PART 1

1) Isosceles trapezoid $ABCD$ has diagonals $\overline{AC}$ and $\overline{BD}$. If $AC = 5x + 13$ and $BD = 11x - 5$, what is the value of x?

(1) 28 (3) 3

(2) $10\frac{3}{4}$ (4) $\frac{1}{2}$

The diagonals of an isosceles trapezoid will be congruent. This means they will be equal in measure. Set up your equation and solve:

5x + 13 = 11x - 5	Subtract 5x from both sides.
13 = 6x - 5	Add 5 to both sides.
18 = 6x	Divide both sides by 6.
3 = x	

ANSWER: (3)

2) ***What is the negation of the statement "The Sun is shining"?***
(1) It is cloudy. (3) It is not raining.
(2) It is daytime. (4) The Sun is not shining.

The negation of "The sun is shining." is "**The sun is not shining**."
The negation of "The sun is not shining." would be "The sun is shining."

ANSWER: (4)

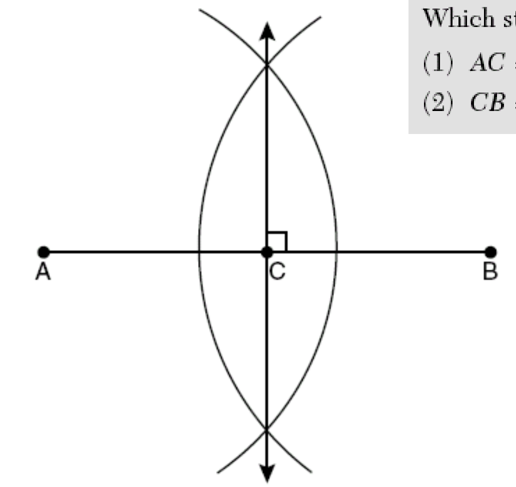
3) ***Triangle ABC has vertices A(1,3), B(0,1), and C(4,0). Under a translation, A', the image point of A, is located at (4,4). Under this same translation, point C' is located at***
(1) (7,1) (3) (3,2)
(2) (5,3) (4) (1,–1)

A translation is a transformation in which a given point is moved a certain distance in the horizontal (x-axis), and a certain distance in the vertical (y-axis). (One of the distances may be 0.)
In the given example, point A with the coordinates (1,3), ends up at (4,4)
The rule for this translation would be: **(x,y) → (x+3, y+1)** or $\mathbf{T_{(3,1)}}$
This tells us that 3 is added to the x-coordinate and 1 is added to the y-coordinate.
This is why A(1,3) became A'(4,4).

Using this rule C(4,0) becomes C'(4+3, 0+1) or **(7,1)**,

ANSWER: (1)

4) ***The diagram below shows the construction of the perpendicular bisector of $\overline{AB}$.***



Which statement is *not* true?

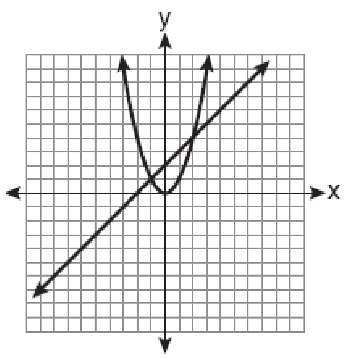
(1) $AC = CB$
(2) $CB = \frac{1}{2}AB$
(3) $AC = 2AB$
(4) $AC + CB = AB$

The perpendicular bisector of a line segment, in this case line segment AB, is perpendicular to the line segment and bisects it.
Choice 3 is the statement that is not true. AC = 1/2 AB, not 2AB

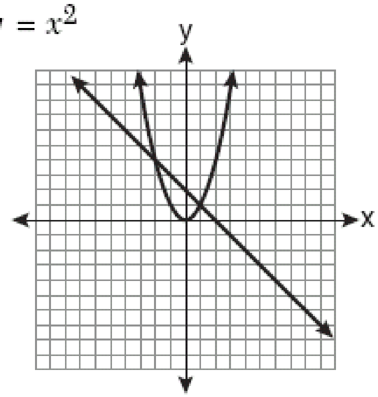
ANSWER (3)

5) ***Which graph could be used to find the solution to the following system of equations?***

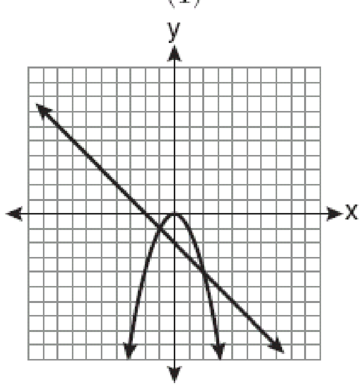
$$y = -x + 2$$
$$y = x^2$$



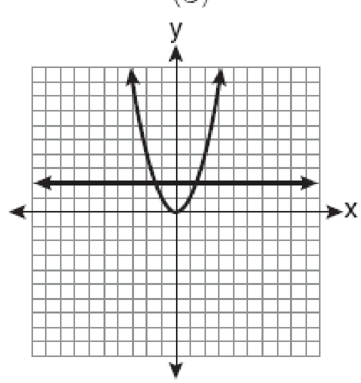
(1)



(3)



(2)



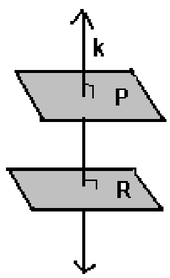
(4)

The easiest way to this problem is to input both equations into your graphing calculator, and graph them. Here is another way to do it.
Since to coefficient of the quadratic is positive, the parabola has to open upwards. This disqualifies choice 2.
The linear equation, the first one, is already in the form of y=mx+b, and m is negative.
This mean that the graph of the linear equation will have a negative slope.
Choice 3 has the parabola opening upwards, and the slope of the graph of the linear equation is negative.

ANSWER: (3)

6) ***Line k is drawn so that it is perpendicular to two distinct planes, P and R. What must be true about planes P and R?***

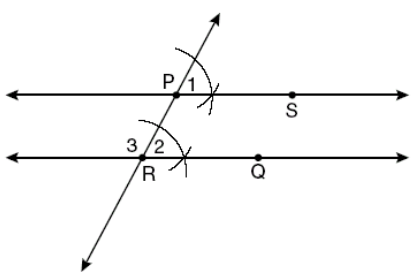
(1) Planes P and R are skew.
(2) Planes P and R are parallel.
(3) Planes P and R are perpendicular.
(4) Plane P intersects plane R but is not perpendicular to plane R.



The two planes will be parallel to each other.
(Two planes are parallel if they do not intersect.)

ANSWER: (2)

7) ***The diagram below illustrates the construction of $\overleftrightarrow{PS}$ parallel to $\overleftrightarrow{RQ}$ through point P.***



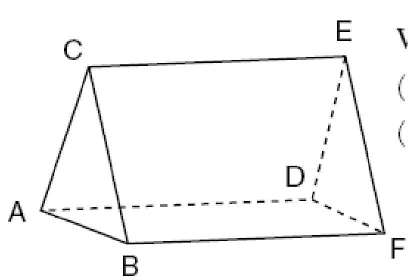
Which statement justifies this construction?

(1) $m\angle 1 = m\angle 2$
(2) $m\angle 1 = m\angle 3$
(3) $\overline{PR} \cong \overline{RQ}$
(4) $\overline{PS} \cong \overline{RQ}$

When two lines are cut by a transversal forming a pair of congruent corresponding angles, the two lines are parallel. Angles 1 and 2 are corresponding angles.

ANSWER: (1)

8) ***The figure in the diagram below is a triangular prism.***



Which statement must be true?

(1) $\overline{DE} \cong \overline{AB}$
(2) $\overline{AD} \cong \overline{BC}$
(3) $\overline{AD} \parallel \overline{CE}$
(4) $\overline{DE} \parallel \overline{BC}$

A triangular prism consists of two triangular and three rectangular sides. In the given triangular prism, one rectangular side happens to be ADEC. As such, choice 3 must be true, as opposite sides of a rectangle are parallel. **ANSWER: (3)**

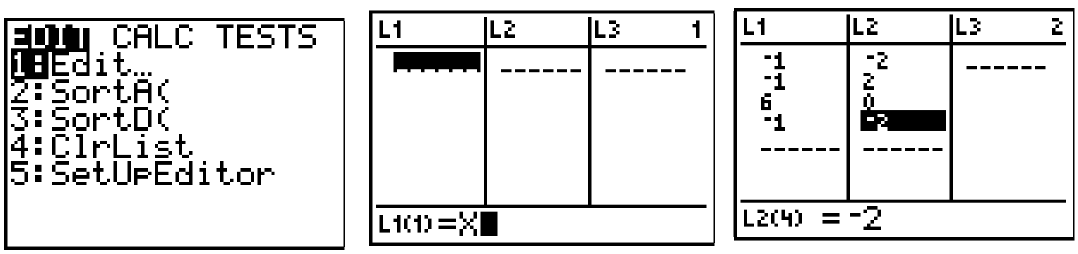
9) ***The vertices of △ ABC are A(–1,–2), B(–1,2), and C(6,0). Which conclusion can be made about the angles of △ ABC?***

(1) $m\angle A = m\angle B$ (3) $m\angle ACB = 90$

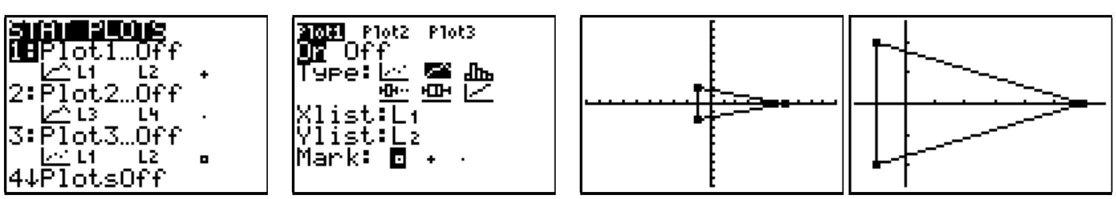
(2) $m\angle A = m\angle C$ (4) $m\angle ABC = 60$

You can sketch the triangle on some scrap and immediately notice that points A and B, form side AB which is perpendicular to the x-axis, and is also bisected by the x-axis. Therefore, the line connecting A to C will be congruent to the line connecting B to C. In other words, an isosceles triangle is formed. The base angles of an isosceles triangle are congruent, making angles A and B congruent.
Just for the fun of it, here is how you can see it using your graphing calculator.

First hit the **STAT** key. Actually first turn on your calculator and then hit the **stat** key. Hit **ENTER** so then you can enter the coordinates into L1 and L2. The x-coordinates go into L1, and the y-coordinates into L2. You will be able to connect this points when you graph them. Point A will be connected to B which will be connected to C. So that the triangle will be complete, again enter the coordinates for A, so that point C will be connected to A.

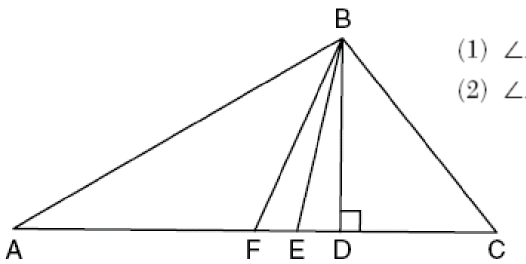


Now your objective is to turn on Plots so that your calculator can plot the data (in this case the coordinates) you entered into L1 and L2. Hit the **2nd** key followed by the **Y=** key to access the STAT PLOT menu. Then hit the **ENTER** key twice to turn Plot1 ON. Next select the type of plot you want by moving the cursor key. The first choice is a scatterplot in which just the points are shown. The second choice, the one you will select is a line plot in which the points will be connected. Also select the type of mark you wish the calculator to use to show your points. Let's select the first choice. Also notice that L1 and L2 are defined as your Xlist and Ylist.
At this point press the **GRAPH** key and you will see the 3rd screen below.
To see it even clearer, hit the **ZOOM** key followed by **9**.



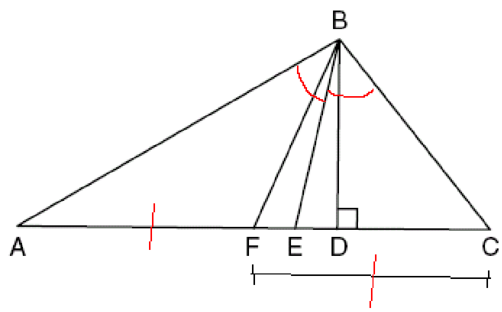
You clearly see that it is an isosceles triangle. (The x-axis is the perpendicular bisector of its base). The base angles A and B are therefore congruent, as base angles of an isosceles triangle are congruent. **ANSWER: (1)**

10) ***Given △ ABC with base $\overline{AFEDC}$, median $\overline{BF}$, altitude $\overline{BD}$, and $\overline{BE}$ bisects ∠ABC, which conclusion is valid?***



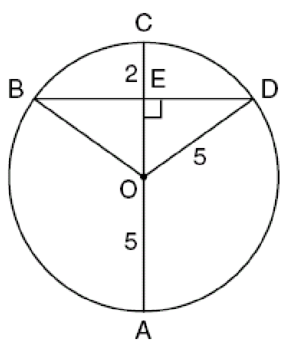
(1) ∠FAB ≅ ∠ABF (3) $\overline{CE} \cong \overline{EA}$
(2) ∠ABF ≅ ∠CBD (4) $\overline{CF} \cong \overline{FA}$

To the right you see the triangle presented for this problem with markings indicating the result of the information given. The answer to this problem is based on the given statement that $\overline{BF}$ is a median. As such, choice 4 is correct. Line segments CF and FA will be congruent, as the median of a line segment divides the line into two congruent parts.



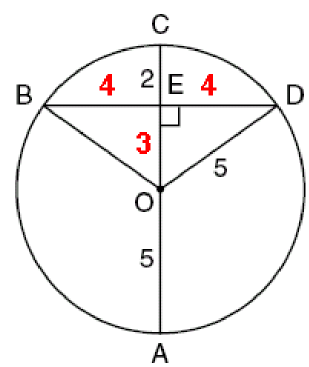
ANSWER: (4)

11) ***In the diagram below, circle O has a radius of 5, and CE = 2. Diameter $\overline{AC}$ is perpendicular to chord $\overline{BD}$ at E.***



What is the length of $\overline{BD}$?

(1) 12 (3) 8
(2) 10 (4) 4



The given circle has a radius of 5. This means that the measure of OC is also 5 as it is a radius as well. As you see at the right, OE will therefore equal 3. (OE+CE)=5
ΔOED is a right triangle. Once you know that one of its sides is 3, and its hypotenuse is 5, the other side has to be 4. You know this because you recall your 3, 4, 5 Pythagorean triple, or because you can use the Pythagorean Theorem to solve for the final leg.
AC is a diameter perpendicular to BD, a chord. It therefore bisects the chord. This means that the measure of BE will be 4 as well. BE + DE = 4 + 4 = 8

ANSWER: (3)

12) ***What is the equation of a line that passes through the point (– 3,–11) and is parallel to the line whose equation is 2x – y = 4?***

(1) $y = 2x + 5$ (3) $y = \frac{1}{2}x + \frac{25}{2}$

(2) $y = 2x - 5$ (4) $y = -\frac{1}{2}x - \frac{25}{2}$

Rather than doing this problem the usual way, let's make it a bit easier by eliminating some of the choices. The key word above is "parallel." Parallel lines will have equal slopes. You are looking for a line parallel to the line whose equation is 2x - y = 4. First let us determine its slope by transforming it into the form of y = mx + b, where m represents the slope.

2x - y = 4	Subtract 2x from both sides.
-y = -2x + 4	Divide both side by -1.
y = **2**x - 4	y=mx+b…..m=2 slope is therefore 2.

Choice 3 represents a line with a slope of 1/2; choice 4 with a slope of -(1/2). That leaves choices 1 and 2. Hopefully, one of them will have the point (-3,-11) as a solution set. Let's see which one.

Using the point (-3,-11).

x = -3 y = -11	x = -3 y = -11
Choice 1	**Choice 2**
y = 2x + 5	**y = 2x - 5**
-11 = 2(-3) + 5	-11 = 2(-3) - 5
-11 = -6 + 5	-11 = -6 -5
-11 = -1	-11 = -11
FALSE	**TRUE**

ANSWER: (2)

13) ***Line segment AB has endpoints A(2,– 3) and B(– 4,6). What are the coordinates of the midpoint of*** $\overline{AB}$ ***?***

(1) $(-2,3)$ (3) $(-1,3)$

(2) $\left(-1,1\frac{1}{2}\right)$ (4) $\left(3,4\frac{1}{2}\right)$

To find the x-coordinate of the midpoint, add the x-coordinates of the two given points and divide by 2.
(2) + (-4) = -2; now divide by 2 and your x-coordinate of the midpoint is **-1**

To find the y-coordinate, we follow the same procedure--add the two y-coordinates and divide the sum by 2.
(-3) + (6) = 3; now divide by 2 and your y-coordinate is $\frac{3}{2}$ or $\mathbf{1\frac{1}{2}}$.

ANSWER: (2)

14) ***What are the center and radius of a circle whose equation is $(x – A)^2 + (y – B)^2 = C$?***

(1) center = (A,B); radius = C

(2) center = $(-A,-B)$; radius = C

(3) center = (A,B); radius = $\sqrt{C}$

(4) center = $(-A,-B)$; radius = $\sqrt{C}$

Here's a little explanation or rather example. If Xc will be the x-coordinate at the center of a circle, and Yc will be the y-coordinate at the center of the circle, then the equation of the circle will be :
$(x - Xc)^2 + (y - YC)^2 = r^2$

Here are some examples:

$(x + 3)^2 + (x + 5)^2 = 6^2$	center: (-3, -5); radius = 6
$(x - 3)^2 + (x + 5)^2 = 36$	center: (+3, -5); radius = $\sqrt{36}$ or 6
$(x + 3)^2 + (x - 5)^2 = 35$	center: (-3, +5); radius = $\sqrt{35}$
$(x - 3)^2 + (x - 5)^2 = 29$	center: (+3, +5); radius = $\sqrt{29}$
$(x - A)^2 + (y - B)^2 = C$	**center: (+A, +B); radius = $\sqrt{C}$**

ANSWER: (3)

15) ***A rectangular prism has a volume of $3x^2 + 18x + 24$. Its base has a length of $x + 2$ and a width of 3. Which expression represents the height of the prism?***

(1) $x + 4$

(2) $x + 2$

(3) 3

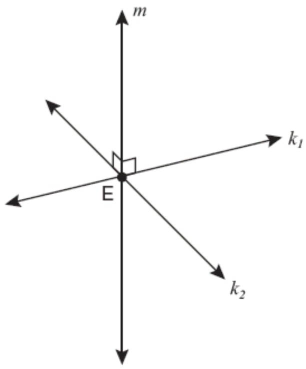
(4) $x^2 + 6x + 8$

The volume of a rectangular prism is found by finding the product of its length, width, and height. In this case, you are given the volume and also the length and width. You are missing the height. Let's factor the volume and see what its factors are:

$3x^2 + 18x + 24$	Factor a 3.
$3 (x^2 + 6x + 8)$	Now factor the trinomial.
$3 (x +2) (x + 4)$	

Choice1 is your answer. Your width is 3, the length is x + 2, and the height must be x + 4, as the product of these factor equals the required volume. **ANSWER: (1)**

16) ***Lines k1 and k2 intersect at point E. Line m is perpendicular to lines k1 and k2 at point E.***



Which statement is always true?

(1) Lines k_1 and k_2 are perpendicular.

(2) Line m is parallel to the plane determined by lines k_1 and k_2.

(3) Line m is perpendicular to the plane determined by lines k_1 and k_2.

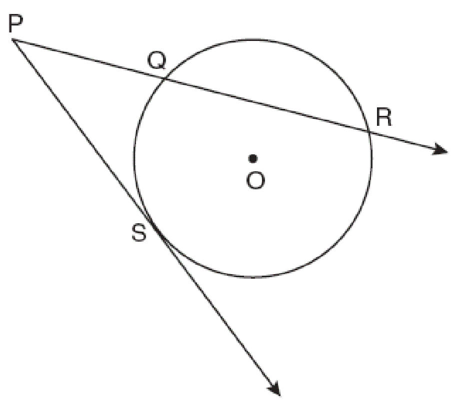
(4) Line m is coplanar with lines k_1 and k_2.

If a line is perpendicular to each of two intersecting lines at their point of intersection, the line is perpendicular to the plane determined by these two intersecting lines.
Choice 3 is the correct answer. **ANSWER: (3)**

17) ***In the diagram below, $\overline{PS}$ is a tangent to circle O at point S, $\overline{PQR}$ is a secant, PS = x, PQ = 3, and PR = x + 18.***

What is the length of $\overline{PS}$?

(1) 6 (3) 3
(2) 9 (4) 27

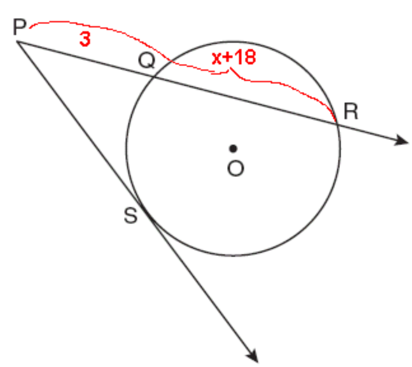


(Not drawn to scale)

To the right is a copy of the diagram with the given information shown in red. In the case given, the length of the tangent, will be the mean proportional between the length of the secant and its external segment.
In other words:

$\frac{PR}{PS} = \frac{PS}{PQ}$ or $(PS)^2 = PR \times PQ$

Substitute the known values and solve for x.



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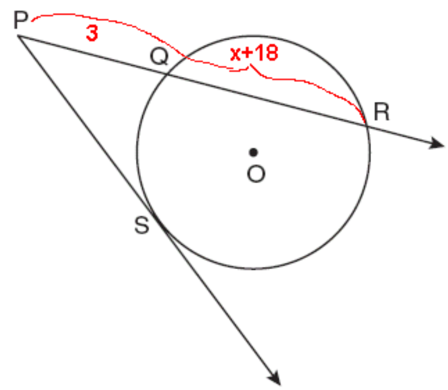
$(PS)^2 = PR \times PQ$

$x^2 = (x + 18)(3)$ Use distributive property.
$x^2 = 3x + 54$ Subtract 3x from both sides.
$x^2 - 3x = 54$ Subtract 54 from both sides.
$x^2 - 3x - 54 = 0$ Factor the trinomial.
$(x - 9)(x + 6) = 0$ Set factors equal to 0 and solve.

$x - 9 = 0$ $x + 6 = 0$
$\mathbf{x = 9}$ $x = -6$ Reject the negative value as length can't be negative.



(Not drawn to scale)

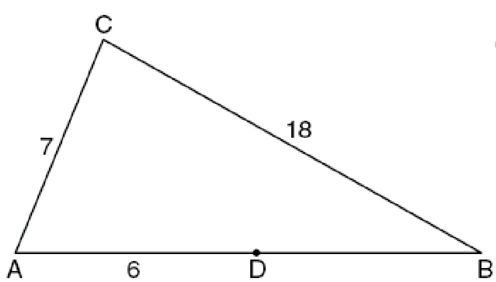
The length of tangent PS is 9. **ANSWER: (2)**

18) ***A polygon is transformed according to the rule: (x, y) (x + 2, y). Every point of the polygon moves two units in which direction?***

(1) up ***(3) left***
(2) down ***(4) right***

Check out question 3 on this regents which deals with translations. This problem again describes a translation. The x+2 indicates that every x-coordinate is moved two units in the plus direction along the x-axis.
This is a move of two units to the right. **ANSWER: (4)**

19) ***In the diagram below of △ABC, D is a point on $\overline{AB}$, AC = 7, AD = 6, and BC = 18.***



(Not drawn to scale)

The length of $\overline{DB}$ could be

(1) 5 (3) 19
(2) 12 (4) 25

To answer this question, you have to know that the sum of any two sides of a triangle has to be greater than the third side. First I will eliminate the wrong choices using this concept.

If DB = 5, then you will have sides of 11, 7, and 18. 11 + 7 is not greater than 18.
If DB = 19, you will have sides of 25, 7, and 18. 7 + 18 is not greater than 25.
If DB = 25, you will have sides of 31, 7, and 18. 7 + 18 is not greater than 31.
If DB = 12, your sides will be18, 7, and 18. The sum of any two sides **will be greater** than the third side. **ANSWER: (2)**

20) ***The diameter of a circle has endpoints at (– 2,3) and (6,3). What is an equation of the circle?***
(1) $(x - 2)^2 + (y - 3)^2 = 16$
(2) $(x - 2)^2 + (y - 3)^2 = 4$
(3) $(x + 2)^2 + (y + 3)^2 = 16$
(4) $(x + 2)^2 + (y + 3)^2 = 4$

This is a multistep problem. In order to know the equation of a circle, you have to know the coordinates of the circle's center, and the circle's radius.
The center of the above circle can easily be found because you are given the coordinates of the endpoints of its diameter. The center of the circle will be the midpoint of the diameter. Now review problem 13 on this Regents. To find the coordinates of the midpoint, the following will be true.

The x-coordinate of the midpoint will be the sum of the x-coordinates of the endpoints divided by 2. $(-2+ 6) \div 2 = 4 \div 2 =$ **2**

The y-coordinate of the midpoint will be the sum of the y-coordinates of the endpoints divided by 2. $(3 + 3) \div 2 = 6 \div 2 =$ **3**

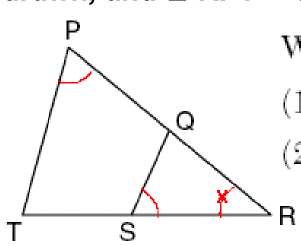
The coordinates of the circle's center are **(2,3).** At this point you can eliminate choices 3 and 4, leaving only choices 1 and 2 as possible answers.

You can now use the distance formula to determine the length of the radius, but it may be easier to use the given set of coordinates to see if they are a solution set for choice 1 or 2. Let's use the given endpoint (6,3) and choice1.

$(x - 2)^2 + (y - 3)^2 = 16$ Substitute for x and y.
$(6 - 2)^2 + (3 - 3)^2 = 16$ Simplify.
$4^2 + 0^2 = 16$ Continue simplifying.
$16 = 16$ The point (6,3) is a solution set. This is the equation of the required circle.

ANSWER: (1)

21) ***In the diagram below of ΔPRT, Q is a point on $\overline{PR}$, S is a point on $\overline{TR}$, $\overline{QS}$ is drawn, and ∠ RPT ≅ ∠ RSQ.***



Which reason justifies the conclusion that $\triangle PRT \sim \triangle SRQ$?

(1) AA
(2) ASA
(3) SAS
(4) SSS

Above is the diagram presented on the Regents with a modification. I have indicated in red the two angles that are congruent. Also notice that ∠R is an angle in both ΔPRT and ΔSRQ You are asked which choice proves the similarity of the two triangles. The answer is choice1, AA. Whenever you have two angles of one triangle congruent to two angles of another triangle, in this case ΔPRT and ΔSRQ, the two triangles are similar.

ANSWER: (1)

22) ***The lines 3y + 1 = 6x + 4 and 2y + 1 = x – 9 are***

(1) parallel
(2) perpendicular
(3) the same line
(4) neither parallel nor perpendicular

First let's transpose each line into the form y=mx+b, where m represents the slope of the line. If the lines are parallel their slopes will be equal. If the lines are perpendicular their slopes will be negative reciprocals.

$3y + 1 = 6x + 4$	Subtract 1 from both sides.	$2y + 1 = x - 9$	Subtract 1 from both sides
$3y = 6x - 3$	Divide both sides by 3.	$2y = x - 10$	Divide both sides by 2.
$y = 2x - 1$	**Slope = 2**	**$y = \frac{1}{2}x - 5$**	**Slope = $\frac{1}{2}$**

The slopes are not equal, nor are they negative reciprocals. ANSWER: (4)

23) ***The endpoints of $\overline{AB}$ are A(3,2) and B(7,1). If $\overline{A''B''}$ is the result of the transformation of $\overline{AB}$ under $D_2 \circ T_{-4,3}$ what are the coordinates of A" and B" ?***

(1) A" (–2,10) and B" (6,8)
(2) A" (–1,5) and B" (3,4)
(3) A" (2,7) and B" (10,5)
(4) A" (14 ,– 2) and B" (22,– 4)

What you are being asked to do here is known as a "composition of transformations." In essence what you will be doing is performing one transformation following another.

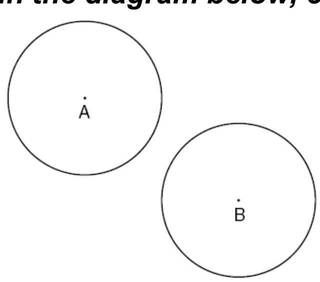
$D_2 \circ T_{-4,3}$ is read as a dilation of 2 **following** a translation of -4,3.

$D_2 \circ T_{-4,3}$ means that the first transformation to be completed will be $T_{-4,3}$ -- a translation. The second transformation will be D_2 -- a dilation of 2.

Under the translation of $T_{-4,3}$ **A(3,2)** becomes A'(3-4,2+3) or **A'(-1, 5)**
B(7,1) becomes B'(7-4,1+3) or **B'(3, 4)**
You now have the new coordinates A' and B'.
Let's transform them using a dilation of 2.

Under a dilation of D_2 **A'(-1, 5)** becomes A"(-1•2, 5•2) or **A"(-2, 10)**
B'(3, 4) becomes B"(3•2, 4•2) or **B"(6, 8)** **ANSWER: (1)**

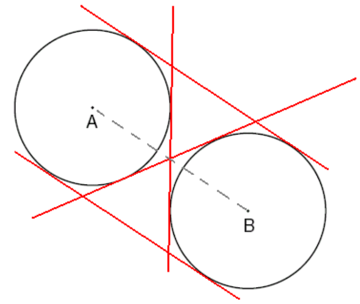
24) ***In the diagram below, circle A and circle B are shown.***



What is the total number of lines of tangency that are common to circle A and circle B?

(1) 1
(2) 2
(3) 3
(4) 4

A tangent to a circle is a line which intersects the circle a only one point. To the right you see lines of tangency that are common to both circles. There are two common internal tangents that form the shape of an X, and intersect their line of center. There are also two common external tangents which do not intersect their line of centers. As you can see, there are a total of 4 lines of tangency common to both circles. **ANSWER: (4)**

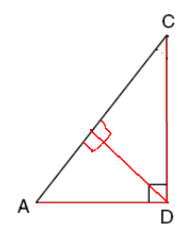


25) ***In which triangle do the three altitudes intersect outside the triangle?***

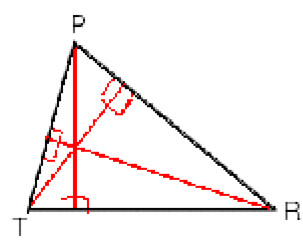
(1) a right triangle
(2) an acute triangle
(3) an obtuse triangle
(4) an equilateral triangle

The altitude of a triangle is the perpendicular line segment drawn from any vertex of the triangle to the opposite side (extended if necessary). In other words, an altitude of a triangle may be outside the triangle.

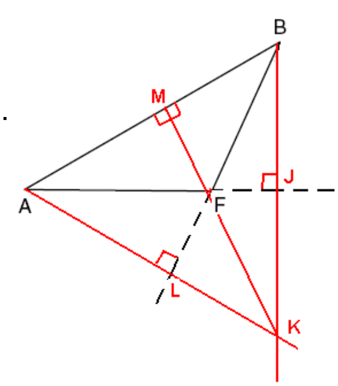
As you see at the right, the altitudes of a right triangle will intersect on one of the vertices of the triangle.



Beneath the right triangle, there is an acute triangle. Notice that its altitudes all intersect within the triangle. The same will be true for an equilateral triangle, as it is acute as well.



The final triangle you see is ABF. It is an obtuse triangle as its angle F is greater than 90 degrees. In order to drop altitudes from vertex B, side AF had to be extended. To drop an altitude from point A, side BF was extended. The altitudes drawn in red all intersect at point K. Point K is outside the triangle.



ANSWER: (3)

26) ***Two triangles are similar, and the ratio of each pair of corresponding sides is 2 : 1. Which statement regarding the two triangles is not true?***

(1) Their areas have a ratio of 4 : 1.
(2) Their altitudes have a ratio of 2 : 1.
(3) Their perimeters have a ratio of 2 : 1.
(4) Their corresponding angles have a ratio of 2 : 1.

The corresponding angles of similar triangles will be congruent. It is their sides that maintain the ratio. Choice 4 is therefore an incorrect statement. **ANSWER: (4)**

27) ***What is the measure of an interior angle of a regular octagon?***

(1) 45 º ***(3) 120 º***
(2) 60 º ***(4) 135 º***

Here is one way to do this problem. You do have to know that an octagon is an eight-sided polygon. Since it is a regular octagon, its sides, as well as its angles, will be congruent.
The sum of the measures of the angles of any triangle is 180 degrees. If you experiment, you will see that any polygon that is not already a triangle can be divided into triangles. A 4 sided polygon can be divided into 2 triangles. A 5 sided polygon (pentagon) can be divided into 3 triangles. The number of triangles contained in the particular polygon will always be 2 less than the number of its sides. So in our case of the octagon which has 8 sides, you will have a total of 8-2 or 6 triangles. As stated earlier, the sum of the degree measures of a triangle is 180, so in the case of our octagon the measure of the sum of the angles will equal 6 times 180, or 1080 degrees. Also as stated earlier, the angles of a regular polygon will be equal. Therefore, the measure of each interior angle of the regular octagon will equal 1080 divided by 8 (the number of its angles) or 135 degrees. **ANSWER: (4)**

28) ***What is the slope of a line perpendicular to the line whose equation is 5x + 3y = 8?***

(1) $\frac{5}{3}$ (3) $-\frac{3}{5}$

(2) $\frac{3}{5}$ (4) $-\frac{5}{3}$

As stated in the explanation to problem 12 of this sample regents, perpendicular lines will have slopes that are negative reciprocals. First let's transpose the given equation into the form of y = mx + b, where m represents the slope.

$5x + 3y = 8$	Subtract 5x from both sides.
$3y = -5x + 8$	Divide both sides by 3.
$y = -\frac{5}{3}x + \frac{8}{3}$	

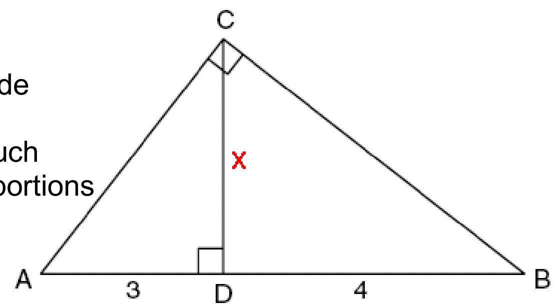
The slope is $-\frac{5}{3}$ **$-\frac{5}{3}$ and $\frac{3}{5}$** are **negative reciprocals** as their product is -1.

ANSWER: (2)

PART II

29) ***In the diagram below of right triangle ACB, altitude $\overline{CD}$ intersects $\overline{AB}$ at D. If AD = 3 and DB = 4, find the length of $\overline{CD}$ in simplest radical form.***

This problem involves your setting up the correct proportion involving right triangles. Shown at the right, the altitude which I have labeled x is drawn to the hypotenuse of right triangle ACB. In such a case, the following is one of the proportions that are true:
The length of the altitude is the mean proportional between the lengths of the segments of the hypotenuse.
In other words, **AD : CD = CD : DB**



You may prefer to write it as follows: $\mathbf{\frac{AD}{CD} = \frac{CD}{DB}}$

Now substitute x for CD and the lengths of AD and DB, and continue to solve.
Remember the product of the means equals the product of the extremes.

$\mathbf{\frac{AD}{CD} = \frac{CD}{DB}}$ Substitute.

$\frac{3}{x} = \frac{x}{4}$ Product of means equals product of extremes.

$x^2 = (3)(4)$ Multiply.
$x^2 = 12$ Take square root of both sides.
$x = \sqrt{12}$ Now simplify the radical.

$\mathbf{\sqrt{12}} = \sqrt{4} \cdot \sqrt{3}$ $\mathbf{= 2\sqrt{3}}$ **ANSWER: $2\sqrt{3}$**

30) ***The vertices of ΔABC are A(3,2), B(6,1), and C(4,6). Identify and graph a transformation of ΔABC such that its image, ΔA'B'C', results in $\overline{AB} \parallel \overline{A'B'}$.***

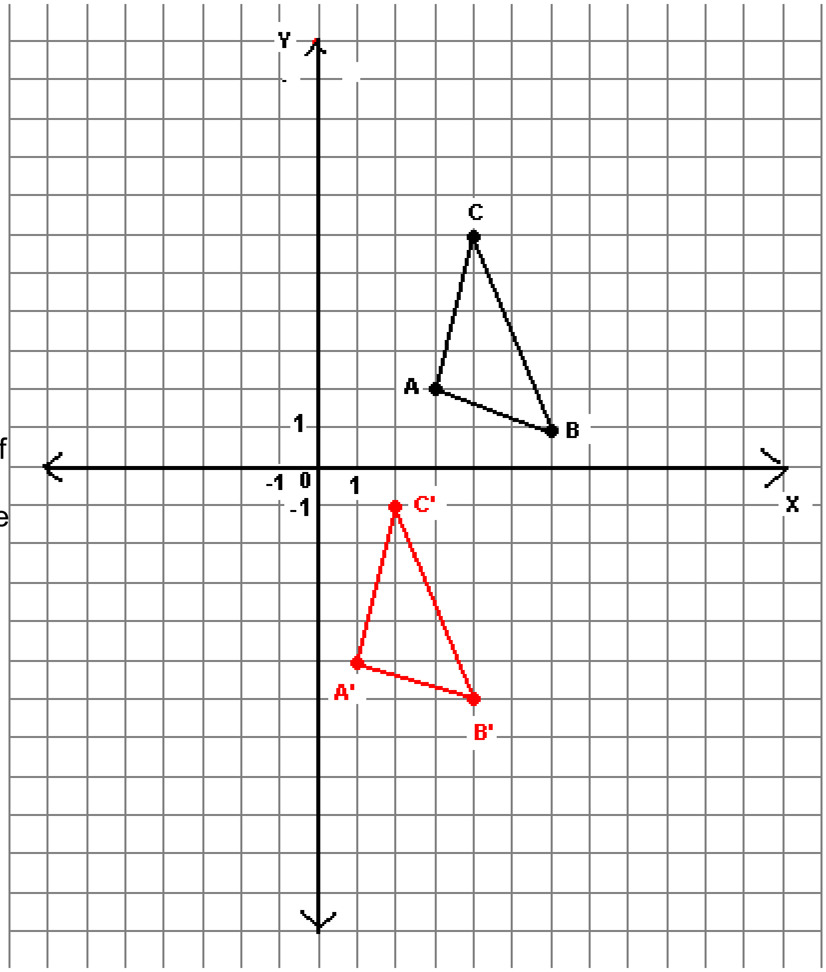
1…Graph triangle ABC.

2…Select **any** point as A', and graph it on the same coordinate axis.

3…Determine the rule of translation used to get from A to A'.

Go to the next page to see how to determine the rule of translation. By the way, the reason you want to use a translation as opposed to a line reflection is because the corresponding line segments of the image obtained when using a translation will be parallel to the segments of the original figure. You could also have used either a dilation or reflection through the origin and the corresponding line segments would also have remained parallel.

To determine the rule of translation used to get from A to A', count the number of units from point A to A'. In the example at the right, first you move 2 units to the left, and then 7 units down. Using this pattern, B' will be the image of B, and C' will be the image of C. That is the red triangle you see at the right. Line segments AB and A'B' will automatically be parallel as their slopes will be equal. Their slopes happen to equal -1/3 -- a movement of 1 down in the y-axis, followed by 3 to the right in the x-axis. For slope we first move in the y and then the x, because slope is the difference in the y-coordinates divided by the difference in the x-coordinates.



You notice that to get from A to its image A' you have moved 2 units to the left, followed by 7 units to the down. That is a translation of $T_{-2,-7}$. To move from B to B', and from C to C' you have completed that same translation.
Look at it mathematically:
A(3,2) after a translation of $T_{-2,-7}$ becomes A'(3-2, 2-7) or **A'(1, -5)**.
B(6,1) becomes B'(6-2, 1-7) or **B'(4, -6)**.
C(4,6) becomes C'(4-2, 6-7) or **C'(2, -1)**.

31) ***The endpoints of $\overline{PQ}$ are P(– 3,1) and Q(4,25). Find the length of $\overline{PQ}$.***

In order to find the length of a line segment when given the coordinates of its endpoints, you can use the distance formula.

$$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

where d represents the distance, X1 and y1 are the coordinates of the first point, while x2 and y2 are the coordinates of the second point.

Using your given points: $x_1= -3$ $x_2= 4$ $y_1= 1$ $y_2= 25$

Substituting you get: $d = \sqrt{(-3-4)^2+(1-25)^2}$ Simplify.

$= \sqrt{(-7)^2+(-24)^2}$ Continue simplifying.

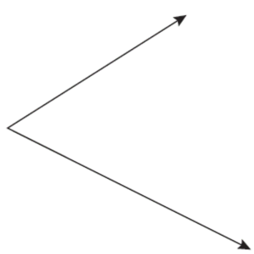
$= \sqrt{49+576}$ Simplify.

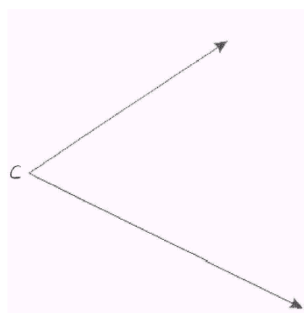
$= \sqrt{625}$ Find square root of 625.

d = 25

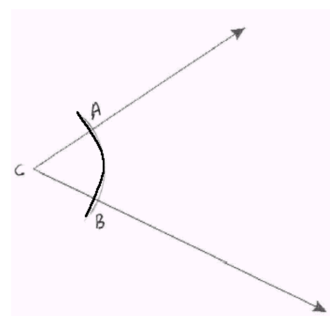
ANSWER: The length of $\overline{PQ}$ is 25.

32) ***Using a compass and straightedge, construct the bisector of the angle shown below. [Leave all construction marks.]***

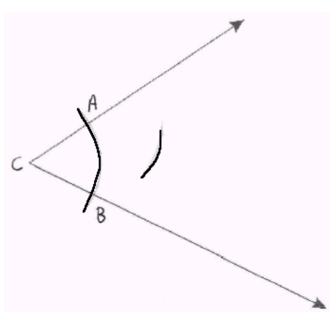




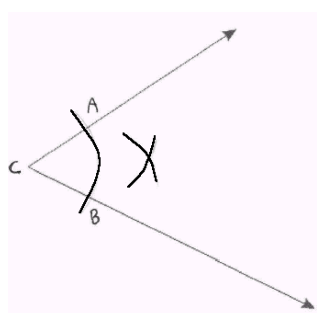
(1)



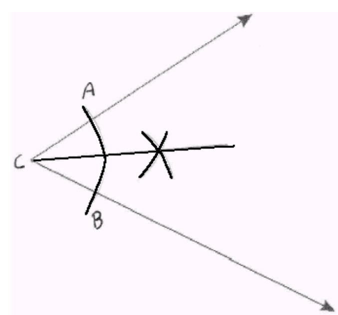
(2)



(3)



(4)



(5)

Begin by putting compass point at C and drawing the arc you see in diagram 2. Keep same radius, put compass point at A and draw the arc you see in diagram 3. Put compass point at B and draw an arc intersecting previous arc as in diagram 4. Finally, complete the construction by drawing a line from C through the intersecting arcs as you see in final diagram 5.

33) ***The volume of a cylinder is 12,566.4 cm^3. The height of the cylinder is 8 cm. Find the radius of the cylinder to the nearest tenth of a centimeter.***

You are presented with the formula for the volume of a cylinder on your reference sheet. It is ***V = Bh*** where *B* is the area of the base
In this problem you are told that the height is 8. You can easily figure out the area of the base (B) of this cylinder.

V = Bh	Substitute.
12,566.4 = B(8)	Divide both sides by 8.
1570.8 = B	**The base of the cylinder has an area of 1570.8 cm.**

The base of a cylinder is a circle. The formula for the area of a circle is:
$A = \pi r^2$ where r represents the radius of the circle. You are being asked to determine the radius.

$A = \pi r^2$	Substitute using the area you just obtained.
$1570.8 = \pi r^2$	Divide both sides by π. Use the "pi" key on your calculator.
$500.0011692 = r^2$	Take square root of both sides.
22.36070592 = r	Round to nearest tenth.
22.4	

ANSWER: The radius to the nearest tenth is 22.4 cm.

34) ***Write a statement that is logically equivalent to the statement "If two sides of a triangle are congruent, the angles opposite those sides are congruent."***

Identify the new statement as the converse, inverse, or contrapositive of the original statement.

The answer to the last question will be the contrapositive.
The contrapositive of a statement will always be logically equivalent to the original statement.

One way of finding the contrapositive, is to first find the inverse. Then find the converse of that inverse.

Original statement: If two sides of a triangle are congruent, the angles opposite those sides are congruent.
Inverse: If two sides of a triangle are not congruent, the angles opposite those sides are not congruent.
Contrapositive: If the angles opposite two sides of a triangle are not congruent, the two sides are not congruent.

This final statement is the contrapositive of the original statement and is its logical equivalent. That means that the truth values of the statements will always be the same.

ANSWER: If the angles opposite two sides of a triangle are not congruent, the two sides are not congruent. This statement is the contrapositive of the original statement.

PART III

35) ***On the set of axes below, graph and label ΔDEF with vertices at D(–4,–4), E(–2,2), and F(8,–2).***

If G is the midpoint of $\overline{EF}$ and H is the midpoint of $\overline{DF}$, state the coordinates of G and H and label each point on your graph.

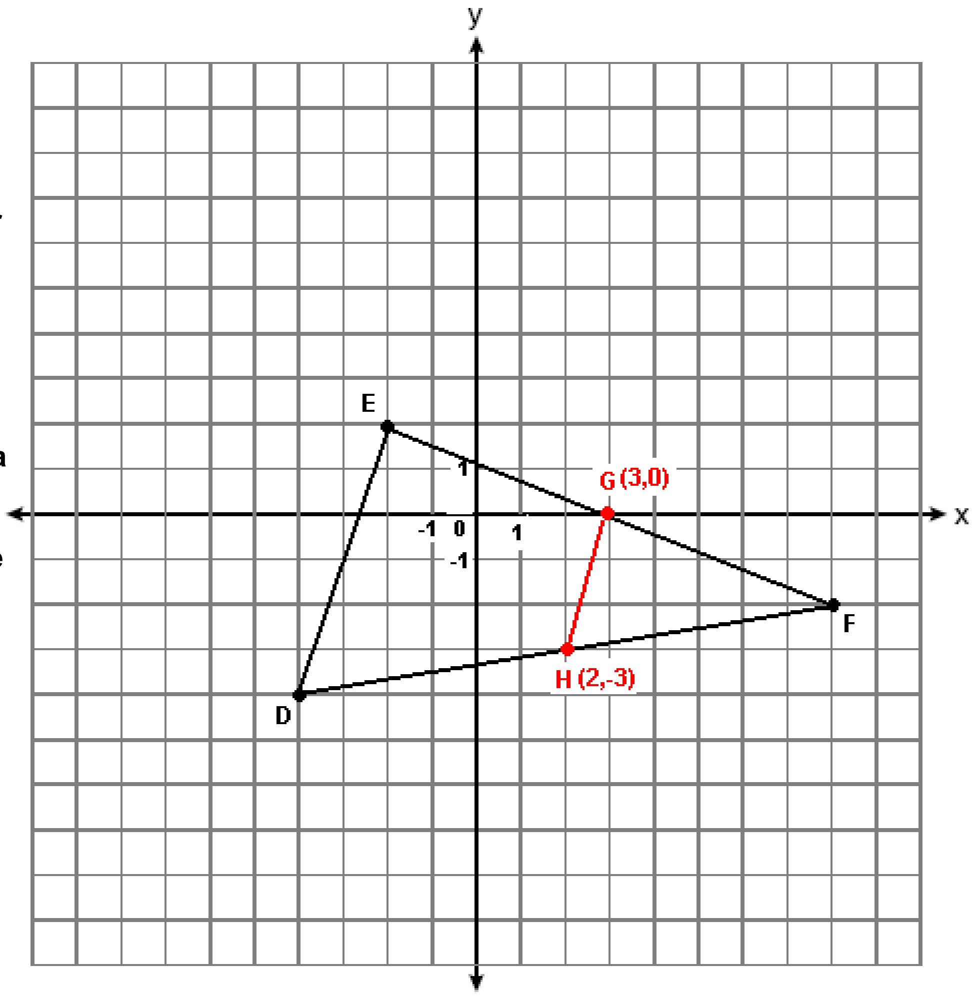
Explain why $\overline{GH} \parallel \overline{DE}$.

Review problem 13 on this regents to refresh your memory on how to determine the coordinates of the midpoint of a line segment, when the coordinates of the endpoints are given.
In the case above, **to find G**, the midpoint of $\overline{EF}$: E(-2,2), F(8,-2)
Add the 2 x-coordinates, divide by 2. Add the 2 y-coordinates, divide by 2.
(-2 + 8) / 2 = 6 / 2 = **3** (2 + -2) / 2 = 0 / 2 = **0** **G (3,0)**

To find H, the midpoint of $\overline{DF}$: D(-4,-4), F(8,-2)
(-4+ 8) / 2 = 4 / 2 = **2** (-4+-2) / 2 = -6 / 2 = **-3** **H(2,-3)**

$\overline{GH} \parallel \overline{DE}$

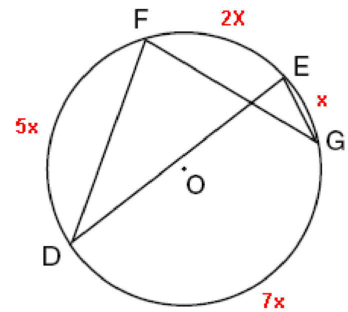
They are parallel because their slopes are equal.
The line segment that connects the midpoints of two sides of a triangle, will always be parallel to the third side.



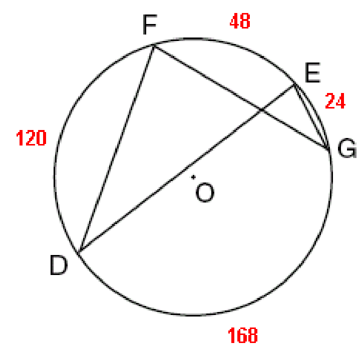
36) In the diagram below of circle O, chords $\overline{DF}$, $\overline{DE}$, $\overline{FG}$, and $\overline{EG}$ are drawn such that $m\overset{\frown}{DF} : m\overset{\frown}{FE} : m\overset{\frown}{EG} : m\overset{\frown}{GD} = 5 : 2 : 1 : 7$. Identify one pair of inscribed angles that are congruent to each other and give their measure.

To the right you see the circle. I have marked in red the measures of the arcs in terms of x, based on the information given.
You know that the sum of the measures of the arcs of a circle equals 360 degrees. You can now solve for x and determine the measure of each given arc.

5x+2x+x+7x=360	Simplify.
15x = 360	Divide both sides by 15.
x = 24	



Once you know that x is 24, you know the measures of the arcs. To the right you again see the circle, but this time with the measures of the arc. (Once you know the value of x, simply substitute it for x. That is why the arc that is 5x becomes 5 (24) which is 120).

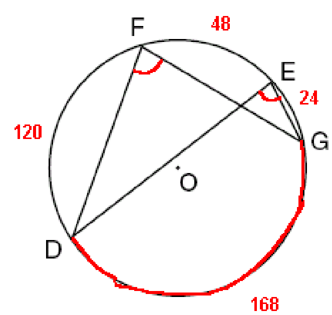


You are asked to identify a pair of congruent inscribed angles. An inscribed angles is an angle whose vertex is on the circumference of the circle, and whose sides contain chords of the circle. Some inscribed angles of the circle at the right are:

∠DFE, ∠DEG, ∠EGF, and ∠EDF.

You can notice at the right, that ∠F and ∠E both intercept the same arc, $\overset{\frown}{GD}$, whose measure is 168.
Inscribed angles that intercept congruent arcs will be congruent.
The measure of an inscribed angle will equal one-half the intercepted arc.
In our case since the arc $\overset{\frown}{GD}$ measures 168,

∠F and ∠E will both equal half of that or 84 degrees.



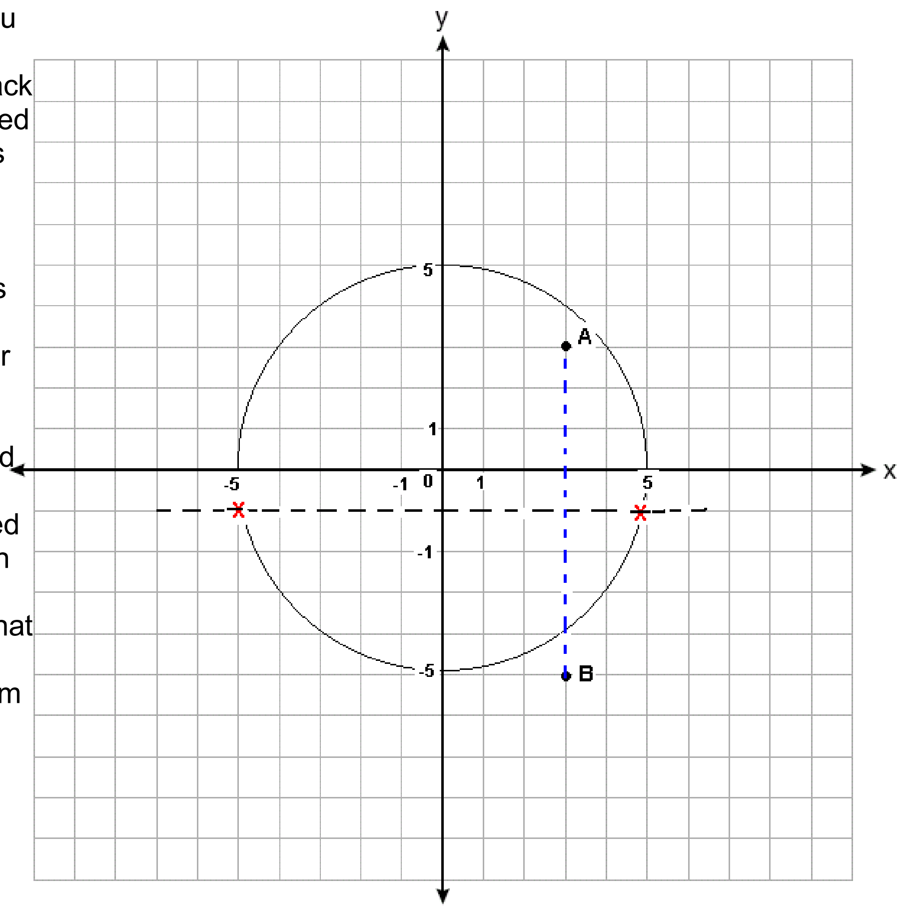
ANSWER: ∠F and ∠E are a pair of congruent inscribed angles.
Their measure is 84º.

(Alternate solution: Angles D and G are congruent. They intercept the same arc-- arc FE, and will be equal to half of that arc. Angles D and G equal 24 degrees, which is half of 48--the measure of arc FE.)

37) ***A city is planning to build a new park. The park must be equidistant from school A at (3,3) and school B at (3,– 5). The park also must be exactly 5 miles from the center of town, which is located at the origin on the coordinate graph. Each unit on the graph represents 1 mile.***

On the set of axes below, sketch the compound loci and label with an X all possible locations for the new park.

To the right you see schools A and B. The black horizontal dotted line represents the locus of points equidistant from both cities. It is the perpendicular bisector of the line segment that connects A and B, the vertical light blue dotted line you see on the set of axis. Any point on that line would be equidistant from A and B.



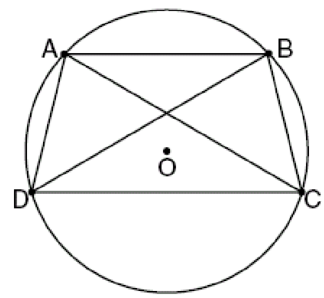
The circle represents the locus of points 5 units from the center of the city, in this case, the origin. Any point on the circle is 5 units from the origin.

The two red X's represent the two possible locations for the park. They are at the locations where all given conditions are satisfied. They are at the intersection of the locus of points equidistant from A and B, and the locus of points 5 units from the origin.

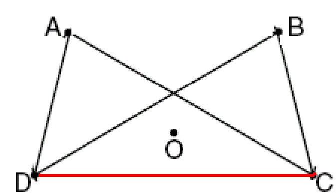
PART IV

38) ***In the diagram below, quadrilateral ABCD is inscribed in circle O, $\overline{AB} \parallel \overline{DC}$, and diagonals $\overline{AC}$ and $\overline{BD}$ are drawn.***

Prove that ΔACD ≅ ΔBDC.

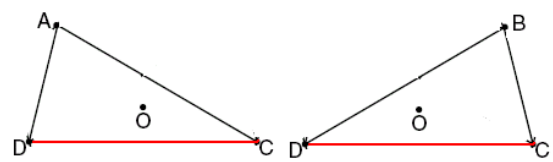


To the right you see the diagram presented on the Regents. It involves what is known as overlapping triangles.

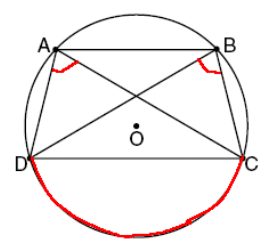


Beneath the first diagram, you see the two triangles you are asked to prove congruent. Side $\overline{DC}$, which is outlined in red, is a side common to both triangles.

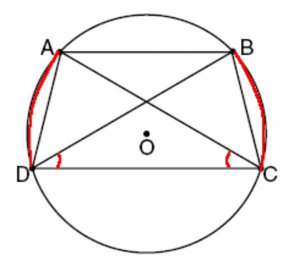
And finally to the right, you see both triangles individually.



Your plan will be to prove the triangles congruent by AAS. You will be able to prove two angles of one triangle and the side opposite one of these angles, congruent to two angles and the side opposite one of these angles in the other triangle. The main concept used will be the one you already used in problem 36 of this Regents, angles that intercept congruent arcs (or the same arc) are themselves congruent.



To the right you again see the original diagram. This time you see in red that both ∠DAC and ∠CBD intercept the same arc, $\overset{\frown}{DC}$, and are therefore congruent.



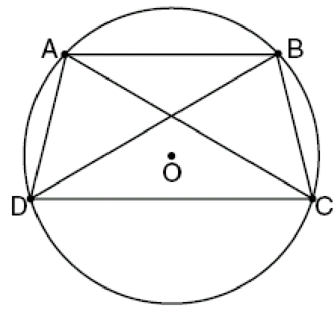
In addition, since chords $\overline{AB}$ and $\overline{DC}$ are parallel, $\overset{\frown}{AD}$ and $\overset{\frown}{BC}$ are equal in measure, because parallel chords intercept equal arcs. Now you know that the two angles you see marked in red in the final diagram, ∠BDC and ∠ACD, are congruent because they intercept equal arcs.

The formal proof appears on the next page…

Here is the question again:

In the diagram below, quadrilateral $ABCD$ is inscribed in circle O, $\overline{AB} \parallel \overline{DC}$, and diagonals $\overline{AC}$ and $\overline{BD}$ are drawn.

Prove that $\triangle ACD \cong \triangle BDC$.



Given: Quadrilateral ABCD is inscribed in circle O.

$\overline{AB} \parallel \overline{DC}$

Prove: $\triangle ACD \cong \triangle BDC$

Statements	Reasons
1. $\overline{AB} \parallel \overline{DC}$	1. Given
2. $\overset{\frown}{AD} = \overset{\frown}{BC}$	2. Parallel lines which intersect a circle intercept equal arcs on that circle.
3. ∠ACD ≅ ∠BDC (a. ≅ a.)	3. Angles that intercept congruent arcs are congruent.
4. ∠DAC ≅ ∠CBD (a. ≅ a.)	4. Angles that intercept the same arc are congruent.
5. $\overline{DC} \cong \overline{DC}$ (s. ≅ s.)	5. Reflexive Property
6. ΔACD ≅ ΔBDC	6. a.a.s. ≅ a.a.s.

One final time, here is the diagram again, this time with the two triangles and what was proven marked in red.