

- 5) **The local ice cream stand offers three flavors of soft-serve ice cream: vanilla, chocolate, and strawberry-two types of cone: sugar and wafer; and three toppings: sprinkles, nuts, and cookie crumbs. If Dawn does not order vanilla ice cream, how many different choices can she make that have one flavor of ice cream, one type of cone, and one topping?**
 (1) 7 (2) 8 (3) 12 (4) 18

To find the answer, you will use the Counting Principle. If one activity can occur in A ways, and another activity can occur in B ways, then both activities can occur in the order given in A times B ways.

Ice cream flavors - 3. **WAIT!** Dawn does not order vanilla, so there are only 2 flavors.

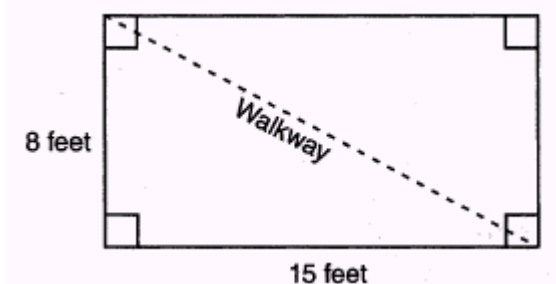
Types of cones - 2

Toppings - 3

Based on the above, using the counting principle, the answer will be (2)(2)(3) or 12.

ANSWER: (3)

- 6) **Nancy's rectangular garden is represented in the diagram below.**



If a diagonal walkway crosses her garden, what is its length, in feet?

- (1) 17 (3) $\sqrt{161}$
 (2) 22 (4) $\sqrt{529}$

Whenever you are presented with a right triangle and are given two of its sides, you can always determine the third side by using the Pythagorean Theorem: The hypotenuse squared, equals the sum of the square of the legs. In the case above, the walkway is the hypotenuse. Set up your equation and solve.

$$x^2 = 8^2 + 15^2 \quad \text{Simplify.}$$

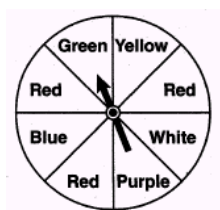
$$x^2 = 64 + 225 \quad \text{Simplify.}$$

$$x^2 = 289 \quad \text{Find square root.}$$

$$x = \sqrt{289} = 17$$

ANSWER: (1)

- 7) **The spinner below is divided into eight equal regions and is spun once. What is the probability of not getting red?**



- (1) $\frac{3}{5}$ (3) $\frac{5}{8}$
 (2) $\frac{3}{8}$ (4) $\frac{7}{8}$

Three of the regions are labeled RED. That means that the probability of "yes" getting RED is $\frac{3}{8}$. The probability, therefore of not getting red is $\frac{5}{8}$. $\frac{3}{8} + \frac{5}{8} = 1$. The probability of an event happening plus the probability of the event not happening always equals 1. **ANSWER: (3)**

- 8) **Which relationship can best be described as causal?**
 (1) *height and intelligence*
 (2) *shoe size and running speed*
 (3) *number of correct answers on a test and test score*
 (4) *number of students in a class and number of students with brown hair*

A causal relationship is one where one event can be considered a direct consequence, cause, of the other event. Choice 3 is your answer. The number of correct answers on a test, does indeed determine the test score. **ANSWER: (3)**

- 9) Solve for x: $\frac{3}{5}(x + 2) = x - 4$
 (1) 8 (2) 13 (3) 15 (4) 23

$\frac{3}{5}(x + 2) = x - 4$	Multiply all terms by the denominator 5.
$3(x + 2) = 5x - 20$	Use the distributive property.
$3x + 6 = 5x - 20$	Subtract 3x from both sides.
$6 = 2x - 20$	Add 20 to both sides.
$26 = 2x$	Divide both sides by 2.
$13 = x$	

ANSWER: (2)

- 10) **Erica is conducting a survey about the proposed increase in the sports budget in the Hometown School District. Which survey method would likely contain the most bias?**
 (1) *Erica asks every third person entering the Hometown Grocery Store.*
 (2) *Erica asks every third person leaving the Hometown Shopping Mall this weekend.*
 (3) *Erica asks every fifth student entering Hometown High School on Monday morning.*
 (4) *Erica asks every fifth person leaving Saturday's Hometown High School football game.*

Choice number 4, surveying people who attended a football game, would likely contain the most bias, and be the least objective. The odds are that individuals attending a football game would certainly be interested in increasing the sports budget since they seem to be interested in sports. **ANSWER: (4)**

- 11) **Which equation represents a line parallel to the x-axis?**
 (1) $y = -5$ (2) $y = -5x$ (3) $x = 3$ (4) $x = 3y$

Choice 1 represents a line that will cross the y-axis at -5. In other words, it will be 5 units below the x-axis and parallel to it. **ANSWER: (1)**

- 12) **Given:**
 $A = \{\text{All even integers from 2 to 20, inclusive}\}$
 $B = \{10, 12, 14, 16, 18\}$
What is the complement of set B within the universe of set A?
 (1) $\{4, 6, 8\}$ (2) $\{2, 4, 6, 8\}$ (3) $\{4, 6, 8, 20\}$ (4) $\{2, 4, 6, 8, 20\}$

The **complement** of set B within the universe of set A consists of all the elements that are in set B but not in the universal set, in this case set A.

The universe or universal set is the main set being specified. In this problem, set A is your universe. It consists of the following numbers shown using set notation:

$$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

$$B = \{10, 12, 14, 16, 18\}$$

The complement of this subset would be all the elements in A that are not in B.

That would be the following numbers: 2, 4, 6, 8, 20.

In set notation this can be written as you see below.

$$\bar{B} = \{2, 4, 6, 8, 20\}$$

(Other symbols that can be used to denote the complement of B are B^c , B' or $\sim B$.)

ANSWER: (4)

- 13) **Which value of x is in the solution set of the inequality $-2(x - 5) < 4$?**
 (1) 0 (2) 2 (3) 3 (4) 5

Solve the inequality:

$$-2(x - 5) < 4 \quad \text{Use the distributive property.}$$

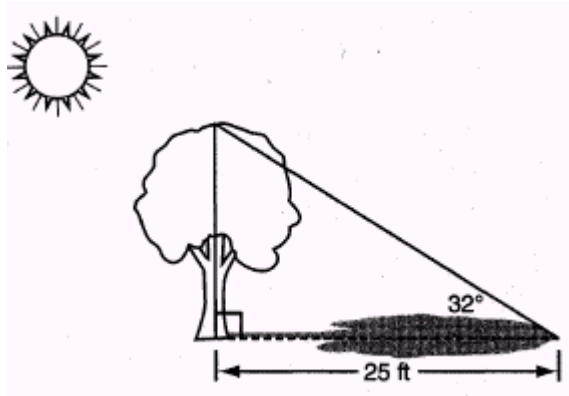
$$-2x + 10 < 4 \quad \text{Subtract 10 from both sides.}$$

$$-2x < -6 \quad \text{Divide by -2. (The inequality symbol switches direction).}$$

$$x > 3 \quad \text{Of the given choices, only choice 4 is correct.}$$

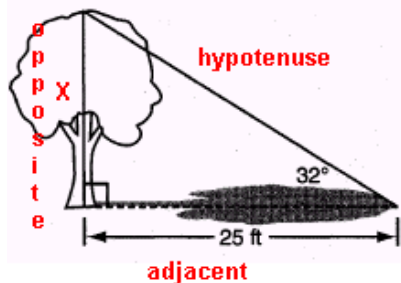
ANSWER: (4)

- 14) **A tree casts a 25-foot shadow on a sunny day, as shown in the diagram below.**



If the angle of elevation from the tip of the shadow to the top of the tree is 32° , what is the height of the tree to the *nearest tenth of a foot*?

- (1) 13.2 (3) 21.2
 (2) 15.6 (4) 40.0



Sine, cosine, and tangent are ratios involving the sides of a right triangle. You are asked for the height of the tree. Let X be that height, as you see in the diagram at the left, where the diagram is labeled relative to the 32 degree angle, the three trigonometric ratios are:

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan = \frac{\text{opposite}}{\text{adjacent}}$$

Relative to the 32 degree angle, the tree (marked x) is an opposite, the shadow is the adjacent. The tangent ratio uses the opposite and adjacent: $\text{tangent} = \frac{\text{opposite}}{\text{adjacent}}$.

Substitute the knowns:

$$\tan 32 = \frac{x}{25}$$

Multiply both sides by 25.

$$25 (\tan 32) = x$$

Use calculator to multiply. $25 * \tan(32) = 15.6217338$

$$x = 15.6217338$$

$x = 15.6$ to nearest tenth.

ANSWER: (2)

- 15) **What is the slope of the line that passes through the points $(-5,4)$ and $(15,-4)$?**

(1) $-\frac{2}{5}$

(3) $-\frac{5}{2}$

(2) 0

(4) undefined

Slope equals the change in y over the change in x .

Consider the first point to be (x_1, y_1) followed by (x_2, y_2) .

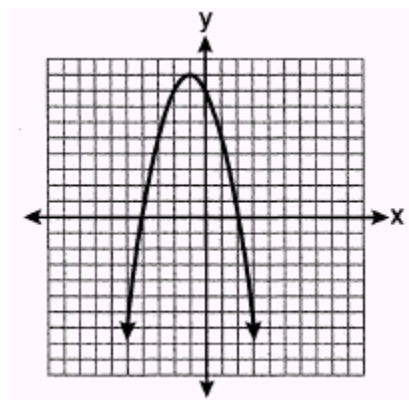
$$x_1 = -5 \quad y_1 = 4 \quad x_2 = 15 \quad y_2 = -4$$

$$\text{Slope} = m = \frac{y_1 - y_2}{x_1 - x_2} \quad \text{Substitute the given coordinates.}$$

$$m = \frac{4 - (-4)}{-5 - 15} = \frac{8}{-20} = -\frac{2}{5}$$

ANSWER: (1)

- 16) **The equation $y = -x^2 - 2x + 8$ is graphed on the set of axes below.**



Based on this graph, what are the roots of the equation $-x^2 - 2x + 8 = 0$?

(1) 8 and 0

(3) 9 and -1

(2) 2 and -4

(4) 4 and -2

The graph is that of a parabola. The roots of the equation as shown by the graph will be the points of its intersection with the x -axis.

It intersects the x -axis at two points: 2 and -4

ANSWER: (2)

- 17) What is the sum of $\frac{3}{2x}$ and $\frac{4}{3x}$ expressed in simplest form?
- (1) $\frac{12}{6x^2}$ (3) $\frac{7}{5x}$
 (2) $\frac{17}{6x}$ (4) $\frac{17}{12x}$

In order to add fractions we need a common denominator. In this case the common denominator can be $6x$.

Multiply the numerator and denominator by the same term to obtain that common denominator.

$$\frac{3}{2x} \cdot \frac{(3)}{(3)} + \frac{4}{3x} \cdot \frac{(2)}{(2)} = \frac{9}{6x} + \frac{8}{6x} = \frac{17}{6x}$$

ANSWER: (2)

- 18) Which value of x makes the expression $\frac{x^2 - 9}{x^2 + 7x + 10}$ undefined?
 (1) -5 (2) 2 (3) 3 (4) -3

A fraction will be undefined if its denominator equals 0.
 Set the denominator equal to 0 and solve for x .

$$x^2 + 7x + 10 = 0 \quad \text{Factor.}$$

$$(x + 5)(x + 2) = 0 \quad \text{Set factors = 0 and solve for } x.$$

$$x + 5 = 0 \quad \text{Subtract 5.} \quad x + 2 = 0 \quad \text{Subtract 2.}$$

$$x = -5 \quad \quad \quad x = -2$$

ANSWER: (1)

- 19) Which relation is not a function?
 (1) $\{(1,5), (2,6), (3,6), (4,7)\}$
 (2) $\{(4,7), (2,1), (-3,6), (3,4)\}$
 (3) $\{(-1,6), (1,3), (2,5), (1,7)\}$
 (4) $\{(-1,2), (0,5), (5,0), (2,-1)\}$

A function is a relation in which every element in the domain will correspond to exactly one element in the range. The domain consists of your x -coordinates; the range consists of your y -coordinates. (x comes before y , and d comes before r). When looking at a set of ordered pairs defining a function, no two different y -coordinates can share the same x -coordinate. A relation that is a function cannot have an element in the domain paired up with more than one unique element in the range. Choice 3 is therefore not a function because when x equals 1, y equals 3 and 7. It is not a function because there are multiple y -values for a given x -value.

ANSWER: (3)

- 20) What is the value of the y -coordinate of the solution to the system of equations $x - 2y = 1$ and $x + 4y = 7$?
 (1) 1 (2) -1 (3) 3 (4) 4

Line up the equations and subtract to eliminate x .

(Multiply 2nd equation terms by -1)

$$x - 2y = 1 \rightarrow x - 2y = 1$$

$$x + 4y = 7 \rightarrow -x - 4y = -7$$

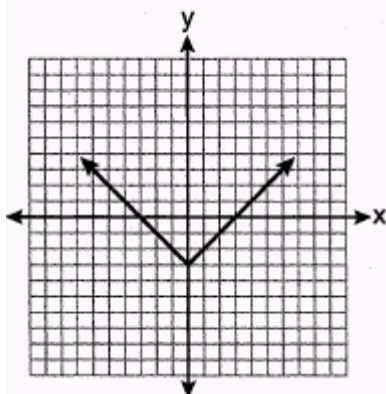
$$\underline{-6y = -6}$$

$$y = 1$$

Divide both sides by -6.

ANSWER: (1)

25) Which equation is represented by the graph below?



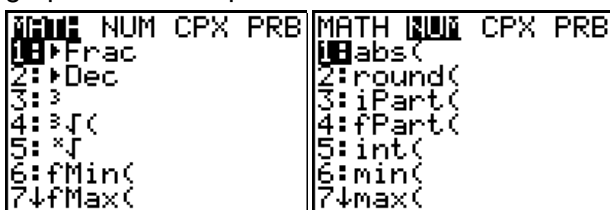
(1) $y = x^2 - 3$

(2) $y = (x - 3)^2$

(3) $y = |x| - 3$

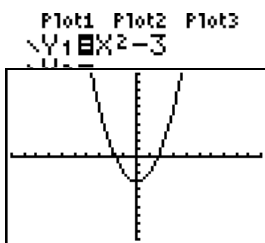
(4) $y = |x - 3|$

You should recognize the graph as being that of an absolute value equation. Choices 3 and 4 are absolute value equations. Choices 1 and 2 are quadratics and their graphs would be parabolas.

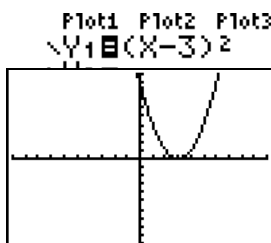


To enter the absolute value function using your calculator, first hit the **MATH** key followed by the right arrow key, to access the num menu. The first choice appearing in that menu is abs (. That stands for absolute value.

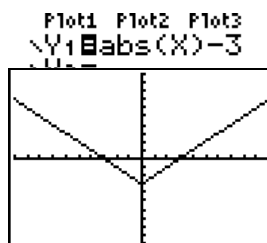
Below are the graphs for each choice:



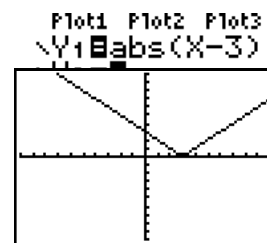
(1)



(2)



(3)



(4)

ANSWER: (3)

26) Carrie bought new carpet for her living room. She calculated the area of the living room to be 174.2 square feet. The actual area was 149.6 square feet. What is the relative error of the area to the nearest ten-thousandth?

- (1) 0.1412 (2) 0.1644 (3) 1.8588 (4) 2.1644

The relative error will be the difference between the two areas (the actual area and Carrie's area) divided by the actual area.

Carrie's area: 174.2

Actual area: 149.6

Difference = 174.2 - 149.6 = 24.6

Relative Error = Difference/Actual = 24.6 ÷ 149.6 = .1644385027

To nearest ten-thousandth = .1644

ANSWER: (2)

- 27) **What is an equation of the line that passes through the point (3,-1), and has a slope of 2?**

(1) $y = 2x + 5$

(3) $y = 2x - 4$

(2) $y = 2x - 1$

(4) $y = 2x - 7$

The easiest method to use for determining an equation of a line when given its slope and one point, is the point slope form of a line. It is $y - y_1 = m(x - x_1)$. m represents the slope, x_1 and y_1 represent the coordinates of the given point, x and y represent the coordinates of another point.

In our problem the slope is 2 and the given point is (3, -1). All you do now is substitute.
 x_1 is 3 y_1 is -1 m is 2

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = 2(x - 3) \quad \text{Simplify.}$$

$$y + 1 = 2(x - 3) \quad \text{Use the distributive property.}$$

$$y + 1 = 2x - 6 \quad \text{Subtract 1 from both sides.}$$

$$y = 2x - 7$$

Alternate solution using slope intercept form of a line: $y = mx + b$

m represents the slope b represents y-intercept point given (3, -1)

$$y = mx + b \quad \text{Substitute all givens} \quad m = 2 \quad b = ? \quad x = 3 \quad y = -1$$

$$-1 = 2(3) + b \quad \text{Simplify}$$

$$-1 = 6 + b \quad \text{Subtract 6 from both sides.}$$

$$-7 = b \quad \text{You now know that } m = 2 \text{ and } b = -7$$

$$y = mx + b \quad \text{Substitute}$$

$$y = 2x - 7 \quad \text{This is your equation.}$$

ANSWER: (4)

- 28) **The ages of three brothers are consecutive even integers. Three times the age of the youngest brother exceeds the oldest brother's age by 48 years. What is the age of the youngest brother?**

(1) 14 (2) 18 (3) 22 (4) 26

Consecutive even integers are two apart. Therefore let

$$x = \text{youngest}$$

$$x + 2 = \text{middle}$$

$$x + 4 = \text{oldest}$$

3 times the age of the youngest is $3x$.

This age of $3x$ exceeds the oldest brother's age by 48. (Exceeds means is more).

$3x$ is 48 more than $x + 4$. Mathematically this is written as:

$$3x = (x + 4) + 48 \quad \text{Simplify.}$$

$$3x = x + 52 \quad \text{Subtract } x \text{ from both sides.}$$

$$2x = 52 \quad \text{Divide both sides by 2.}$$

$$x = 26 \quad \text{The youngest brother is 26 years old.}$$

ANSWER: (4)

- 29) **Cassandra bought an antique dresser for \$500. If the value of her dresser increases 6% annually, what will be the value of Cassandra's dresser at the end of 3 years to the nearest dollar?**

(1) \$415 (3) \$596

(2) \$590 (4) \$770

6% as a decimal is .06. The value of her dresser after 1 year will be \$500 plus a 6% increase, or $500 + .06(500)$, which equals 530.

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PART II

- 31) **Chad complained to his friend that he had five equations to solve for homework. Are all of the homework problems equations? Justify your answer.**

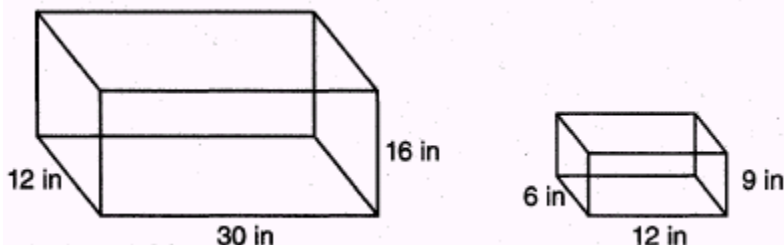
Math Homework	
1.	$3x^2 \cdot 2x^4$
2.	$5 - 2x = 3x$
3.	$3(2x + 7)$
4.	$7x^2 + 2x - 3x^2 - 9$
5.	$\frac{2}{3} = \frac{x+2}{6}$
Name <u>Chad</u>	

In mathematics, an equation is a mathematical sentence which states that two expressions are equal. It also contains the "=" symbol.

At the left, only items 2 and 5 are equations and contain the "=" symbol.

ANSWER: All his homework problems are not equations. Items 1,3, and 4 do not contain the "equal" symbol.

- 32) **The diagram below represents Joe's two fish tanks.**



Joe's larger tank is completely filled with water. He takes water from it to completely fill the small tank. Determine how many cubic inches of water will remain in the larger tank.

Step 1. Determine the volume of both tanks. Volume = Length X Width X Height.

Larger Tank Volume = $12 \times 30 \times 16 = 5760$ cubic inches.

Smaller Tank Volume = $6 \times 12 \times 9 = 648$ cubic inches.

Step 2. Pour 648 from the 5760 into the smaller tank. In other words, subtract 648 from the larger tank of 5760 and your answer will be what remains in the larger tank.

$$5760 - 648 = 5,112$$

ANSWER: 5,112 cubic inches of water will remain in the larger tank.

- 33) **Clayton has three fair coins. Find the probability that he gets two tails and one head when he flips the three coins.**

One way of doing this problem is to complete a sample space of all possible outcomes when flipping three coins.

H will stand for Heads, and T for Tails.

H T T	T H H
H T H	T H T
H H H	T T T
H H T	T T H

Above you see that there are 8 possible outcomes when flipping three coins. 3 of those outcomes (the ones that are in bold) show two tails and one head.

ANSWER: The probability of getting two tails and one head is $\frac{3}{8}$.

- 34) **Find algebraically the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $y = -2x^2 - 8x + 3$.**

The formula for finding the axis of symmetry is $x = \frac{-b}{2a}$

As you recall, a quadratic equation can be represented in the form of $ax^2 + bx + c = 0$. In our equation, **$y = -2x^2 - 8x + 3$** , $a = -2$ and $b = -8$. Let us substitute:

$$x = \frac{-b}{2a} = \frac{-(-8)}{2(-2)} = \frac{8}{-4} = -2 \quad \mathbf{x = -2 \text{ is the equation of the axis of symmetry.}}$$

Once you know the axis of symmetry you can now substitute its value in the equation to obtain the corresponding y-coordinate to determine the vertex, or turning point.

$$\mathbf{y = -2x^2 - 8x + 3} \quad \text{Substitute -2 for x.}$$

$$y = -2(-2)^2 - 8(-2) + 3 \quad \text{Simplify.}$$

$$y = -2(4) + 16 + 3 \quad \text{Simplify.}$$

$$y = -8 + 16 + 3$$

$$\mathbf{y = 11}$$

The vertex, or turning point will be (-2,11)

ANSWER:

Equation for the axis of symmetry is $x = -2$.

The vertex is (-2,11).

#35 begins on the next page...

- 35) ***At the end of week one, a stock had increased in value from \$5.75 a share to \$7.50 a share. Find the percent of increase at the end of week one to the nearest tenth of a percent.***
At the end of week two, the same stock had decreased in value from \$7.50 to 5.75. Is the percent of decrease at the end of week two the same as the percent of increase at the end of week one?
Justify your answer.

To find the percent of increase, first find the difference (subtract) in the price of the shares, and then divide by the starting price.

Starting price = \$5.75

New price = \$7.50

Difference = $7.50 - 5.75 = 1.75$

% increase = Difference / Starting price = $1.75 \div 5.75 = .304378261$

.304378261 (move decimal 2 to right) **30.4% increase to nearest tenth.**

To find the percent of decrease, use the same method.

First find the difference in the price of the shares and divide by the starting price.

Starting price = \$7.50

New price \$5.75

Difference = $7.50 - 5.75 = 1.75$

% decrease = Difference / Starting price = $1.75 \div 7.50 = .2333\dots$

.2333... (move decimal 2 to right) **23.3% decrease to nearest tenth.**

ANSWER:

The percent increase after the end of week one is 30.4% to the nearest tenth.

The percent increase and decrease are not equal. This is because although the difference in share price is the same, the starting price differs.

- 36) ***The chart below compares two runners.***

Runner	Distance, in miles	Time, in hours
Greg	11	2
Dave	16	3

Based on the information in this chart, state which runner has the faster rate. Justify your answer.

You are asked to determine which runner has the faster rate.

Distance = Rate x Time Rate = Distance / time

Greg's rate = $11 \div 2 = 5.5$ miles/hour.

Dave's rate = $16 \div 3 = 5.3$ miles/hour.

Greg has the faster rate. 5.5 is greater than 5.3.

37) **Express in simplest form:** $\frac{2x^2 - 8x - 42}{6x^2} + \frac{x^2 - 9}{x^2 - 3x}$

Dividing by a fraction is the same as multiplying by the reciprocal of that fraction.

$$\begin{aligned} \frac{2x^2 - 8x - 42}{6x^2} + \frac{x^2 - 9}{x^2 - 3x} &= \frac{2x^2 - 8x - 42}{6x^2} \cdot \frac{x^2 - 3x}{x^2 - 9} = \frac{2(x^2 - 4x - 21)}{6x^2} \cdot \frac{x^2 - 3x}{x^2 - 9} = \\ \frac{2(x-7)(x+3)}{6x^2} \cdot \frac{x(x-3)}{(x+3)(x-3)} &= \frac{\cancel{2}(x-7)\cancel{(x+3)}}{\cancel{6}x^2} \cdot \frac{1\cancel{(x-3)}}{\cancel{(x+3)}\cancel{(x-3)}} = \\ &= \frac{(x-7)}{3x} \cdot 1 = \frac{x-7}{3x} \end{aligned}$$

ANSWER: $\frac{x-7}{3x}$

38) **On the grid below, solve the system of equations graphically for x and y.**

$$4x - 2y = 10$$

$$y = -2x - 1$$

When presented with equations to solve graphically, you have to graph them. The two above equations are linear equations and will be represented by straight lines. You can simply select 3 values for either x or y in each equation, and solve for each corresponding coordinate.

Or you can use the slope intercept form of a line to help you do the graphing. First change the first equation into the form of $y = mx + b$. (The second equation is already presented in that form).

"m" represents the slope and "b" represents the y-intercept.

$$4x - 2y = 10 \quad \text{Subtract } 4x \text{ from both sides.}$$

$$-2y = -4x + 10 \quad \text{Divide both sides by } -2.$$

$$y = 2x - 5$$

The two equations you are asked to graph are therefore:

$$y = 2x - 5 \quad \text{and} \quad y = -2x - 1.$$

At this point you know the slopes of both graphs, as well as their y-intercepts. That is enough information to allow you to graph the system of equations and find their point of intersection. The graph of the first equation has a y-intercept of -5 and a slope of 2. The graph of the second equation has a y-intercept of -1 and a slope of -2.

Plot1	Plot2	Plot3
\Y1=2X-5		
\Y2=-2X-1		
\Y3=		
\Y4=		
\Y5=		
\Y6=		
\Y7=		

You can also use your graphing calculator to set up a table of values for each equation. Enter the first equation into your calculator using the **Y=** editor key. Hit **ENTER** and then enter your second equation. For the second equation remember to use the minus in parentheses (-) which is used to indicate the negative 2x.

Next hit the **2nd** key followed by the **GRAPH** key. This will access the **TABLE** function.

X	Y1	Y2
-3	-11	5
-2	-9	3
-1	-7	1
0	-5	-1
1	-3	-3
2	-1	-5
3	1	-7
X=-3		

The screen on your calculator may not look like the screen capture at the left. It makes no difference. You can actually scroll up or down to any value you wish. What you see on the first line of my screen capture is that in equation 1, when $x = -3$, $y = -11$. In equation 2 when $x = -3$, $y = 5$.

X	Y1	Y2
0	-5	-1
1	-3	-3
2	-1	-5

X = -3

Based on the screen capture at the left you even see the solution set as being the point (1,-3), because in both equations when x=1 y=-3.

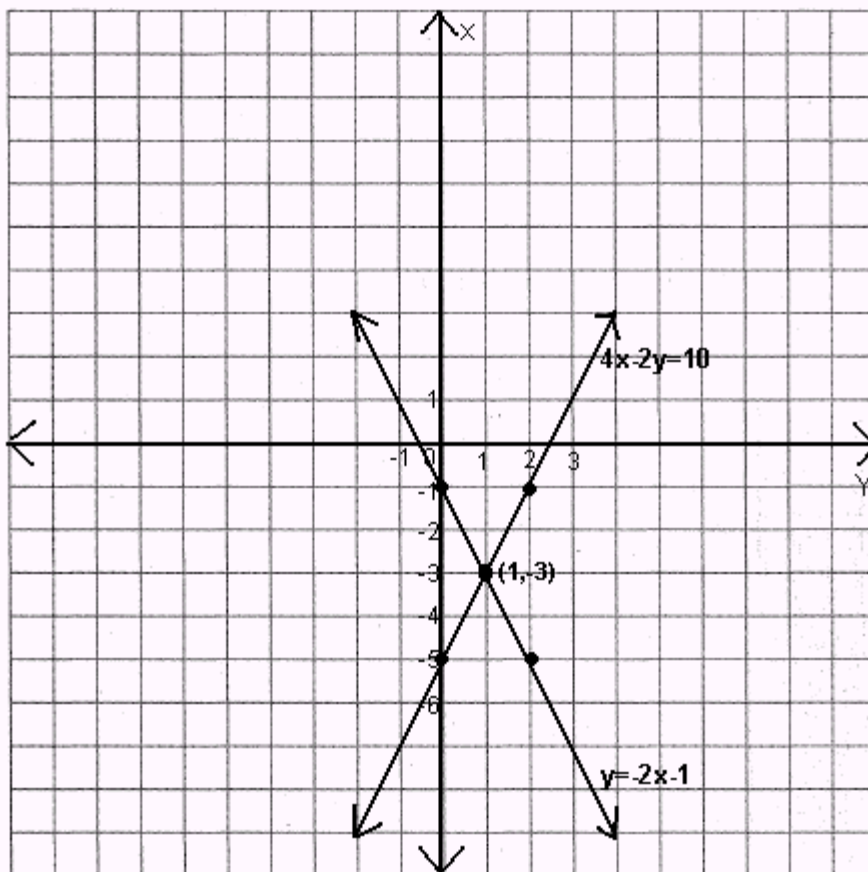
Here are the coordinates I will use for this problem.

$y = 2x - 5$

x	0	1	2
y	-5	-3	-1

$y = -2x - 1$

x	0	1	2
y	-1	-3	-5



At the left is the graph of both equations and their solution.

The solution is (1,-3). It is the point where both lines intersect. The point of intersection is labeled on the graph.

Although I entered $y = 2x - 5$ to generate the table for the second equation, I entered the original equation on the graph at the left. $4x - 2y = 10$, and $y = 2x - 5$ are equivalent equations.

