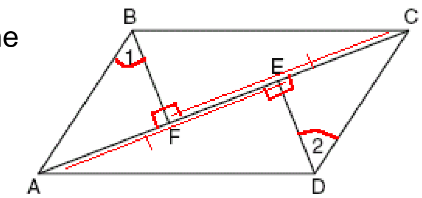
The new coordinates are:
D'(-1,1)
E'(-1,5)
G'(-4,5)

The new transformation preserves distance because it is a rotation, and all rotations preserve distance.

38) ***Given: Quadrilateral ABCD, diagonal $\overline{AFEC}$, $\overline{AE} \cong \overline{FC}$, $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$, $\angle 1 \cong \angle 2$.***
***Prove: ABCD is a parallelogram*.**



To the right is the diagram again, with the givens marked in red.



Plan: You will prove $\Delta BAF \cong \Delta DCE$ using AAS. One set of congruent angles are given. The other set of congruent angles will be the right angles. You will be able to use the subtraction postulate to prove $\overline{AF}$ and $\overline{CE}$ congruent.
Once you've proven the two triangles congruent, you will know $\overline{BA} \cong \overline{DC}$ because corresponding parts of congruent triangles are congruent.
Also $\angle BAF \cong \angle DCE$ for the same reason. These two angles are a pair of congruent alternate interior angles. This makes $\overline{BA}$ parallel to $\overline{DC}$.
Quadrilateral ABCD is therefore a parallelogram because it contains a pair of opposite sides that are both congruent and parallel.

Given: Quadrilateral $ABCD$, diagonal $\overline{AFEC}, \overline{AE} \cong \overline{FC}, \overline{BF} \perp \overline{AC}, \overline{DE} \perp \overline{AC}, \angle 1 \cong \angle 2$
`
Prove: ABCD is a parallelogram.

Statements	Reasons
1. $\overline{AE} \cong \overline{FC}$	1. Given
2. $\overline{FE} \cong \overline{FE}$	2. Reflexive Property
3. $\overline{AE} - \overline{FE} \cong \overline{FC} - \overline{FE}$ or $\overline{AF} \cong \overline{CE}$ (s. ≅ s.)	3. Subtraction postulate
4, BF ⊥ AC and DE ⊥ AC	4. Given
5. ∠BFA and ∠DEC are rt. angles.	5. Perpendicular lines form right angles.
6. ∠BFA ≅ ∠DEC (a. ≅ a.)	6. All right angles are congruent.
7. ∠1≅ ∠2 (a. ≅ a.)	7. Given
8, $\Delta BAF \cong \Delta DCE$	8. a.a.s. ≅ a.a.s.
9. $\overline{BA} \cong \overline{DC}$	9. Corresponding parts of congruent Δ's are ≅.
10. ∠BAF ≅ ∠DEC	10. Corresponding parts of congruent Δ's are ≅.
11. ∠BAF and ∠DEC are alternate interior angles.	11. Definition of alternate interior angles.
12. $\overline{BA} \parallel \overline{DC}$	12. If 2 lines are cut by a transversal forming a pair of congruent alternate interior angles, the 2 lines are parallel.
13. ABCD is a parallelogram.	13. A quadrilateral is a parallelogram if two opposite sides are both congruent and parallel.